SCOTTISH
QUALIFICATIONS
AUTHORITY

APPLIED MATHEMATICS
Advanced Higher

Second edition – published May 2007

National Course Specification

APPLIED MATHEMATICS (ADVANCED HIGHER)

COURSE CODES

C202 13 Applied Mathematics: Statistics
C204 13 Applied Mathematics: Mechanics

COURSE STRUCTURE

C202 13 Applied Mathematics: Statistics

This course consists of three mandatory units as follows:

D326 13	***Statistics 1 (AH)***	***1 credit (40 hours)***
D330 13	***Statistics 2 (AH)***	***1 credit (40 hours)***
DE8Y 13	***Mathematics for Applied Mathematics (AH)***	***1 credit (40 hours)***

Administrative Information

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Additional copies of this course specification (including unit specifications) can be purchased from the Scottish Qualifications Authority for £7.50. **Note:** Unit specifications can be purchased individually for £2.50 (minimum order £5).

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

C204 13 Applied Mathematics: Mechanics

This course consists of three mandatory units as follows:

D327 13	***Mechanics 1 (AH)***	***1 credit (40 hours)***
D331 13	***Mechanics 2 (AH)***	***1 credit (40 hours)***
DE8Y 13	***Mathematics for Applied Mathematics (AH)***	***1 credit (40 hours)***

All courses include a further 40 hours for induction, extending the range of learning and teaching approaches, additional support, consolidation, integration of learning and preparation for external assessment. This time is an important element of the course and advice on the use of the overall 160 hours is included in the course details.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained one of the following:

- Higher Mathematics course award or its component units
- equivalent

Statistics 1 (AH) assumes knowledge of Outcome 2 of Statistics (H), while experience of Outcomes 1 and 4 of Statistics (H) would be advantageous.

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CORE SKILLS

The course Applied Mathematics: Statistics (Advanced Higher)
gives automatic certification of the following:

Complete core skills for the course	Numeracy	H
Additional core skills components for the course	Critical Thinking	H

This course Applied Mathematics: Mechanics (Advanced Higher)
gives automatic certification of the following:

Complete core skills for the course	Numeracy	H
Additional core skills components for the course	Critical Thinking	H

For information about the automatic certification of core skills for any individual unit in this course, please refer to the general information section at the beginning of the unit.

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

RATIONALE

As with all mathematics courses, Advanced Higher Applied Mathematics courses aim to build upon and extend candidates' mathematical skills, knowledge and understanding in a way that recognises problem solving as an essential skill. Through the study of units on Statistics, and Mechanics, the focus within the course is placed firmly on applications of mathematics to real-life contexts and the formulation and interpretation of mathematical models. Each applied mathematics topic brings its own unique brand of application and extended activity to the course and provides the opportunity to demonstrate the advantages to be gained from the power of calculators and computer software in real-life situations.

Because of the importance of these features, the grade descriptions for Advanced Higher Applied Mathematics courses emphasise the need for candidates to undertake extended thinking and decision making so as to be able to model, solve problems and apply mathematical knowledge. The use of coursework tasks, therefore, to practise problem solving as set out in the grade descriptions and the involvement in recommended practical activities, are both strongly encouraged.

The choice of courses available at this level is wider than at lower levels. This breadth of choice is provided in response to the variety of candidates' aspirations in higher education studies or areas of employment. The courses satisfy the needs of a wide range of interests, regardless of whether or not a specialism in mathematics is the primary intention. The courses offer depth of applied mathematical experience and, thereby, achieve relevance to further study or employment in the areas of mathematical and physical sciences, computer science, engineering, biological and social sciences, medicine, accounting, business and management. For these purposes, the courses can be taken as an alternative to the Advanced Higher Mathematics course. However, when an Advanced Higher Applied Mathematics course is taken in addition to the Advanced Higher Mathematics course, an opportunity is offered for the candidate to acquire exceptional breadth and depth of mathematical experience.

The courses offer candidates, in an interesting and enjoyable manner, an enhanced awareness of the range and power of mathematics and the importance of mathematical applications to society in general.

COURSE CONTENT

The syllabi are designed to allow candidates the opportunity to study one area of applied mathematics in depth. The opportunity to specialise, introduced at Higher level through the optional statistics unit, is now extended to three areas of applied mathematics. Two units in each of Statistics and Mechanics provide the opportunity to study one of these areas in depth. Whatever applied course is chosen, the two units have been designed in a progressive way, with the first unit of each course providing a rounded experience of the topic for those candidates who choose it as a free-standing unit and do not wish to proceed to study the topic further in the second unit.

National Course Specification: course details

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

Additionally, the courses make demands over and above the requirements of individual units. Some of the 40 hours of flexibility time should be used to ensure that candidates satisfy the grade descriptions for mathematics courses that involve solving problems/carrying out assignments and undertaking practical activities, thereby encouraging more extended thinking and decision making. Candidates should be exposed to coursework tasks which require them to interpret problems, select appropriate strategies, come to conclusions and communicate the conclusions intelligibly.

In assessments candidates should be required to show their working in carrying out algorithms and processes.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)

Detailed content

The content listed below should be covered in teaching the course. All of this content will be subject to sampling in the external assessment. Where comment is offered, this is intended to help in the effective teaching of the course.

References in this style indicate the depth of treatment appropriate to grades A and B.

CONTENT	COMMENT
Statistics 1 (AH) **Conditional probability and expectation**	
appreciate the existence of dependent events calculate conditional probability solve problems using conditional probability	Conditional probability can be calculated from first principles in every case but the following formulae may prove useful in solving problems involving dependent events. $P(E\|F) = \dfrac{P(E \text{ and } F)}{P(F)}, \ P(F) > 0$ $P(E \text{ and } F) = P(E\|F)P(F) = P(F\|E)P(E)$ $P(F) = \sum_i P(F \mid E_i)P(E_i)$ $P(E_j \mid F) = \dfrac{P(F \mid E_j)P(E_j)}{\sum_i P(F \mid E_i)P(E_i)}$ for mutually exclusive and exhaustive events E_i [A/B]
solve a problem involving Bayes' theorem [A/B]	Bayes' theorem is used to 'reverse the dependence', so that if the probability of F given that E_j has occurred is known then the probability of E_j given that F has occurred can be found.

CONTENT	COMMENT
apply the laws of expectation and variance: use the results $E(aX + b) = aE(X) + b$ and $E(X \pm Y) = E(X) \pm E(Y)$	When considering the laws of expectation and variance an intuitive approach may be taken, perhaps as follows: It should be clear from graphical considerations that if a random variable has a scale factor applied followed by a translation then so must the corresponding expectation (or mean) giving $E(aX + b) = aE(X) + b$.
use the results $V(aX + b) = a^2V(X)$ and $V(X \pm Y) = V(X) + V(Y)$ if X and Y are independent	It may be equally intuitive that variability (dispersion) is not affected by translation and a little numerical investigation should demonstrate that $V(aX + b) = a^2V(X)$. It follows that $V(-X) = V(X)$
Probability distributions calculate probabilities using the binomial, Poisson and normal distributions	The difference between discrete and continuous distributions should be discussed. The former may be exemplified by the uniform, binomial and Poisson distributions and the latter by the uniform (a, b) and normal distributions. Candidates should be able to calculate probabilities with or without tables. The process of standardising data to produce standard scores is an important one and should be discussed.
use standard results for the mean and standard deviation of binomial, Poisson and normal distributions	Candidates should be introduced to the probability functions and the formulae for the mean and standard deviation for each of the discrete distributions.

CONTENT	COMMENT
apply an appropriate approximation (normal or Poisson) to a binomial distribution and demonstrate the correct use of a continuity correction if appropriate [A/B]	It is often desirable to approximate binomial probabilities. The reasons for using either the Poisson or normal should be exemplified, particularly the use of a continuity correction in the latter case. One rule of thumb in current use is to use the normal approximation if np and nq are >5 and the Poisson otherwise, as long as p is small.
solve problems involving the sum or difference of two independent normal variates [A/B]	
Sampling methods distinguish between different sampling methods: appreciate the difference between sample and census, and describe random, stratified, cluster, systematic and quota sampling	The difference between a sample and a census should be stressed. The advantages and disadvantages of each type of sampling should be discussed, noting quota is not an example of random sampling.
use the central limit theorem and distribution of the sample mean and sample proportion	The central limit theorem states that for sufficiently large n the distribution of the sample mean is normal, irrespective of the distribution of the sample variates. Candidates should be able to show that sample mean has expectation μ and standard error (standard deviation) $\frac{\sigma}{\sqrt{n}}$ and that the sample proportion has mean p and standard error (deviation) $\sqrt{\frac{pq}{n}}$ for large n and $q = 1 - p$.
use the sample mean and standard deviation to estimate the population mean and standard deviation and the sample proportion to estimate the population proportion	In elementary sampling theory the population mean/proportion is estimated by the sample mean/proportion and the population variance by the sample variance given by $s^2 = \frac{1}{n-1}\sum(x_i - \overline{x})^2$.

CONTENT	COMMENT
obtain a confidence interval for the population mean [A/B]	An approximate 95% confidence interval for the population mean is given by $\bar{x} \pm 1.96 \frac{s}{\sqrt{n}}$ for large n. We may interpret this as saying that if we take a large number of samples and compute a confidence interval for each then 95% of these intervals would be expected to contain the population mean. It is also common practice to use a 99% confidence interval where 1.96 is replaced by 2.58.
obtain a confidence interval for the population proportion [A/B]	An approximate 95% confidence interval for the population proportion p is given by $\hat{p} \pm 1.96 \sqrt{\frac{\hat{p}\hat{q}}{n}}$ for large n, where $\hat{q} = 1 - \hat{p}$ and $\hat{p}$ is the sample proportion.
Hypothesis Testing understand the terms null hypothesis, alternative hypothesis, one-tail test, two-tail test, distribution under H_0, test statistic, critical region, level of significance, p-value, reject H_0 and accept H_0	In a statistical investigation we often put forward a hypothesis – the null hypothesis H_0 – about a population parameter. In order to test this hypothesis we take a sample from the population and perform a statistical test. If we decide to reject H_0, we do so in favour of an alternative hypothesis H_1.
use a z-test on a statistical hypothesis: decide on a significance level state one-tail or two-tail test determine the p-value draw appropriate conclusions	A z-test should be used to test whether the population mean is equal to some specific value, where the population variable is normally distributed with known variance. eg H_o: $\mu=\mu_o$ versus H_1: $\mu\neq\mu_o$ for a two-tail test.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)

CONTENT	COMMENT
Statistics 2 (AH)	
Simple control charts construct a control chart for the sample mean or proportion	Two basic charts should be used as an introduction to the subject of control.
use and interpret such a chart	A 3-sigma chart has action limits when a sample mean or proportion goes beyond three standard deviations from the expected value. The process should be investigated for special causes of variation. 2-sigma warning limits should also be discussed. A 3-sigma Shewhart chart has control limits drawn 3 standard deviations either side of the expected value. The control limits for the sample mean are $\mu \pm 3\frac{\sigma}{\sqrt{n}}$ Similarly, a p-chart for the proportion has control limits $p \pm 3\sqrt{\frac{pq}{n}}$ where $q = 1\text{-}p$
Further hypothesis testing carry out a chi-squared goodness-of-fit test carry out a chi-squared test for association in a contingency table	The chi-squared statistic can be used to test goodness-of-fit ie to establish whether a set of observed frequencies differ significantly from those of a specified discrete probability distribution. It can also be used to test for association between two factors in a contingency table. Tables of the chi-squared distribution are required for this test. Candidates require to be introduced to the notion of degrees of freedom and to the restriction of 1 being the minimum allowable expected frequency with no more than 20% of expected frequencies being less than 5 as a general guideline for valid application without loss of degrees of freedom. Candidates should be aware that conclusions should be made with caution when any frequencies are low.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)

CONTENT	COMMENT
carry out a sign test	The sign test can be used as a non-parametric test to make inferences about the median of a single population. It can also be used with paired data to test that the population median difference is zero. The test relies on binomial probabilities since a sign can be either positive or negative.
carry out a Mann-Whitney test compare two different methods of applying the Mann-Whitney test [A/B]	The Mann-Whitney test is a non-parametric test used to compare the medians of two populations using independent samples. Candidates should be able to find the p-value from first principles, from tables or by using the normal approximation (formulae given) with continuity correction. This test assumes that the two distributions have the same shape and hence the same variance.
t-distribution determine a confidence interval for the population mean, given a random sample from a normal population with unknown variance use the appropriate number of degrees of freedom	The t-distribution was discovered by William S. Gosset (*c*.1900) during his time as a scientist in the Guinness breweries in Dublin. He published his findings under the pseudonym of 'student' which explains why the distribution is often called the student's t-distribution.
carry out a one sample t-test for the population mean	The t-distribution is used for small samples from a normal population with unknown variance which can be estimated by the sample variance. It can also be used with paired data to test that the population mean difference is zero. Without the underlying assumption of normality there is very little statistical analysis that can be done with a small sample.

CONTENT	COMMENT
Analyse the relationship between two variables test the significance of a product moment correlation coefficient	Assuming that a plot of y against x shows that a linear model is appropriate, we calculate the sample product moment correlation coefficient (r) and test the null hypothesis that the population product moment correlation coefficient $\rho = 0$ using the test statistic $t = \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}}$
consider the linear model $Y_i = \alpha + \beta x_i + \in_i$ use a residual plot to check the model assumptions $E(\in_i) = 0$ and $V(\in_i) = \sigma^2$ by calculating the residuals $(y_i - (a + bx_i))$ and plotting them against fitted values $(a + bx_i)$	The model $Y_i = \alpha + \beta x_i + \in_i$ assumes that $\in_i$ are independent and that the $E(\in_i) = 0$ and $V(\in_i) = \sigma^2$ which is constant for all x_i. Ideally the plot of residuals against fitted values should show a random scatter centred on zero. If this is not the case then the model may be inappropriate (perhaps non-linear) or the data may require to be transformed to restore constant variance. A further assumption is that $\in_i \sim N(0, \sigma^2)$ permitting the construction of both i) a prediction interval for an individual response and ii) a confidence interval for a mean response. It can be shown that the sum of squared residuals, $SSR = S_{yy} - \dfrac{(S_{xy})^2}{S_{xx}}$ and that an estimate of σ^2 is $s^2 = \dfrac{SSR}{n-2}$.
use a t-test to test the fit of this model	We may also use a t-test to test whether the slope parameter β in the model is zero, using the test statistic $t = \dfrac{b}{s/\sqrt{S_{xx}}}$

CONTENT	COMMENT
construct a prediction interval for an individual response or confidence interval for mean response, as appropriate	In a prediction or confidence interval the reliability of the estimate depends on the sample size, the variability in the sample and the value of x. A $100(1-\alpha)\%$ prediction interval for $Y_i \vert x_i$ is given by $\hat{Y}_i \pm t_{n-2,\, 1-\alpha/2}\, s\sqrt{1+\frac{1}{n}+\frac{(x_i-\bar{x})^2}{S_{xx}}}$ A $100(1-\alpha)\%$ confidence interval for $E(Y_i \vert x_i)$ is given by $\hat{Y}_i \pm t_{n-2,\, 1-\alpha/2}\, s\sqrt{\frac{1}{n}+\frac{(x_i-\bar{x})^2}{S_{xx}}}$

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
Mechanics 1 (AH)	
Motion in a straight line know the meaning of position, displacement, velocity, acceleration, uniform speed, uniform acceleration, scalar quantity, vector quantity	Concepts of position, velocity and acceleration should be introduced using vectors. Candidates should be very aware of the distinction between scalar and vector quantities, particularly in the case of speed and velocity.
draw, interpret and use distance/time, velocity/time and acceleration/time graphs	Candidates should be able to draw these graphs from numerical or graphical data.
know that the area under a velocity/time graph represents the distance travelled	
know the rates of change $v = \frac{dx}{dt} = \dot{x}$ and $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{d\dot{x}}{dt} = \dot{v} = \ddot{x}$	Candidates should be familiar with the dot notation for differentiation with respect to time.
derive, by calculus methods, and use the equations governing motion in a straight line with constant acceleration, namely: $v = u + at,\ s = ut + \frac{1}{2}at^2$ and from these, $v^2 = u^2 + 2as,\ \ s = \frac{(u+v)t}{2}$	Candidates need to appreciate that these equations are for motion with <u>constant</u> acceleration only. The general technique is to use calculus.
solve analytically problems involving motion in one dimension under constant acceleration, including vertical motion under constant gravity	

CONTENT	COMMENT
solve problems involving motion in one dimension where the acceleration is dependent on time, ie $a = \frac{dv}{dt} = f(t)$	
Position, velocity and acceleration vectors including relative motion know the meaning of the terms relative position, relative velocity and relative acceleration, air speed, ground speed and nearest approach	
be familiar with the notation: $\underline{r}_P$ for the position vector of P $\underline{v}_P = \dot{\underline{r}}_P$ for the velocity vector of P $\underline{a}_P = \dot{\underline{v}}_P = \ddot{\underline{r}}_P$ for the acceleration vector of P $\overrightarrow{PQ} = \underline{r}_Q - \underline{r}_P$ for the position vector of Q relative to P $\underline{v}_Q - \underline{v}_P = \dot{\underline{r}}_Q - \dot{\underline{r}}_P$ for the velocity of Q relative to P $\underline{a}_Q - \underline{a}_P = \dot{\underline{v}}_Q - \dot{\underline{v}}_P = \ddot{\underline{r}}_Q - \ddot{\underline{r}}_P$ for the acceleration of Q relative to P	
resolve vectors into components in two and three dimensions	This requires emphasis.
differentiate and integrate vector functions of time	
use position, velocity and acceleration vectors and their components in two and three dimensions; these vectors may be functions of time	
apply position, velocity and acceleration vectors to solve practical problems, including problems on the navigation of ships and aircraft and on the effect of winds and currents	Candidates should be able to solve such problems both by using trigonometric calculations in triangles and by vector components. Solutions by scale drawing would not be accepted
solve problems involving collision courses and nearest approach	

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
Motion of projectiles in a vertical plane know the meaning of the terms projectile, velocity and angle of projection, trajectory, time of flight, range and constant gravity	Candidates also require to know how to resolve velocity into its horizontal and vertical components.
solve the vector equation $\ddot{\underline{r}} = -g\,\underline{j}$ to obtain $\underline{r}$ in terms of its horizontal and vertical components	The vector approach is particularly recommended.
obtain and solve the equations of motion $\ddot{x} = 0, \ddot{y} = -g$, obtaining expressions for $\dot{x}$, $\dot{y}$, x and y in any particular case	
find the time of flight, greatest height reached and range of a projectile	Only range on the horizontal plane through the point of projection is required.
find the maximum range of a projectile on a horizontal plane and the angle of projection to achieve this	
find, and use, the equation of the trajectory of a projectile	Candidates should appreciate that this trajectory is a parabola.
solve problems in two-dimensional motion involving projectiles under a constant gravitational force and neglecting air resistance	Applications from ballistics and sport may be included and vector approaches should be used where appropriate.
Force and Newton's laws of motion understand the terms mass, force, weight, momentum, balanced and unbalanced forces, resultant force, equilibrium, resistive forces	
know Newton's first and third laws of motion	
resolve forces in two dimensions to find their components	Resolution of velocities, etc., has been covered in previous sections.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
combine forces to find resultant force	
understand the concept of static and dynamic friction and limiting friction	
understand the terms frictional force, normal reaction, coefficient of friction μ, angle of friction λ, and know the equations $F = \mu R$ and $\mu = \tan \lambda$	Balanced, unbalanced forces and equilibrium could arise here. Candidates should understand that for stationary bodies, $F \leq \mu R$.
solve problems involving a particle or body in equilibrium under the action of certain forces	Forces could include weight, normal reaction, friction, tension in an elastic string, etc.
know Newton's second law of motion, that force is the rate of change of momentum, and derive the equation $\underline{F} = m\underline{a}$	
use this equation to form equations of motion to model practical problems on motion in a straight line	
solve such equations modelling motion in one dimension, including cases where the acceleration is dependent on time	
solve problems involving friction and problems on inclined planes	Both rough and smooth planes are required.

CONTENT	COMMENT
Mechanics 2 (AH) **Motion in a horizontal circle with uniform angular velocity** know the meaning of the terms angular velocity and angular acceleration	
Know that for motion in a circle of radius *r*, the radial and tangential components of velocity are $\underline{0}$ and $r\dot{\theta}\,\underline{e}_\theta$ respectively, and of acceleration are $-r\dot{\theta}^2\underline{e}_r$ and $r\ddot{\theta}\,\underline{e}_\theta$ respectively, where $\underline{e}_r = \cos\theta\underline{i} + \sin\theta\underline{j}$ and $\underline{e}_\theta = -\sin\theta\underline{i} + \cos\theta\underline{j}$ are the unit vectors in the radial and tangential directions, respectively	Vectors should be used to establish these, starting from $\underline{r} = r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j}$, where *r* is constant and θ is varying.
know the particular case where $\dot{\theta} = \omega t$, ω being constant, when the equations are $\underline{r} = r\cos(\omega t)\underline{i} + r\sin(\omega t)\underline{j}$ $\underline{v} = -r\omega\sin(\omega t)\underline{i} + r\omega\cos(\omega t)\underline{j}$ $\underline{a} = -r\omega^2\cos(\omega t)\underline{i} - r\omega^2\sin(\omega t)\underline{j}$ from which $v = r\omega = r\dot{\theta}$ $a = r\omega^2 = r\dot{\theta}^2 = \frac{v^2}{r}$ and $\underline{a} = -\omega^2\underline{r}$	
apply these equations to motion in a horizontal circle with uniform angular velocity including skidding and banking and other applications	Examples should include motion of cars round circular bends, with skidding and banking, the 'wall of death', the conical pendulum, etc.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
know Newton's inverse square law of gravitation, namely that the magnitude of the gravitational force of attraction between two particles is inversely proportional to the square of the distance between the two particles	
apply this to simplified examples of motion of satellites and moons	Circular orbits only.
find the time for one orbit, height above surface, etc	
Simple harmonic motion know the definition of simple harmonic motion (SHM) and the meaning of the terms oscillation, centre of oscillation, period, amplitude, frequency	
know that SHM can be modelled by the equation $\ddot{x} = -\omega^2 x$	
know the solutions $x = a\sin(\omega t + \alpha)$ and the special cases $x = a\sin(\omega t)$ and $x = a\cos(\omega t)$, of the SHM equation	At this stage these solutions can be verified or established from $\underline{r} = a\cos(\omega\ t)\underline{i} + a\sin(\omega\ t)\underline{j}$ rotating round a circle. Solution of second order differential equations is not required.
know and be able to verify that $v^2 = \omega^2(a^2 - x^2)$, where $v = \dot{x}$ $T = \dfrac{2\pi}{\omega}$ maximum speed is ωa, the magnitude of the maximum acceleration is $\omega^2 a$ and when and where these arise	Proof using differential equations is not required here but will arise in the section of work on motion in a straight line later in this unit.
know the meaning of the term tension in the context of elastic strings and springs	

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
know Hooke's law, the meaning of the terms natural length, l, modulus of elasticity, λ, and stiffness constant, k, and the connection between them, $\lambda = kl$	
know the equation of motion of an oscillating mass and the meaning of the term position of equilibrium	
apply the above to the solution of problems involving SHM	These will include problems involving elastic strings and springs, and small amplitude oscillations of a simple pendulum but not the compound pendulum.
Principles of momentum and impulse know that force is the rate of change of momentum	This was introduced in Mechanics 1 (AH).
know that impulse is change in momentum ie $\underline{I} = m\underline{v} - m\underline{u} = \int \underline{F} dt$	
understand the concept of conservation of linear momentum	
solve problems on linear motion such as motion in lifts, recoil of a gun, pile-drivers, etc.	The equation $\underline{F} = m\underline{a}$ is again involved here. Equations of motion with constant acceleration could recur.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
Principles of work, power and energy know the meaning of the terms work, power, potential energy, kinetic energy	
understand the concept of work	Candidates should appreciate that work can be done by or against a force.
calculate the work done by a constant force in one and two dimensions, ie, $W = Fd$ (one dimension); $W = \underline{F}.\underline{d}$ (two dimensions)	
calculate the work done in rectilinear motion by a variable force using integration, ie $W = \int \underline{F}.\underline{i}dx$; $W = \int \underline{F}.\underline{v}dt$, where $\underline{v} = \frac{dx}{dt}\underline{i}$	
understand the concept of power as the rate of doing work, ie, $P = \frac{dW}{dt} = \underline{F}.\underline{v}$ (constant force), and apply this in practical examples	Examples can be taken from transport, sport, fairgrounds, etc.
understand the concept of energy and the difference between kinetic (E_K) and potential (E_P) energy	
know that $E_K = \frac{1}{2}mv^2$	
know that the potential energy associated with: a. a uniform gravitational field is $E_P = mgh$ b. Hooke's law is $E_P = \frac{1}{2}k$ (extension)^2 c. Newton's inverse square law is $E_P = \frac{GMm}{r}$	 Link with simple harmonic motion. Link with motion in a horizontal circle.

CONTENT	COMMENT
understand and apply the work–energy principle understand the meaning of conservative forces like gravity, and non-conservative forces like friction know and apply the energy equation $E_K + E_P$ = constant, including to the situation of motion in a vertical circle **Motion in a straight line, where the solution of first order differential equations is required** know that $a = v\frac{dv}{dx}$ as well as $\frac{dv}{dt}$ use Newton's law of motion, $\underline{F} = m\underline{a}$, to form first order differential equations to model practical problems, where the acceleration is dependent on displacement or velocity, ie $\frac{dv}{dt} = f(v)$, $v\frac{dv}{dx} = f(x)$, $v\frac{dv}{dx} = f(v)$	
solve such differential equations by the method of separation of variables	It may be necessary to teach this solution technique, depending on the mathematical background of the candidates. Examples will be straightforward with integrals which are covered in Mathematics 1, 2 (AH). If more complex, then the anti-derivative will be given.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT
derive the equation $v^2 = \omega^2(a^2 - x^2)$ by solving $v\frac{dv}{dx} = -\omega^2 x$	
know the meaning of the terms terminal velocity, escape velocity and resistance per unit mass and solve problems involving differential equations and incorporating any of these terms or making use of $F = \frac{p}{v}$	This section can involve knowledge and skills from other topics within this unit.

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
Mathematics for Applied Mathematics (AH) **Algebra**		
know and use the notation $n!$, ${}^nC_r$ and $\binom{n}{r}$	eg Calculate $\binom{8}{5}$ eg Solve, for $n \in \mathbf{N}$, $\binom{n}{2} = 15$	Candidates should be aware of the size of $n!$ for small values of n, and that calculator results consequently are often inaccurate (especially if the formula ${}^nC_r = \dfrac{n!}{r!(n-r)!}$ is used).
know and use the Σ notation		
know Pascal's triangle	Pascal's triangle should be extended up to $n = 7$.	For assessment purposes $n \le 5$.
know and use the binomial theorem $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$, for $r, n \in \mathbf{N}$	eg Expand $(x + 3)^4$ eg Expand $(2u - 3v)^5$ [A/B]	

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
know the formulae $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$, apply the above results to prove by direct methods results concerning other sums	eg $\sum_{r=1}^{n}(ar+b) = a\sum_{r=1}^{n} r + \sum_{r=1}^{n} b$	
express a proper rational function as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	eg Express $\frac{5-10x}{1-3x-4x^2}$ in partial fractions.	The denominator may include a repeated linear factor or an irreducible quadratic factor. This is also required for integration of rational functions and useful for graph sketching when asymptotes are present.
include cases where an improper rational function is reduced to a polynomial and a proper rational function by division or otherwise [A/B]	eg Express $\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}$ in partial fractions [A/B].	When the degree of the numerator of the rational function exceeds that of the denominator by 1, non-vertical asymptotes occur.

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Matrix algebra know the meaning of the terms matrix, element, row, column, order, identity matrix, inverse, determinant, singular, non-singular		
perform matrix operations: addition, subtraction, multiplication by a scalar, multiplication, establish equality of matrices		Link with Systems of linear equations in Mathematics 1 (AH).
calculate the determinant of 2×2 and 3×3 matrices		
know the relationship of the determinant to invertability		
find inverses, where they exist, of 2×2 matrices and 3×3 matrices		The result $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ should be known.
find the inverse, where it exists, of a 3×3 matrix by elementary row operations		
use inverse matrices in solving linear systems		

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
Differentiation know the meaning of the terms limit, derivative, differentiable at a point, differentiable on an interval, derived function, second derivative		
use the notation: $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$		
recall the derivatives of x^{α} (α rational), sin x and cos x		
know and use the rules for differentiating linear sums, products, quotients and composition of functions: $(f(x) + g(x))' = f'(x) + g'(x)$ $(kf(x))' = kf'(x)$, where k is a constant the chain rule: $(f(g(x)))' = f'(g(x)).g'(x)$ the product rule: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ the quotient rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$		Candidates should be exposed to formal proofs of differentiation, although proofs will not be required for assessment purposes. Once the rules for differentiation have been learned, computer algebra systems (CAS) may be used for consolidation/extension. However, when CAS are being used for difficult/real examples the emphasis should be on the understanding of concepts rather than routine computation. When software is used for differentiation in difficult cases, candidates should be able to say which rules were used.
differentiate given functions which require more than one application of one or more of the chain rule, product rule and the quotient rule [A/B]		

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
know the derivative of tan x the definitions and derivatives of sec x, cosec x and cot x the derivatives of e^x (exp x) and ln x		Link with the graphs of these functions. The definitions of e^x and ln x should be revised and examples given of their occurrence.
apply differentiation to simple rates of change eg	eg Find the acceleration of a particle whose displacement s metres from a certain point at time t seconds is given by $s = 8 - 75t + t^3$.	
rectilinear motion, optimisation problems		Optimisation problems should be linked with graph sketching.
understand how a function can be defined parametrically		

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
understand simple applications of parametrically defined functions	eg $x^2 + y^2 = r^2$, $x = r\cos\theta$, $y = r\sin\theta$	
use parametric differentiation to find first and second derivatives [A/B], and apply to motion in a plane	eg If the position is given by $x = f(t)$, $y = g(t)$ then the velocity components are given by $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and the speed by $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ The instantaneous direction of motion is given by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	
apply differentiation to related rates in problems where the functional relationship is given explicitly or implicitly	eg Explicitly: $V = \frac{1}{3}\pi r^2 h$; given $\frac{dh}{dt}$, *find* $\frac{dV}{dt}$. Implicitly: $x^2 + y^2 = r^2$ where x, y are functions of t; given $\frac{dx}{dt}$, find $\frac{dy}{dt}$ using $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.	
solve practical related rates by first establishing a functional relationship between appropriate variables [A/B]		

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
Integration know the meaning of the terms integrate, integrable, integral, indefinite integral, definite integral and constant of integration		CAS may be used for consolidation/extension. However, when CAS are being used for difficult/real examples the emphasis should be on understanding of the concepts rather than routine computation. When software is being used for integration in difficult cases, candidates should be able to say which rules were used.
recall standard integrals of x^{α} $(\alpha \in Q, \alpha \neq -1)$, sin x and cos x, and know the folllowing: $\int (af(x) + bg(x))dx = a\int f(x)dx + b\int g(x)dx,\ a, b \in \mathbf{R}$ $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx\ ,\ a < c < b$ $\int_b^a f(x)dx = -\int_a^b f(x)dx,\ b \neq a$ $\int_a^b f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$ know the integrals of e^x, x^{-1}, $\sec^2 x$		

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
integrate by substitution: expressions requiring a simple substitution	Candidates are expected to integrate simple functions on sight. eg $\int xe^{x^2}\,dx$	Where substitutions are given they will be of the form $x = g(u)$ or $u = g(x)$.
expressions where the substitution will be given	eg $\int \cos^3 x \sin x\,dx,\ u = \cos x$	
the following special cases of substitution		
$\int f(ax+b)\,dx$	eg $\int \sin(3x+2)\,dx$	
$\int \frac{f'(x)}{f(x)}\,dx$	eg $\int \frac{2x}{x^2+3}\,dx$	
apply integration to the evaluation of areas	Other applications may include volumes of simple solids of revolution (disc/washer method) [A/B].	

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
integrate rational functions, both proper and improper, by means of partial fractions; the degree of the denominator being ≤ 3 the denominator may include: (i) two separate or repeated linear factors (ii) three linear factors [A/B]		
integrate by parts with one application	eg $\int x \sin x dx$	
integrate by parts involving repeated applications [A/B]	eg $\int x^2 e^{3x} dx$ [A/B].	

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Ordinary differential equations know the definition of differential equation and the meaning of the terms linear, order, general solution, arbitrary constants, integrating factor particular solution, initial condition		Link with differentiation. Begin by verifying that a particular function satisfies a given differential equation. Candidates should know that differential equations arise in modelling of physical situations, such as electrical circuits and vibrating systems, and that the differential equation describes how the system will change with time so that initial conditions are required to determine the complete solution.
solve first order differential equations (variables separable)	ie, equations that can be written in the form $\frac{dy}{dx} = \frac{g(x)}{h(y)}$	
solve first order linear differential equations using the integrating factor method	eg Write the linear equation $a(x)\frac{dy}{dx} + b(x)y = g(x)$ in the standard form $\frac{dy}{dx} + P(x)y = f(x)$ and hence as $\frac{d}{dx}\left(e^{\int P(x)dx} y\right) = e^{\int P(x)dx} f(x)$	
formulate a simple statement involving rate of change as a simple separable first order differential equation		

National Course Specification: course details (cont)

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
know the laws of growth and decay: applications in practical contexts		Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good motivating examples and can build on the knowledge and use of logarithms.
find general solutions and solve initial value problems	eg Mixing problems, such as salt water entering a tank of clear water which is then draining at a given rate. eg Growth and decay problems, an alternative method of solution to separation of variables. eg Simple electrical circuits.	

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

ASSESSMENT

To gain the award of the course, the candidate must pass all unit assessments as well as the external assessment. External assessment will provide the basis for grading attainment in the course award.

Where units are taken as component parts of a course, candidates will have the opportunity to achieve at levels beyond that required to attain each of the unit outcomes. This attainment may, where appropriate, be recorded and used to contribute towards course estimates and to provide evidence for appeals. Additional details are provided, where appropriate, with the exemplar assessment materials. Further information on the key principles of assessment are provided in the paper *Assessment*, published by HSDU in May 1996.

DETAILS OF THE INSTRUMENTS FOR EXTERNAL ASSESSMENT

The external assessment will take the form of an examination of up to three hours' duration. Candidates will sit an examination assessing *Statistics 1 (AH), Statistics 2 (AH)* and *Mathematics for Applied Mathematics (AH)* **or** one assessing *Mechanics 1 (AH), Mechanics 2 (AH)* and *Mathematics for Applied Mathematics (AH).* Each examination will contain a balance of short questions designed mainly to test knowledge and understanding and extended response questions which also test problem solving skills. These two styles of questions will include ones which are set in more complex contexts to provide evidence for performance at grades A and B.

GRADE DESCRIPTIONS FOR ADVANCED HIGHER APPLIED MATHEMATICS

Advanced Higher Applied Mathematics courses should enable candidates to solve problems which integrate mathematical knowledge across performance criteria, outcomes and units, and which require extended thinking and decision making. The award of grades A, B and C is determined by the candidate's demonstration of the ability to apply knowledge and understanding to problem solving. To achieve grades A and B in particular, this demonstration will involve more complex contexts including the depth of treatment indicated in the detailed content tables.

In solving these problems, candidates should be able to:

a) interpret the problem and consider what might be relevant;
b) decide how to proceed by selecting an appropriate strategy;
c) implement the strategy through applying mathematical knowledge and understanding and come to a conclusion;
d) decide on the most appropriate way of communicating the solution to the problem in an intelligible form.

Familiarity and complexity affect the level of difficulty of problems/assignments. It is generally easier to interpret and communicate information in contexts where the relevant variables are obvious and where their inter-relationships are known. It is usually more straightforward to apply a known strategy than to modify one or devise a new one. Some concepts are harder to grasp and some techniques more difficult to apply if they have to be used in combination.

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

Exemplification at grade C and grade A

a) ***Interpret the problem and consider what might be relevant***

At grade C candidates should be able to interpret and model qualitative and quantitative information as it arises within:

- the description of real-life situations
- the context of other subjects
- the context of familiar areas of mathematics

Grade A performance is demonstrated through coping with the interpretation of more complex contexts requiring a higher degree of reasoning ability in the areas described above.

b) ***Decide how to proceed by selecting an appropriate strategy***

At grade C candidates should be able to tackle problems by selecting algorithms drawn from related areas of mathematics or apply a heuristic strategy.

Grade A performance is demonstrated through an ability to decide on and apply a more extended sequence of algorithms to more complex contexts.

c) ***Implement the strategy through applying mathematical knowledge and understanding, and come to a conclusion***

At grade C candidates should be able to use their knowledge and understanding to carry through their chosen strategies and come to a conclusion. They should be able to process data in numerical and symbolic form with appropriate regard for accuracy, marshal facts, sustain logical reasoning and appreciate the requirements of proof.

Grade A performance is demonstrated through an ability to cope with processing data in more complex situations and sustaining logical reasoning, where the situation is less readily identifiable with a standard form.

d) ***Decide on the most appropriate way of communicating the solution to the problem in an intelligible form***

At grade C candidates should be able to communicate qualitative and quantitative mathematical information intelligibly and to express the solution in language appropriate to the situation.

Grade A performance is demonstrated through an ability to communicate intelligibly in more complex situations and unfamiliar contexts.

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

APPROACHES TO LEARNING AND TEACHING

The approaches to learning and teaching recommended for Higher level should be continued and reinforced, whenever possible. Exposition to a group or class remains an essential technique. However, candidates should be more actively involved in their own learning in preparation for future study in higher education. Opportunities for discussion, problem solving, practical activities and investigation should abound in Advanced Higher Applied Mathematics courses. There also exists much greater scope to harness the power of technology in the form of mathematical and graphical calculators and computer software packages.

Independent learning is further encouraged in the grade descriptions for the courses. Coursework tasks and projects/assignments are recommended as vehicles for the introduction of new topics, for illustration or reinforcement of mathematics in context and for the development of extended problem solving, practical and investigative skills as well as adding interest to the courses.

In particular:

- in statistics, it is important that illustrative examples should, as far as possible, be drawn from real life, emphasising the relevance of the subject in the modern world and it is also highly desirable that much of the experimental work, data analysis and simulation should be undertaken with suitable computer software

- in mechanics, the demonstration to candidates of the application to real problems through modelling is fundamental. The tackling of realistic problems is essential to acquiring a better understanding of the physical nature of the world. The teaching may be enhanced with the aid of computer software

National Course Specification: course details (cont)

COURSES Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

SPECIAL NEEDS

This course specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/4, September

SCOTTISH QUALIFICATIONS AUTHORITY

National Unit Specification: general information

UNIT Statistics 1 (Advanced Higher)

NUMBER D326 13

COURSE Applied Mathematics: Statistics (Advanced Higher)

SUMMARY

This unit is the first of two Advanced Higher units which, together with the Mathematics for Applied Mathematics (AH) unit, comprise the Advanced Higher Applied Mathematics: Statistics course. It builds on the work of Statistics (H) and introduces special distributions, sampling, estimation and hypothesis testing. The unit provides a basis for progression to Statistics 2 (AH).

OUTCOMES

1 Use conditional probability and the algebra of expectation and variance.
2 Use probability distributions in simple situations.
3 Identify sampling methods and estimate population parameters.
4 Use a z-test on a statistical hypothesis where the significance level is given.
5 Undertake a statistical assignment.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Higher Mathematics award including Statistics (H)

Administrative Information

Superclass: RB

Publication date: **March 2001**

Source: Scottish Qualifications Authority

Version: 03

National Unit Specification: general information (cont)

UNIT Statistics 1 (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher.

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	Numeracy	H
Additional core skills components for the unit	None	

National Unit Specification: statement of standards

UNIT Statistics 1 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use conditional probability and the algebra of expectation and variance.

Performance criteria

(a) Calculate a conditional probability.
(b) Apply the laws of expectation and variance in simple cases.

OUTCOME 2

Use probability distributions in simple situations.

Performance criteria

(a) Use the binomial distribution.
(b) Use the Poisson distribution.
(c) Use the normal distribution.

OUTCOME 3

Identify sampling methods and estimate population parameters.

Performance criteria

(a) Identify a given sampling method.
(b) Estimate a population parameter from a sample statistic.

OUTCOME 4

Use a z-test on a statistical hypothesis where the significance level is given.

Performance criteria

(a) State null and alternative hypotheses.
(b) State one-tail or two-tail test.
(c) Determine the p-value.
(d) State and justify an appropriate conclusion.

National Unit Specification: statement of standards (cont)

UNIT Statistics 1 (Advanced Higher)

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

OUTCOME 5

Undertake a statistical assignment.

Performance criteria

(a) Pose the question that the assignment addresses.
(b) Collect (generate) the relevant data.
(c) Analyse the data.
(d) Interpret and communicate the conclusions.

Evidence requirements

The assignment must satisfy the performance criteria, using the statistical content of the unit. A full report is to be written by the candidate individually. This report may include sets of data, graphs, computer printout, calculated statistics, consideration of probability and a conclusion.

National Unit Specification: support notes

UNIT Statistics 1 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills with the emphasis on the application of mathematical ideas and techniques to relevant and accessible problems. This unit is designed with the two-fold objective of providing a rounded experience of statistics for candidates who take the unit free-standing and, at the same time, forming a sound basis for progression to Statistics 2 (AH) for candidates specialising in statistics in the Advanced Higher Applied Mathematics: Statistics course.

The first outcome of this unit extends the earlier work on probability to conditional probability and allows candidates the opportunity to study the algebra of expectation and variance.

Outcome 2 assumes knowledge of the Statistics (H) outcome on discrete probability distributions and extends the study to special distributions.

Outcome 3 requires demonstration of competence in sampling methods and estimation of population parameters, for which experience of the Statistics (H) outcome on exploratory data analysis is of some advantage. Outcome 4 introduces the concept of hypothesis testing. Candidates are given the opportunity to apply statistical processes in Outcome 5 where competence will be demonstrated by completing an assignment.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Statistics 1 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level.

Where appropriate, statistical topics should be taught and skills in applying statistics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical, statistical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

The fifth outcome is assessed by means of a statistical assignment. Examples of such assignments, with marking schemes, are contained within the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/4, September 2003).

SCOTTISH QUALIFICATIONS AUTHORITY

National Unit Specification: general information

UNIT Statistics 2 (Advanced Higher)

NUMBER D330 13

COURSE Applied Mathematics: Statistics (Advanced Higher)

SUMMARY

This unit is the second of two Advanced Higher units which, together with the Mathematics for Applied Mathematics (AH) unit, comprise the Advanced Higher Applied Mathematics: Statistics course. It builds on the work of Statistics 1 (AH) and introduces control charts and the t-distribution.

OUTCOMES

1. Use simple control charts.
2. Test a statistical hypothesis.
3. Use the t-distribution.
4. Analyse the relationship between two variables.
5. Undertake a statistical investigation.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Statistics 1 (AH)

CREDIT VALUE

1 credit at Advanced Higher.

Administrative Information

Superclass: RB

Publication date: **March 2001**

Source: Scottish Qualifications Authority

Version: 03

National Unit Specification: general information (cont)

UNIT Statistics 2 (Advanced Higher)

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	Numeracy	H
Additional core skills components for the unit	None	

National Unit Specification: statement of standards

UNIT Statistics 2 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use simple control charts.

Performance criteria

(a) Construct a control chart.
(b) Interpret a control chart.

OUTCOME 2

Test a statistical hypothesis.

Performance criteria

(a) Carry out a chi-squared test.
(b) Carry out a sign test.
(c) Carry out a Mann-Whitney test.

OUTCOME 3

Use the t-distribution.

Performance criteria

(a) Determine a confidence interval for a population mean.
(b) Carry out a one sample t-test for the population mean.

National Unit Specification: statement of standards (cont)

UNIT Statistics 2 (Advanced Higher)

OUTCOME 4

Analyse the relationship between two variables.

Performance criteria

(a) Test the significance of the strength of a linear relationship.
(b) Use a residual plot to check model assumptions.
(c) Construct an interval estimate for a given response.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed-book test. Tests should be carried out under supervision. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

OUTCOME 5

Undertake a statistical investigation.

Performance criteria

(a) Pose the question that the investigation addresses.
(b) Collect (generate) the relevant data.
(c) Analyse the data.
(d) Interpret and communicate the conclusions.

Evidence requirements

The investigation must satisfy the performance criteria, using the statistical content of the unit. A full report on the investigation is to be written by the candidate individually. This report may include sets of data, graphs, computer printout, calculated statistics, consideration of probability and a conclusion.

National Unit Specification: support notes

UNIT Statistics 2 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills with the emphasis on the application of mathematical ideas and techniques to relevant and accessible problems. This unit is designed to build on the content of Statistics 1(AH) and extend the specialism in this topic to a wider experience and to a more advanced level.

In this unit, Outcomes 1 and 3 introduce the statistical concepts of control charts and the t-distribution respectively.

Hypothesis testing, introduced in Statistics 1(AH), is now extended in Outcome 2 to include chi-squared, sign and Mann-Whitney tests.

Regression analysis studied in Statistics (H) is extended in Outcome 4 to include the significance of the product moment correlation coefficient and the consideration of interval estimates.

To reinforce the practical nature of the subject, candidates, in Outcome 5, are required to demonstrate competence in all the stages of undertaking a statistical investigation.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Statistics 2 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level.

Where appropriate, statistical topics should be taught and skills in applying statistics developed through real-life contexts. Candidates should be encouraged, throughout this unit, to make efficient use of the arithmetical, mathematical, statistical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

The fifth outcome is assessed by means of a statistical investigation. This investigation should be a substantial piece of work, taking up to ten hours, in which candidates collect their own data by, for example, carrying out an experiment. The analysis, interpretation and communication of the conclusions should be included in each candidate's report. Examples of such investigations, are contained within the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes (cont)

UNIT Statistics 2 (Advanced Higher)

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/4, September 2003).

SCOTTISH QUALIFICATIONS AUTHORITY

National Unit Specification: general information

UNIT Mechanics 1 (Advanced Higher)

NUMBER D327 13

COURSE Applied Mathematics: Mechanics (Advanced Higher)

SUMMARY

This unit is the first of two Advanced Higher units which, together with the Mathematics for Applied Mathematics (AH) unit, comprise the Advanced Higher Applied Mathematics: Mechanics course. This unit uses some of the skills of Higher Mathematics applied to problems in mechanics. The unit provides a basis for progression to Mechanics 2 (AH).

OUTCOMES

1 Interpret and solve problems on motion in a straight line.
2 Solve problems involving position, velocity and acceleration vectors, including relative motion.
3 Solve problems on the motion of projectiles in a plane, under constant gravity and ignoring resistances.
4 Solve problems on forces in equilibrium and on linear motion involving forces.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Higher Mathematics award

Administrative Information

Superclass: RB

Publication date: **March 2001**

Source: Scottish Qualifications Authority

Version: 03

National Unit Specification: general information (cont)

UNIT Mechanics 1 (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher.

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	Numeracy	H
Additional core skills components for the unit	None	

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications* (BA0906, August 2001).

National Unit Specification: statement of standards

UNIT Mechanics 1 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Interpret and solve problems on motion in a straight line.

Performance criteria

(a) Draw a velocity-time graph and use it to calculate distance.
(b) Use an equation of motion in a simple situation.
(c) Given an expression for displacement, find corresponding expressions for velocity and acceleration (where these are functions of time).

OUTCOME 2

Solve problems involving position, velocity and acceleration vectors, including relative motion.

Performance criteria

(a) Find the relative velocity of one body with respect to another (a simple trigonometric approach is allowed).
(b) Solve a simple problem involving collision.

OUTCOME 3

Solve problems on the motion of projectiles in a plane, under constant gravity and ignoring resistances.

Performance criterion

(a) Given appropriate information about initial velocity, find the time of flight, the greatest height reached and the range of a projectile.

National Unit Specification: statement of standards (cont)

UNIT Mechanics 1 (Advanced Higher)

OUTCOME 4

Solve problems on forces in equilibrium and on linear motion involving forces.

Performance criteria

(a) Resolve forces in two dimensions.
(b) Find the resultant of two forces.
(c) Given a set of three forces in equilibrium, derive appropriate results by resolving horizontally and vertically
(d) Solve a simple problem involving the coefficient of friction, μ.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mechanics 1 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills with the emphasis on the application of mathematical ideas and techniques to relevant and accessible problems. This unit is designed with the two-fold objective of providing a rounded experience of mechanics for candidates who take the unit free-standing and, at the same time, forming a sound basis for progression to Mechanics 2 (AH) for candidates specialising in mechanics in the Advanced Higher Applied Mathematics: Mechanics course.

The four outcomes of this unit provide the candidate with the opportunity to demonstrate competence in the solution of problems in a range of aspects of motion. Outcome 1 introduces motion in a straight line and Outcome 2 extends this concept to position, velocity and acceleration vectors including relative motion. Outcome 3 requires demonstration of ability to solve projectile problems and Outcome 4 the ability to deal with forces in equilibrium and linear motion involving forces.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Mechanics 1 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/3, December 2001).

National Unit Specification: general information

UNIT Mechanics 2 (Advanced Higher)

NUMBER D331 13

COURSE Applied Mathematics: Mechanics (Advanced Higher)

SUMMARY

This unit is the second of two Advanced Higher units which, together with the Mathematics for Applied Mathematics (AH) unit, comprise the Advanced Higher Applied Mathematics: Mechanics course. It builds on the work of Mechanics 1 (AH) and introduces circular motion and simple harmonic motion. It also applies some further calculus from Mathematics 1 and 2 (AH) to problems in mechanics.

OUTCOMES

1 Interpret and solve problems involving motion in a horizontal circle with uniform angular velocity.
2 Solve problems involving simple harmonic motion.
3 Apply the principles of momentum and impulse.
4 Apply the principles of work, power and energy.
5 Interpret and solve problems involving first order differential equations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained:

- Mechanics 1 (AH)

Candidates will also require to have knowledge of the calculus content of Mathematics 1 and 2 (AH).

Administrative Information

Superclass: RB

Publication date: March 2001

Source: Scottish Qualifications Authority

Version: 03

National Unit Specification: general information (cont)

UNIT Mechanics 2 (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher.

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	Numeracy	H
Additional core skills components for the unit	None	

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications* (BA0906, August 2001).

National Unit Specification: statement of standards

UNIT Mechanics 2 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Interpret and solve problems involving motion in a horizontal circle with uniform angular velocity.

Performance criteria

(a) Solve a simple problem involving the inverse square law of gravitation.
(b) Solve a simple problem involving motion in a horizontal circle.

OUTCOME 2

Solve problems involving simple harmonic motion.

Performance criteria

(a) Solve a simple problem involving simple harmonic motion in a straight line, making use of basic equations, period, amplitude, maximum velocity or maximum acceleration as appropriate.

OUTCOME 3

Apply the principles of momentum and impulse.

Performance criteria

(a) Use the concept of conservation of linear momentum.
(b) Use impulse appropriately in a simple situation.

OUTCOME 4

Apply the principles of work, power and energy.

Performance criteria

(a) Evaluate work done appropriately.
(b) Use $P = \underline{F}.\underline{v}$ appropriately.
(c) Use the concept of conservation of energy.

National Unit Specification: statement of standards (cont)

UNIT Mechanics 2 (Advanced Higher)

OUTCOME 5

Interpret and solve problems involving first order differential equations.

Performance criterion

(a) Form and solve a differential equation of motion, modelling a simple practical problem where the acceleration is a function of the velocity.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mechanics 2 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills with the emphasis on the application of mathematical ideas and techniques to relevant and accessible problems. This unit is designed to build on the content of Mechanics 1(AH) and extend the specialism in this topic to a wider experience and to a more advanced level.

In this unit, the study of motion and vector methods in Mechanics 1(AH) and the emphasis on problem solving throughout the unit is continued and extended to circular motion in Outcome 1 and simple harmonic motion in Outcome 2.

In Outcome 3, competence in solution of problems on impulse and conservation of linear momentum requires to be demonstrated and likewise for problems on work, power and energy in Outcome 4.

Outcome 5 involves the modelling and solution of practical problems involving first order differential equations. The calculus involved will be within the content of Mathematics 1 (AH) and 2 (AH).

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Mechanics 2 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/3, December 2001).

National Unit Specification: general information

UNIT	Mathematics for Applied Mathematics (Advanced Higher)
NUMBER	DE8Y 13
COURSE	Applied Mathematics: Statistics (Advanced Higher) Applied Mathematics: Mechanics (Advanced Higher)

SUMMARY

This unit is the third unit of each of the Advanced Higher Applied Mathematics courses. This unit extends the algebra and calculus work from Higher level and introduces matrix algebra which is applied to solving systems of linear equations.

OUTCOMES

1 Use algebraic skills.
2 Use matrix algebra.
3 Use the rules of differentiation on the elementary functions x^n ($n \in Q$), sin x, cos x, e^x and ln x and their composites.
4 Integrate using substitution and partial fractions and by parts.
5 Solve first order ordinary differential equations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained :

- Higher Mathematics award, including Mathematics 3 (H)

Administrative Information

Superclass: RB

Publication date: May 2007

Source: Scottish Qualifications Authority

Version: 02

National Unit Specification: general information (cont)

UNIT Mathematics for Applied Mathematics (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher (8 SCQF credit points at SCQF level 7*)

**SCQF credit points are used to allocate credit to qualifications in the Scottish Credit and Qualifications Framework (SCQF). Each qualification in the Framework is allocated a number of SCQF credit points at an SCQF level. There are 12 SCQF levels, ranging from Access 1 to Doctorates.*

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	None	
Additional core skills components for the unit	Using Number	H

National Unit Specification: statement of standards

UNIT Mathematics for Applied Mathematics (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

This unit is the third component unit of the three Advanced Higher Applied Mathematics Courses.

In this unit, you are required to demonstrate competence in five outcomes (topics) by achieving the performance criteria listed for each outcome.

OUTCOME 1

Use algebraic skills.

Performance criteria

(a) Expand an expression of the form $(x + y)^n$, $n \in \mathbf{N}$ and $n \leq 5$.

(b) Evaluate a simple sum of the form $\sum_{r=1}^{n}(ar+b)$; $r, n \in \mathbf{N}$

(c) Express a proper rational function as a sum of partial fractions where the denominator is a quadratic in factorised form.

OUTCOME 2

Use matrix algebra

Performance criteria

(a) Perform matrix operations of addition, subtraction and multiplication
(b) Calculate the determinant of a 3×3 matrix
(c) Find the inverse of a 2×2 matrix

OUTCOME 3

Use the rules of differentiation on the elementary functions x^n, $(n \in \mathbf{Q})$, $\sin x$, $\cos x$, e^x and $\ln x$ and their composites.

Performance criteria

(a) Differentiate a product.
(b) Differentiate a quotient.
(c) Find the first derivative of a function defined parametrically.

National Unit Specification: statement of standards (cont)

UNIT Mathematics for Applied Mathematics (Advanced Higher)

OUTCOME 4

Integrate using substitution and partial fractions and by parts.

Performance criteria

(a) Integrate using a substitution when the substitution is given.
(b) Integrate a proper rational function where the denominator is a factorised quadratic.

OUTCOME 5

Solve first order ordinary differential equations

Performance criteria

(a) Find a general solution of a first order differential equation (variables separable type).
(b) Solve a simple first order linear differential equation using an integrating factor.

Evidence requirements
Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics for Applied Mathematics (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated and for checking of accuracy and whether or not an answer is sensible to be ever present.

In this unit the algebraic skills learnt at Higher level are extended in Outcome 1 to binomial expansions, sigma notation and partial fractions.

Outcome 2, provides an introduction to matrix algebra.

In Outcomes 3 and 4, the elementary calculus studied at Higher level is extended to differentiation of sums, products, quotients and composites of elementary functions and to integration using substitution and partial fractions and by parts. In both of Outcomes 3 and 4, computer algebra systems can be used extensively for consolidation and extension.

Outcome 5 introduces first order ordinary differential equations of the variables separable type and where an integrating factor is used.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/4, September 2003).