National Qualifications

Standard Grade Arrangements in Mathematics

Foundation, General and Credit Levels
in and after 2001

STANDARD GRADE ARRANGEMENTS IN MATHEMATICS

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Introduction

This document sets out the arrangements for examinations in Standard Grade Mathematics which have been developed in the light of a two-stage consultation exercise carried out between April 1997 and June 1998. The consultation, initially sought the views of interested bodies and presenting centres on the introduction of some statistical content and the consequent deletion of some other content, and latterly on the following package of proposals:

(i) Introduction of non-calculator papers into the external examinations

(ii) Introduction of statistical content

(iii) Reduction of the number of assessable elements to two (removal of the element Investigating).

Section 1

Mathematical Education: Aims and Objectives

1 Mathematical Education: Aims and Objectives

1 1 Introduction

The essential aim of mathematical education is to help pupils to learn how to describe, tackle and ultimately solve problems which require the use of mathematical knowledge and techniques. This essential aim is not peculiar to a particular section of the school population and, consequently, it is recommended that the main approach to teaching and learning mathematics should be firmly based on problem solving.

Since the development of problem solving abilities is the declared aim of many other areas of the curriculum it is worth making the point here that the types of problems which should be selected for use in the mathematics classroom are those which require a mathematical strategy and which will be amenable to solution by use of mathematical knowledge, skills and concepts. The range of problems is great – from those relying largely on the straightforward application of known mathematical techniques to those where a considerable amount of interpretation and analysis is required and where a particular line of investigation may call for a review of earlier stages in the problem solving process.

1 2 Aims

Mathematics courses at all stages and all levels should seek to:

- contribute to pupils' personal development and overall education;
- enable pupils to develop to the limit of their capability the mathematical skills and understanding required for their present needs, both in and out of school, and for future demands of adult life, employment, further study and training;
- develop an appreciation and enjoyment of mathematics and an awareness of its importance in society and in the development of technology.

These broad aims are most likely to be fulfilled by a course which is designed to encourage pupil participation, which illustrates the need for what is being learned, and which sees mathematics as arising out of real or realistic contexts and those arousing curiosity, which pupils can appreciate. While exposition by the teacher and the working of practice exercises by pupils will have a prominent place, these activities alone would not allow the aims to be achieved.

The objectives set out below provide a suitable rationale. It is recommended that Mathematics courses should be based on these objectives and that course planning, teaching approach, classroom activities, choice of resources and procedures for assessment should be consciously concerned with achieving them.

1 3 Objectives

1 3 1 Objectives Related to Problem Solving

Published works offer analyses of problem solving of varying complexity, but a list of objectives should not be so lengthy as to hinder the conscious achievement of its components. The four processes associated with problem solving which are given below have been identified as particularly significant and helpful in designing mathematics courses. These processes are presented in the order in which they are frequently met in problem solving. First, the problem has to be understood: the requirements of the problem and the vocabulary and concepts in which it is expressed have to be interpreted. Second, how to go about solving the problem has to be considered and a strategy must be chosen. Third, the strategy has to be carried out and this may require the processing of data. Finally, the solution of the problem, or the conclusions reached if there is no solution, has to be communicated. This order should not be taken to imply a steady progression through the processes: there are many problems where the performance at one stage may lead to a review of an earlier stage. It would be misleading, therefore, to view these processes as independent of each other.

In order to make this model as complete as possible, other processes which are sometimes presented as separate stages have been incorporated into these four. Thus, decisions on evaluating or checking solutions have been included under "selecting a strategy", although the consequences of these decisions may involve the pupil in, say, "interpreting information" or "processing data".

a) Interpreting the information in a problem

Information appears in a wide variety of forms – in writing and in diagrams, tables, graphs, codes, algebraic formulae or may be expressed orally. Underlying these forms are many facts and concepts with which pupils will have to be familiar. Not all of these concepts will, however, be accessible to all pupils. On some occasions there will be excess information and pupils will have to learn to select what is likely to be relevant; on others, where little or no information is supplied, pupils will have to make decisions on what is needed and how this can be obtained. In some instances the information may be able to be gathered by the pupils, for example, where measurements are involved, but often it will have to be provided by the teacher.

Once they have identified what is requird, pupils may need to translate the information into a form which is helpful for tackling the problem, eg jotting down relevant facts, sketching a diagram, or writing down an equation.

b) Selecting a strategy to tackle a problem

The strategy required may be to choose the appropriate operation or algorithm, eg to find a hire purchase charge or an amount of interest; to apply Pythagoras' Theorem or the formula for solving a quadratic equation. But there are problems for which there is no obvious algorithm and for which pupils have to decide their own strategies. These strategies include deciding the steps to be taken and the order in which to carry them out; experimenting; trial and improvement; looking for a pattern; making an exhaustive list; reduction to a simpler problem; working backwards; and using deductive reasoning.

1 3 1 *(continued)*

All pupils should be encouraged to evaluate and check their solutions and to learn to reject those which are inappropriate. For some, the review stage should involve looking for possible generalisations and hidden assumptions. Such activities should provide opportunities for abler pupils to develop the idea of proof.

c) Processing data

Some data to be processed will be numerical, requiring the application of the rules of computation as well as other skills such as expressing data in a form suitable for calculation; making rough estimates; mental calculation; and appropriate use of the calculator.

Other data to be processed will be in the form of symbols requiring the application of the four rules and other procedures such as re-arranging algebraic terms; simplifying mathematical expressions; changing the subject of formulae; and carrying out algorithms.

d) Communicating

In trying to reach a solution to a problem pupils will require to communicate. Often this communication will be for their own use and can be in any form they feel is helpful. Sometimes this will involve communication with others engaged on the same task. The communication of the outcome will, however, require pupils to take account of the audience for whom it is intended and will, therefore, require decisions about the most effective form of communication to be used. This may require forms such as words, numerals, formulae, graphs, tables or scale drawings. Consideration should be given to the presentation of an oral report whenever practicable.

Note: In the objectives above the use of practical skills is seen as contributing to "problem solving". There are many problems in the real world which depend on information obtained by measurement or estimation of length, area, volume, weight, angle and time and where the making of a model or sketching or drawing to scale will support the communication of a solution. (See also Section 5.)

1 3 2 Development of Desirable Social and Personal Qualities

a) Working together

Working cooperatively with others is a powerful way of tackling problems. Moreover, the exchange of ideas through discussion is an essential part of learning. Activities are required to develop the ability to work with others towards a common goal, or for a common purpose. This can often be achieved through discussing findings or ideas; planning a project; playing and analysing games, solving puzzles, or carrying out coursework tasks/investigations; sharing an otherwise routine task (eg collecting data); or doing practical work where the task is difficult for one person.

1 3 2 *(continued)*

b) Showing initiative

The teacher should not predetermine the range and scope of every task. There should be opportunities for all pupils to show initiative and resourcefulness. These could arise in situations such as problems where there is more than one acceptable solution; open-ended questions and coursework tasks/investigations; the exploration of a social context or the historical background of a mathematical idea; and an assignment or project in which decisions have to be made.

1 3 3 Choice of Content and Approach

a) Work should be relevant

The aims stated in 1 2 include references to pupils' present and future needs and the importance of mathematics in society and technology. Account must be taken of this in preparing courses.

Trying to solve a real-life problem which has meaning for the pupils can be a very good starting point for a topic. The context, or the methods used to illustrate the problem, may both be motivating. Contexts include, for example, other school subjects, the home, leisure, employment and more advanced applications. Every chance should be taken for pupils to apply their learning in appropriate contexts, which should provide ample opportunity to reinforce understanding of concepts and to practise skills. For some pupils almost everything they do should be related to situations familiar to them; for other pupils some work, particularly the development of facility in manipulative skills, should be included because of its importance for later studies.

b) Work should be interesting

In deciding on the approach and content of courses it should be kept in mind that one of the aims is to develop an appreciation and enjoyment of mathematics. Frequently, the content is selected for other, more utilitarian reasons and interest may hinge on the approach taken. Sometimes, however, topics can be chosen which, while new to the pupils, revise earlier work without being associated with previous lack of success. Many pupils find work such as model-making, patterns in number and shape and games and puzzles stimulating. Others may, in addition, find pleasure in aspects of abstract mathematics.

Section 2

Approach

2 Approach

2 1 The aims and objectives outlined in Section 1 imply an approach to the teaching and learning of mathematics that is varied and flexible. The wide differences evident in the interests, attainment and pace of learning of pupils have to be taken into account, while at the same time the demands and challenges of content have to be met imaginatively.

2 2 The following questions should enable the teacher to identify worthwhile learning activities with the necessary variety and flexibility.

a) Is the teaching and learning participative and interactive?

b) Have all aspects of the problem solving objectives been covered?

c) Have there been opportunities for all pupils

- to engage in problem solving,
- to work together,
- to display initiative?

d) Has sufficient account been taken of individual differences?

e) Has the content been set in contexts which the pupils can understand and which are likely to motivate them?

f) Has the work been varied in terms of

- class organisation (eg whole class, group, individual);
- resources (eg commercial, teacher-produced, text-books, cards, worksheets, TV programmes, advanced calculators/computers);
- approach (eg teacher-led discussion, discussion among pupils, practical, open-ended);
- and output (eg individual work, models, wall displays)?

g) On what occasions has a practical approach been taken?

h) On what occasions have real-life materials been used to support the work?

i) On what occasions have pupil or staff interests been taken in account?

2 3 Foundation Level courses should be concerned with helping pupils to understand thoroughly and to apply competently and confidently a small amount of content. Teaching and learning should be set in contexts familiar to the pupils, including some which they are likely to meet as part of everyday adult life. Practice in basic skills should normally occur within such contexts.

In the case of General and Credit Level courses also, every effort should be made to introduce new concepts and skills through an approach which illustrates their usefulness, which gives opportunities for discussion and which allows for the use of apparatus where it would help pupils. Exploration of new ideas and development of understanding should, where meaningful and appropriate, precede practising of skills.

2 3 *(continued)*

Care should be taken to ensure that the level of skills does not go beyond the pupils' understanding.

It is important to realise that the skills to be developed cannot always be completely justified in terms of the immediate problem presented to pupils. Manipulative skills are, however, an important part of the mathematician's toolkit and some pupils, particularly those following Credit Level courses, should be able to appreciate the need to develop these to a high degree of fluency.

2 4 Understanding and confidence in mathematics depend on the pupil developing a strong interrelated network of concepts. It is rarely possible for the teacher to know for each pupil what connections are required to embed a new concept in already acquired knowledge. Presenting pupils with an investigative task to carry out is a useful way of allowing pupils to see for themselves how a new idea relates to their own existing knowledge and understanding. Such an approach, particularly if it is followed by discussion, should help the teacher to identify weaknesses in a pupil's understanding.

Consequently, it is recommended that whenever possible, and particularly in topics where unfamiliar concepts are likely to be met, an investigative approach should be taken.

2 5 One of the aims of Mathematics courses is to prepare pupils for the future demands of adult life, employment, further study and training. Such an aim implies that all pupils should be encouraged to tackle problems as they appear in the real world. Such problems are not always well defined and may require information to be sought and decisions to be made. Moreover they are frequently set in a practical context. Work of this kind will allow pupils to see the relevance of what they are learning and should encourage mathematical thinking. For some pupils further study will involve more abstract mathematics, and many will be ready to develop their mathematical thinking through coursework tasks/investigations where generalisation is possible, and where rigour is encouraged in communicating the outcome. For these pupils, some coursework tasks/investigations may be presented in an abstract form.

The use of coursework tasks/investigations should provide opportunities for decision making, for exploring a range of strategies, for sustained thinking and for more extended forms of communication.

2 6 In the years ahead it is likely that there will be increased provision of advanced calculators and computers in schools. As these become more readily available, consideration will need to be given to the part they can play in encouraging mathematical learning.

In the knowledge that the position will require to be reviewed as circumstances change, schools should decide now whether, with existing resources and expertise, technology could be used effectively in any of the following ways:

- to develop a fuller understanding of mathematics;
- to enable the teacher to use realistic contexts (eg real data in statistics);
- to investigate geometric properties of functions and their graphs;
- as an aid to problem solving, for example in modelling statistical experiments, the use of iterative methods to solve equations, or the investigation of a large number of cases for relationships and generalisation;
- to provide computer assisted learning.

Section 3

Assessment for Certification

3 Assessment for Certification

3 1 Certification

Candidates will be assessed by a system common to all Levels.

The Certificate will record an overall award on a 7-point scale of grades, Grade 1 being the highest. The Certificate will also record attainment in each assessable element. The overall award will be derived from a weighted mean of the element grades (see 3 2 1).

For any element, Grade 7 will indicate that the candidate has, in the element concerned, completed the course but has not demonstrated achievement of any specified level of performance as defined by the Grade Related Criteria. Grade 7 in an element will not be available to external candidates.

The Scottish Qualifications Authority (SQA) will regard the submission of an estimate grade for an externally assessed element as evidence that the course has been completed in that element.

The attainment of this course will lead to the automatic award of some of the Core Skills components. A final statement will be provided at a later date by SQA, once full validation procedures are complete.

3 2 Assessable Elements

3 2 1 There will be two assessable elements: Knowledge and Understanding (KU), and Reasoning and Enquiry (RE). The elements will be weighted for the overall award in the ratio of 1:1. Where the calculation of the overall grade results in a mean half way between two grades, the better of the two grades will be awarded. For example, element grades of 2 and 3 for KU and RE will result in an overall grade of 2. Each of these elements covers the problem-solving processes (see 1 3 1), the key aspects of Mathematics; they also take into account the other syllabus objectives which are essential experiences for a worthwhile course (see 1 3 2 and 1 3 3).

Knowledge and Understanding, and Reasoning and Enquiry will be assessed externally.

The overall award to Mathematics will reflect the candidate's ability to solve mathematical problems.

3 2 2 Knowledge and Understanding

This element covers the facts, concepts and skills needed to solve mathematical problems, including the use of appropriate mathematical notation and symbols. Included in this element is the ability to carry out routine procedures and to solve routine problems where the candidate is expected to know the approach to be used. It may be helpful to think of the items of this element in terms of a mathematical "toolkit".

3 2 3 Reasoning and Enquiry

Identification of this element recognises that more than Knowledge and Understanding is needed to solve a mathematical problem. Also required is the ability to make decisions about how to start the problem and what skills to apply, the reasoning and application of the skills needed to continue and complete the problem and, where appropriate, decisions about how best to present the solution. This element encourages candidates to show initiative and resourcefulness. In the assessment of this element candidates will be expected to apply the appropriate level of Knowledge and Understanding in situations where the approach to be used is not immediately apparent.

3 3 External Assessment – Knowledge and Understanding and Reasoning and Enquiry

At each Level, two papers will be set, one where a calculator may not be used and one where a calculator may be used. Each paper will assess both Knowledge and Understanding (KU) and Reasoning and Enquiry (RE).

Paper	*Grades Assessed*	*Use of Calculator*	*Elements Assesssed*	*Time Allocation*
Foundation I	6, 5	Now allowed	KU and RE	20 minutes
Foundation II	6, 5	Allowed	KU and RE	40 minutes
General I	4, 3	Not allowed	KU and RE	35 minutes
General II	4, 3	Allowed	KU and RE	55 minutes
Credit I	2, 1	Not allowed	KU and RE	55 minutes
Credit II	2, 1	Allowed	KU and RE	80 minutes

Papers at each Level will cover the problem-solving processes: interpreting, selecting a strategy, processing data and communicating. Papers will consist of a mixture of short-response questions and extended-response questions, some of which will involve sustained thinking. Most questions will assess one element only but some will assess both elements, with, for example, Knowledge and Understanding being assessed in the early part(s) and Reasoning and Enquiry being assessed in the later part(s). Where appropriate, questions will be set in context.

Marks will be allocated to each question and a total mark obtained for each element. The two grades associated with each Level will be distinguished by setting two cut-off scores. The lower cut-off score, to reflect a satisfactory overall standard of performance, will be set around 55% of the available marks; the upper cut-off score, to reflect a high overall standard of performance, will be set around 75% of the available marks. (Flexibility in the cut-off scores is necessary since some questions in examinations do not always perform as intended.)

3 4 Estimates

Presenting centres must submit to SQA, by 31 March of the year of the examination, an estimate grade for each candidate for Knowledge and Understanding, and Reasoning and Enquiry. The teacher should determine the estimate grades on the basis of each candidate’s work. Estimates may be used by SQA for its internal procedures, including such cases as absence from external examinations, adverse circumstances and appeal.

3 4 *(continued)*

In arriving at their estimate grades teachers will require to decide between two grades at a particular Level. The lower grade should be allocated to those candidates who have achieved success at around the 55% level and the upper grade to those who have achieved success at around the 75% level. Since the work of the course will vary in level teachers will have to exercise their professional judgement in deciding the precise percentages to use.

Evidence in support of these estimates should be retained by centres for submission to SQA if required.

3 5 Levels of Presentation and External Papers

3 5 1 Towards the middle of the second year of their Standard Grade course candidates will be required to decide about their Levels of presentation. At this stage no final commitment is necessary since candidates are only required to choose from Foundation and General, or General and Credit.

3 5 2 Towards the end of the course candidates should consider which papers to attempt in order to achieve the best award of which they are capable. It must be stressed that candidates presented at two Levels are not obliged to attempt the examination papers at both of the Levels. However, other than as a result of an appeal, candidates can only be awarded one of the grades asssessed by the papers attempted, or Grade 7.

The intention of the flexibility in the arrangements is to allow for enhancement or compensation. For example, a candidate presented at Foundation and General Levels and who has done well in a course which included some extension work might wish to attempt, along with the Foundation Level papers, the General Level papers in order to gain an award at Grade 4 or 3. On the other hand a candidate presented at General and Credit Levels but who has had difficulty with some aspects of the Credit Level syllabus might wish to attempt, along with the Credit Level papers, the General Level papers as a safety net. It is also open to a candidate presented at Foundation and General Levels to attempt only the General Level papers or only the Foundation Level papers.

3 5 3 Candidates who attempt papers at two Levels will be given the better of the two grades achieved on these papers. Performance at one Level will **not** be taken into account in grading at the other Level.

3 5 4 Candidates who attempt examination papers at a second Level solely in order to have the chance of gaining an enhanced award will not have covered all the content of the syllabus at the upper Level. Indeed they should not be expected to have done so. Certain items in the General and Credit content lists have been italicised. Questions in the external examination papers covering these items will not appear in the early part of the examination papers or in the early part of a question. For instance, the italicised items in the Credit Level list have been chosen on the basis that they do not link up easily with items in the General Level list and/or because they are readily accessible only to the ablest candidates.

Section 4

Grade Related Criteria

4 Grade Related Criteria

4 1 Definition

Grade Related Criteria (GRC) are positive descriptions of performance against which a candidate's achievement is measured.

4 2 Application of GRC

GRC are defined at three Levels of performance; Foundation, General and Credit. Awards will be reported on six grades, two grades being distinguished at each Level. The upper of the two grades at a given Level will be awarded to candidates who meet the stated criteria demonstrating a high standard of performance: the lower grade to those who demonstrate a lower, but still satisfactory, standard of performance.

There will be a further grade, Grade 7, for candidates who have completed the course but have not demonstrated achievement of any specified level of performance as defined by the Grade Related Criteria.

4 3 Types of GRC

Summary GRC are broad descriptions of performance. They are published as an aid to the interpretation of the profile of attainment by candidates, parents, employers and other users of the Certificate.

Extended GRC are more detailed descriptions of performance. They are to be used mainly to set standards for courses and, along with the Content Checklist (see 5 2 3), to ensure that resources are at an appropriate level of breadth and challenge. They will also be used in the design of external examination papers to ensure the papers are balanced in respect of different aspects of the elements and are set at the right levels.

4 4 Format of the Extended GRC

Each of the Extended GRC is expressed in the form of a statement followed by discriminators which allow performance between Levels to be distinguished. Normally the criteria apply to all three Levels unless indicated otherwise. The criteria for Credit Level are to be understood to include those for General and Foundation Levels; and the criteria for General Level include those for Foundation Level. The criteria have been coded for ease of identification, thus: K for Knowledge and Understanding, R for Reasoning and Enquiry.

At the Level shown, a row of asterisks indicates that the criterion does not apply; a blank indicates that the criterion applies without further description or exemplification.

4 5 Knowledge and Understanding – Summary GRC

Foundation Level (Grades 6, 5)

In familiar everyday contexts, the candidate has demonstrated ability to interpret and communicate mathematical information in the form of straightforward tables, graphs and scale drawings; and to make sensible use of a calculator in carrrying out whole number, decimal and percentage calculations in routine problems.

4 5 *(continued)*

General Level (Grades 4, 3)

In everyday and straightforward mathematical contexts, the candidate has demonstrated ability to interpret and communicate mathematical information in the form of tables, graphs and mathematical diagrams; to make sensible use of calculators and to tackle routine problems such as those involving elementary trigonometry, statistics and algebra.

Credit Level (Grades 2, 1)

In everyday and theoretical contexts, the candidate has demonstrated ability to interpret and communicate mathematical information in tabular, graphical and more abstract forms; to make sensible use of calculators and to tackle routine problems such as those involving trigonometry, statistics and algebra.

4 6 Reasoning and Enquiry – Summary GRC

Foundation Level (Grades 6, 5)

In familiar everyday contexts, the candidate has demonstrated ability to select and apply strategies such as trial and error and to decide the steps and the order of carrying them out; to explain the solution to a problem by referring to specific values in the problem; and to reason and to apply the appropriate knowledge and understanding to solve clearly defined straightforward problems.

General Level (Grades 4, 3)

In everyday and straightforward mathematical contexts, the candidate has demonstrated ability to select and apply strategies such as adopting a systematic approach and making deductions from a number of facts; to explain the solution to a problem, rejecting inappropriate results; and to reason and to apply the appropriate knowledge and understanding to solve problems.

Credit Level (Grades 2, 1)

In everyday and theoretical contexts, the candidate has demonstrated ability to select and apply strategies such as adopting an efficient approach; to argue in a logical manner; and to reason and to apply the appropriate knowledge and understanding to solve problems.

4 7 Descriptions of Grades

These describe performance within Levels. They apply to each element.

Grade 6 The candidate has met the criteria for Foundation Level, demonstrating a satisfactory overall standard of performance.

Grade 5 The candidate has met the criteria for Foundation Level, demonstrating a high overall standard of performance.

Grade 4 The candidate has met the criteria for General Level, demonstrating a satisfactory overall standard of performance.

4 7 *(continued)*

Grade 3	The candidate has met the criteria for General Level, demonstrating a high overall standard of performance.
Grade 2	The candidate has met the criteria for Credit Level, demonstrating a satisfactory overall standard of performance.
Grade 1	The candidate has met the criteria for Credit Level, demonstrating a high overall standard of performance.

4 8 Knowledge and Understanding – Extended GRC

The candidate can:		Foundation Level (Grades 6, 5)	General Level (Grades 4, 3)	Credit Level (Grades 2, 1)
K 1	interpret mathematical notation, terminology and information presented in statements, formulae and codes and follow instructions presented in the form of a structured diagram;	simple statements and formulae in words in a familiar everyday context	everyday and straightforward mathematical contexts; formulae in symbols	everyday and theoretical contexts; functional notation
K 2	extract information from tables, graphs, scale drawings, maps, mathematical diagrams;	familiar context; simple tables with up to 3 categories of data; straightforward scales on graphs; scales expressed in words	everyday context; tables with up to 5 categories of data; scales expressed as a ratio, RF or scaled line	eg graphs with misleading scales
K 3	recognise relationship between connected quantities in familiar circumstances;	eg trend in a line graph where there is one main trend	eg direct variation; trends or changes of features in graphs; know that $y = ax + b$ is the equation of a straight line and vice versa	eg inverse variation; the effect on the subject of a formula of a change in a variable; know that $y = mx + c$ is the equation of a straight line gradient m, intercept c and vice versa
K 4	read off values on scales in measuring instruments;	familiar instruments with straightforward scales	everyday instruments including those with imperial scales; able to interpolate	
K 5	recognise three-dimensional shapes from their two-dimensional representations or nets;	familiar shapes		
K 6	communicate information in tabular, graphical or diagrammatic form or by means of a formula;	the structure would have to be specified; scales and formulae in words	eg graphs where scale has to be selected	

4 8 Knowledge and Understanding – Extended GRC *(continued)*

The candidate can:		Foundation Level (Grades 6, 5)	General Level (Grades 4, 3)	Credit Level (Grades 2, 1)
K 7	use the properties of shapes to calculate angles, lengths, areas and volumes;	eg find third angle of triangle	eg solve right-angled triangle	eg solve scalene triangle
K 8	select the steps required to solve routine problems;	eg problems on earnings, saving and spending and direct proportion	eg problems on inverse proportion and direct variation	eg problems on inverse and joint variation or maximising ax + by under given constraints
K 9	solve equations;	* * *	simple equations with non-negative solutions	quadratic equations; iterative method to improve an initial approximation to a root of an equation
K 10	solve pairs of simultaneous equations;	* * *	* * *	
K 11	solve inequalities;	* * *	$ax \geq b, x \pm b \geq c$ $a, b, c \in N$ $ax \leq b, x \pm b \geq c$ $a, b, c \in N$	$ax \pm b \geq cx \pm d$ $a, b, c, d \in Q$ $ax \pm b \leq cx \pm d$ $a, b, c, d \in Q$
K 12	carry out accurately appropriate calculations in number, money and measure including the evaluation of formulae;	calculate mentally the change due in a simple shopping situation; add, subtract, multiply and divide whole numbers and decimals; calculate simple percentages and fractions of quantities; round to the nearest unit or in the case of money to the nearest penny	add and subtract integers in practical contexts; express one quantity as a percentage or fraction of another; add, subtract and multiply simple fractions; find the number of fractional parts in a mixed number; express numbers in scientific notation; round to a required number of decimal places	add, subtract, multiply and divide real numbers; divide fractions; simplify expressions involving surds; use laws of indices; round to a required number of significant figures

4 8 Knowledge and Understanding – Extended GRC *(continued)*

The candidate can:		Foundation Level (Grades 6, 5)	General Level (Grades 4, 3)	Credit Level (Grades 2, 1)
K 13	make sensible use of calculators;	the four operations and simple percentages	square roots; sine, cosine and tangent of an acute angle; acute angle given its sine, cosine or tangent	powers of numbers; sine, cosine and tangent of any angle; angles with particular sine, cosine or tangent
K 14	process data in symbolic forms;	* * *	collect like terms; remove brackets, eg 2 (x – 3); extract common factor, eg 4x + 6y	(a + b) (c + d) = ac + ad + bc + bd; $a^2 - b^2 = (a - b)(a + b)$; factorise $ax^2 + bx + c$; determine sum, difference, product, quotient of expressions of the form f(x)/g(x); change subject of formula
K 15	use simple statistics.	calculate mean and mode of a data set; calculate mode of data presented in an ungrouped frequency table	calculate median and range of a data set; calculate mean, median and range of data presented in an ungrouped frequency table: draw a best-fitting line by eye on a scattergraph and use it to estimate the value of one variable given the other; state the probability of a simple outcome interpret calculated statistics eg compare mean and range for two sets of data	calculate the quartiles and semi-interquartile range from a data set or ungrouped frequency table calculate given the formula, the standard deviation of a data set determine the equation of a best-fitting straight line (drawn by eye) on a scattergraph and use it to estimate a y-value given the x-value; find probability defined as: $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ where all the outcomes are equally likely

Description of grades are given in **4 7**.

4 9 Reasoning and Enquiry – Extended GRC

The candidate can:		Foundation Level (Grades 6, 5)	General Level (Grades 4, 3)	Credit Level (Grades 2, 1)
R 1	interpret information when presented as a collection of related statements, possibly involving tables, graphs and diagrams;	familiar everyday context	everyday and straightforward mathematical contexts possibly involving excess information or information in a random layout; graphs could display qualitative relationships	everyday and theoretical contexts
R 2	interpret the solution of a problem in the context of the problem;	* * *	eg round to nearest appropriate penny; interpret intersection of two graphs; reject inappropriate results	eg round to appropriate degree of accuracy; reject invalid solutions
R 3	interpret a mathematical model of a problem;	* * *	eg simple equation or inequality	eg polynomial, exponential or trigonometric function
R 4	select a form in which to communicate information (table, graph, diagram, formula, written statement);	guidance with structure/scales required	the form is appropriate	the form is appropriate and may be novel
R 5	express a relationship in the form of a statement or formula;	simple relationship and formula expressed in words (with guidance)	simple relationship including simple equation or inequality	relationship including system of equations or inequalities
R 6	explain the solution to a problem;	in concrete terms, ie by reference to specific values in the problem	in general terms, displaying an awareness of the overall significance of the steps	in general terms, displaying an awareness of the overall strategy used
R 7	set out the solution to a problem;	* * *	in such a way that the reader can follow the steps carried out	select relevant information and highlight what is important; explain clearly a logical line of thought

4 9 Reasoning and Enquiry – Extended GRC *(continued)*

The candidate can:		Foundation Level (Grades 6, 5)	General Level (Grades 4, 3)	Credit Level (Grades 2, 1)
R 8	communicate information qualitatively;	* * *	with guidance	
R 9	combine information presented in various ways and draw inferences where appropriate;	* * *	* * *	
R 10	recognise when to translate from one form of representation to another;	* * *	with guidance	
R 11	appreciate the need to check solutions;	eg add in different order	eg check the solution to an equation by substitution	
R 12	appreciate requirements of a proof;	* * *	* * *	illustrate results; test conjectures for further cases and prove/disprove conjectures
R 13	experiment;	try a few cases	with guidance, examine particular cases to clarify the nature of the task; in an informed way; introduce a diagram	decide to examine special cases or reduce the task to a simpler one to clarify the nature of the task, possibly by making a temporary assumption
R 14	recognise patterns;	continue simple patterns; extend simple number patterns; simple line symmetry	continue patterns; extend simple patterns; generalise some simple number patterns; recognise some general features of a pattern; line and rotational symmetry	continue complex patterns; extend patterns; use symbols to make a conjecture about the general pattern

4 9 Reasoning and Enquiry – Extended GRC *(continued)*

The candidate can:		Foundation Level (Grades 6, 5)	General Level (Grades 4, 3)	Credit Level (Grades 2, 1)
R 15	enumerate possibilities;	given some, find some more	try to get all by listing them	try to get all by findng the general pattern
R 16	reason and draw valid conclusions where appropriate;	make simple deductions from 2 or 3 given facts	make deductions from a number of facts which may first of all have to be identified	reason in a logical manner; apply sustained reasoning; work backwards; identify crucial aspects of a problem; introduce symbols: assess the significance of the results
R 17	take an organised approach.	decide the steps and their order (2 or 3); use trial and error; use trial and improvement	decide the steps and their order (4 or 5); select a systematic approach eg record results in such a way that any patterns may readily be spotted; list possibilities in such a way that none is missed and none is counted twice	decide the steps and their order; select a systematic approach, eg introduce a table to organise the facts; explore all possible routes with a view to eliminating those that do not work; select an efficient approach, eg note symmetry in results; plan ahead and minimise the number of steps; spot "blind alleys"

Description of grades are given in **4 7**.

Section 5

Content of Courses

5 Content of Courses

5 1 Approaches to Broad Areas of Content

5 1 1 Although this document is primarily concerned with S3 and S4, it is important that mathematics education in the secondary school should be seen as a continuous course from S1 to S4 for all pupils.

The broad areas of content which are considered appropriate for Standard Grade are similar to those in the 5-14 Mathematics guidelines.

It is intended to exclude from external examinations questions based specifically on set theory since this would place undue emphasis on theory to the detriment of practice. Teachers should, however, continue to use the language and ideas of sets as a framework within which to discuss other areas of mathematics.

5 1 2 Number and Money

The importance of the content in this broad area of the syllabus needs no justification here. It is concerned with fundamental aspects of numeracy with which all pupils must cope if they are to make progress in the world outside school.

The ability to carry out straightforward calculations mentally should be stressed, as this is an important skill required in everyday life. On some occasions pupils should be taught using paper and pencil only and on others sensible use of calculators should be encouraged. Calculators should not be allowed to provide unnecessary support, nor substitute for personal proficiency. Pupils should be encouraged to check answers by repeating the calculation perhaps in another way, by approximation taking numbers correct, say, to one significant figure, or by estimating the range within which the answer must lie.

Computational skills should, whenever possible, be practised in real life contexts involving quantities, measures and money. Number puzzles, which can be a motivational factor for some pupils, are an aid in developing computational skills and particular problem solving strategies such as looking for a pattern and making exhaustive listing. Number puzzles may also be used to introduce pupils to making conjectures; to symbolic representation and generalised arithmetic and, for some pupils, to proof.

All pupils should be able to use the notation for describing depths below sea level and temperatures below 0ºC and be able to calculate rises and falls in temperature. However, not all pupils need to manipulate negative integers and only some of them require to proceed to the multiplication of negative integers.

Although the decimal fraction has replaced the vulgar fraction in terms of usefulness, there is still a need for pupils to be familiar with common fractions, at least at a conversational level, and to understand the notation of vulgar fractions and the principle of equivalence. Words such as half, quarter and three-quarters remain in our language: the notation for vulgar fractions conveys more clearly than that of decimal fractions the idea of the whole being divided into a number of equal parts and the notation for ratios relies heavily on that for vulgar fractions. For many pupils the vulgar fraction can be thought of as an operator, being used in contexts such as “half of a litre” and “three-quarters of an hour”. Only for some will it be necessary to proceed to the division of vulgar fractions although the majority of pupils should be able to state, for instance, the number of quarter-metre lengths that can be cut from a piece of material 2½ metres long.

5 1 2 *(continued)*

Home and personal budgeting, leisure and recreation provide familiar and motivating contexts for pupils. Class discussion may establish that there is no single correct answer to a particular problem, as in rent/buy situations, but that calculations can be a considerable aid to sensible decision making. The syllabuses of other school subjects taken by some pupils will contain references to the skills associated with home and personal budgeting and this should be taken into account in the design of a school's mathematics syllabus. For those pupils who have not met these topics, their mathematics course should provide them with sufficient information to understand the context of the problems they tackle.

5 1 3 Measure and Shape

From prior experience, both in and out of school, pupils have a knowledge of the properties of shapes. This knowledge is primarily visual and should be developed, along with the appropriate measurement and deductive skills, so that the pupil can tackle real life practical problems with confidence. Pupils should be provided with opportunities to tackle problems from a range of real life contexts – practical activities in and around the house, at work and while at leisure – which require a grasp of spatial concepts, knowledge and ability to use skills relating to measurement and shape. Wherever possible, pupils should be encouraged to use practical materials in establishing the necessary concepts and skills.

In order to emphasise the practical nature of the topics, the titles "Measure" and "Shape" are preferred to "Geometry", which has been associated mainly with formal deductive arguments. Shapes offer a rich source for investigative work by pupils. For example, to develop the ability to draw and measure angles, pupils could be set the task of drawing a number of angles in semicircles and measuring them. Allowing for experimental error the pupils could infer that "an angle in a semicircle is a right angle" – a result which is not obvious at first sight. For some pupils the discovery would be an end in itself, whereas for others it would be the start of a further investigation into proving the result to be true for all semicircles.

Problems associated with measurement and shape can usually be tackled either in a practical or in a theoretical manner. The approximate area of a circle, for instance, could be found by "counting squares" or by using the formula, and height and distance problems could be solved by scale drawings or elementary trigonometry. The method of tackling the problem depends on the stage reached in the pupils' development or, for those pupils who are aware of a variety of methods, the choice of the most appropriate method in the prevailing circumstances.

The aspects of shape work outlined in the preceding three paragraphs indicate that certain topics in measurement and shape are ideally suited for areas of syllabus overlap. Working with shapes encourages visual thought, has a practical and an aesthetic appeal and can provide opportunities for displaying initiative and creativity and demonstrating the relevance of mathematics.

Squared paper is an invaluable aid in the early work with shapes and its use leads naturally to the introduction of a coordinate system to identify the vertices of figures. The use of coordinates is a valuable strategy to adopt when exploring further properties of shapes, eg the concurrency of the medians of a triangle, the coordinates of the points dividing each median in the ratio 2:1 being found by using proportion. Plotting points can also help identify the relationship between ordered pairs of numbers by highlighting patterns.

5 1 3 *(continued)*

Shape and size are familiar aspects of the environment and the pupils' study of them should be such as to develop an awareness of the links between the real world and mathematics. Wherever possible consideration of topics should stem from an examination of the shapes used by man in the environment. For example:

Packaging	An examination of commercial packaging could lead to questions such as "Why are most boxes cuboids?", "Why are most cans cylinders?" and develop into investigations of the nets and packing of solids and the concepts of volume and surface area.
Tiling	Investigations of tiling and patterns in and around the school (carpet tiles, paving slabs, brickwork) could raise the issue "Why are there no tiles in the shape of regular pentagons?" Examination of tessellations can also lead to consideration of parallel lines and similarity and so develop the concepts of perimeter and area.
Frameworks	Linkages and frameworks can be used to illustrate the uses made of the properties of specific shapes.
Symmetry	Rotational symmetry could be introduced through a discussion of the uses made of certain shapes. "Why is a hexagon the most common shape for a nut?" "Why is the hexagon preferred to a square, to a pentagon?"

Many pupils confuse area with perimeter and so the emphasis, initially, should be on aiding the pupil form the appropriate concept. Pupils should progress from evaluating the areas of both regular and irregular figures drawn on grids to finding the areas of figures upon which they can place or visualise a grid. Only when pupils have a firm grasp of the concept should they proceed to finding areas which require the use of a formula.

The real world is three-dimensional, and physical models are used to investigate it and to communicate information about particular aspects of simple shapes. Communication is also achieved by means of diagrams – maps, plans, instructions for assembling models or furniture. Thus pupils have to develop the ability to relate three-dimensional objects with their two-dimensional representations; for example, pupils should experience viewing familiar objects from different angles.

In order to interpret/draw/discuss scale drawings and plans, and to describe and make models pupils will need to recognise and know the names of the common shapes and their properties and will require to use in this context words such as side, corner, net, diagonal. They will need to be able to draw, measure and name lines and angles, and to enlarge and reduce diagrams. If the solution of problems in navigation/orienteering are to be achieved other than by drawing accurate diagrams and taking measurements, pupils will need to know the angle facts associated with parallel lines (eg to calculate a back bearing) and the angle sum of a triangle. An extension of mapwork and scale drawings would be the introduction of cartesian coordinates and the Theorem of Pythagoras along with the sine, cosine and tangent of acute angles in the solution of, for example, right-angled triangles. To solve problems involving finding the length of a line or size of an angle in a three-dimensional situation pupils will require to develop the ability to pick out the appropriate triangle.

5 1 3 *(continued)*

Pupils should have an appreciation of symmetry, both bilateral and rotational, and of the fact that patterns can be produced using the reflection, rotation and translation of a basic unit. The work should be of a practical nature and use should be made, for example, of mirrors, paper folding and the turning over of shapes in the examination of bilateral symmetry. The circle is an important non-rectilinear shape with many uses in everyday life and pupils should learn its basic properties by exploration.

Navigation and orienteering are not concerned merely with the position of points, but also with movement between points. This displacement can be described in terms of bearings and distance, or in terms of eastings and northings, which provide a foundation on which can be built the development of vectors, for some pupils, at a later stage.

In some contexts it will be more realistic to use contemporary imperial units so that pupils will become familiar with those imperial units that are still in common use, eg miles, to express the distance between towns. However, the emphasis in measurement activities should be on metric units and their interrelationships.

Most pupils should be aware that exact measurements of continuous quantities are not possible and that the accuracy of the measurement is dependent upon the measuring device used. They should be encouraged to expresss a measure as lying within a certain range corresponding to the numbered, or marked, divisions on the scale. If appropriate, some pupils may proceed to interpolation.

5 1 4 Relationships

The development of an understanding of algebra should be built on the pre-algebra and algebra activities in the 5-14 Mathematics programme. The best way for pupils to understand algebra is to go through the development process of carrying out experiments, spotting rules, writing them down, and then making suitable abbreviations. The ability to detect a pattern or spot a rule is a prerequisite for further development in mathematics and so should be nurtured. For some pupils the comparison of different rules which express the same relationship can lead to the need for algebraic manipulation.

Relationships can be expressed in the forms of tables, graphs and formulae expressed in words. Those pupils who can cope with abstract symbols can proceed to formulae expressed in symbols and some can carry on to study equations and functional notation.

Particular types of relationships are important and, for instance, by examining the graphs of a variety of relationships and noting similarities and differences the concept of direct (then inverse and joint) variation can be developed. In a similar manner the common features of the graphs of linear (then quadratic and trigonometric) functions can be established.

Pupils should recognise that the expression of a relationship in the form of a formula, equation or graph is not merely an aid to communicating the relationship, but also a powerful tool to use in the solution of problems.

5 1 4 *(continued)*

Some problems can be solved by pupils of a wide range of ability. For example, in the case of the well known problem of determining the dimensions of the tray of maximum capacity which can be formed by removing a square from each corner of a rectangular sheet of cardboard and folding: pupils in a Foundation course may be able to exhaust all possibilities using integral values for the length of the side of the square; pupils in a General course can extend this process by using a number of non-integral values; pupils in a Credit course should be able to produce the general mathematical model and to solve the problem graphically to any required degree of accuracy.

It is important that the level of symbolic representation and of algebraic manipulation demanded should match the level of intellectual development of individual pupils. No matter which of the three Levels a pupil is aiming for, the degree of sophistication required in the problem-solving process must be determined by the pupil's experience and by the stage of mathematical development reached.

5 1 5 Statistics

The statistics content provides continuity and progression from the Information Handling in 5-14 Mathematics, leading to more advanced study of statistical methods in Higher level and Advanced Higher level courses. Although the Content Checklist emphasises the construction and interpretation of various data displays and the calculation and interpretation of numerical summaries, it is important to recognise the variety of contexts in which these skills are acquired and used – personal, social, vocational, other school subjects and future learning.

A practical approach to learning statistics is recommended and pupils will benefit from opportunities to collect their own data. Such activities follow a simple pattern – pose a question, collect some data, analyse the data, interpret the results. This approach promotes an understanding of the inferential nature of statistics. In any application of mathematics, it is vital to distinguish between the observations we make – the data – and the mathematical explanation we give of these – the model. In a statistical investigation, the data are usually a sample, and the model describes the population from which the sample was drawn. At this stage, it is assumed that the (small) data sets met by pupils are samples.

The variability of the collected data leads to an inevitable difficulty – how sure are we that our subjective interpretation of the data is valid? Future learning in statistics will address this problem, when mathematical models which describe uncertainty are studied. At General and Credit Levels, the idea of quantifying uncertainty and the calculation of simple probabilities are introduced.

In drawing up a course it is important to try to strike a balance between the teaching of skills of immediate necessity, and those of a future, but not too distant, demand. Some topics may claim a place in the courses for the learning of more able pupils because of needs beyond S4. For example the facility in algebraic manipulation required for Higher level would be difficult to achieve if pupils had not started to develop it well before S5.

5 1 5 *(continued)*

In this context, the demands of other school subject areas (the “users” of mathematics) should be considered. For example:

- the use of rough sketch graphs and graphs with scaled axes (Geography, Biology, Chemistry, Physics, Economics);
- the use of simple formulae and possible manipulation of formulae (Physics, Technical Subjects).

5 2 Checklist of Content

5 2 1 Introduction

To assist teachers in selecting material for courses at each Level the content referred to above is presented in three lists. The content for Credit and General Level courses is to be understood to include the items of content listed under the lower Level(s).

When designing courses, contexts should be sought within which pupils can gain the necessary knowledge, form appropriate concepts and develop the essential skills to solve problems. The standards of performance required at each Level are described in the Grade Related Criteria in Section 4.

Knowledge of the mathematical vocabulary listed in 5 3 will be assumed in the external examinations.

5 2 2 Format of the Checklist

Some of the items at General and Credit Level have been italicised; for details see 3 5 4.

At the Level shown, a row of asterisks indicates that the item of content does not apply; a blank indicates that the item of content applies without further description or exemplification.

5 2 3 Content Checklist

Number

appropriate calculations involving:	Foundation Level	General Level	Credit Level
• whole numbers	addition, subtraction, multiplication, division; estimate to check calculation		
• integers	negative integers in context eg temperatures	addition, subtraction; multiplication of a single digit integer by a single digit whole number eg $3 \times (-4)$	multiplication, division
• reading/locating decimal numbers on scales	to hundredths only		
• decimals	addition, subtraction, multiplication, division; rounding to the nearest unit; estimate to check calculation	rounding to a given number of decimal places	rounding to a given number of significant figures
• fractions	unitary fraction of a quantity eg $\frac{1}{6}$ of £24	fraction of a quantity; equivalence of simple fractions eg $\frac{3}{6} = \frac{1}{2}$, $\frac{9}{12} = \frac{3}{4}$; addition, subtraction; multiplication of commonly used fractions and mixed numbers	four operators applied to fractions, including mixed numbers
• percentages	finding a whole number percentage of a quantity	finding a percentage of a quantity; expressing one quantity as a percentage of another	
• equivalence of percentages, decimals and fractions	$50\% = 0.5 = \frac{1}{2}$, $25\% = 0.25 = \frac{1}{4}$	any commonly used equivalence eg $75\% = 0.75 = \frac{3}{4}$, $40\% = 0.4 = \frac{2}{5}$	

Number *(continued)*	Foundation Level	General Level	Credit Level
• square roots	* * *		*simplification of surds,* eg $\sqrt{ab} = \sqrt{a}\sqrt{b}$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$;
• use of index notation	* * *	a^n, $n \in N$, eg find 2^5 conversion of large/small numbers to scientific notation	a^n, $n \in Q$, eg find $8^{\frac{1}{3}}$ multiplication, division of numbers in scientific notation
• ratio	* * *	splitting a quantity in a given ratio eg 3:1, 1:5	eg 3:7
• rate	eg articles produced per hour, miles per gallon		
• distance, speed, time	finding distance given speed and time, simple cases only	calculating one, given the other two	

5 2 3 Content Checklist *(continued)*

Money	Foundation Level	General Level	Credit Level
• interest	simple interest (integral rate, whole year)	simple interest (including fraction of a year)	compound interest
• calculations involving money in appropriate contexts	home and personal budgeting		depreciation/appreciation
	earnings: wage rise (as a whole number percentage); take home pay: deductions; calculate overtime (double time)	wage rise (added to initial wage); commission; calculations involving deductions; bonus; calculate overtime	
	savings: constant amount eg 6 months at £12.50 per month		
	expenditure: profit/loss in buying/ selling (structured); shopping bills; household bills eg fuel bills (500 units at 7p)	profit/loss (unstructured); insurance premiums; foreign exchange	
	borrowing: hire purchase (structured); loans	hire purchase (unstructured eg deposit plus repayments)	
	surcharges: calculate VAT (structured with a whole number percentage)	VAT (unstructured eg know to add on); discount – know to subtract	given a quantity after a percentage change calculate the original quantity eg given price including VAT, calculate basic price (work backwards)

5 2 3 **Content Checklist** *(continued)*

Measure	Foundation Level	General Level	Credit Level
• estimating length, weight, area, volume and angle			
• measuring length, volume, weight, angle, time and temperature	to a reasonable degree of accuracy	to a required degree of accuracy	to an appropriate degree of accuracy
• tolerance	* * *	interpret tolerance notation eg $(25 \pm 2)°$	
• the interrelationships among the units	10mm = 1cm, 1000mm = 1m, 100cm = 1m, 1000m = 1km, 1000g = 1kg, 1000ml = 1ℓ	1000kg = 1 tonne, $1000cm^3 = 1\ell$ conversion of units of length; conversion of cm^3 to ℓ or vice versa	conversion of units
• telling and recording the time	12-hour and 24-hour clock; the units second, minute, hour, day, week, month, year and leap year, and their interrelationships;		
• calculating time intervals	time interval within a given half day on the 12-hour clock eg 2.35 pm to 5.10 pm or within the same day on the 24-hour clock, eg 0920 to 1705	time interval over midnight or midday on the 12-hour clock; change decimal time into actual time	

5 2 3 Content Checklist *(continued)*

Shape

Scale Drawings

	Foundation Level	General Level	Credit Level
• interpreting and constructing scale drawings (including simple maps and plans)	simple scale expressed in words eg 1cm represents 10 metres	scale expressed as a ratio, scaled line or *Representative Fraction*; in construction of scale drawings the scale may or may not be given	
• enlarging and reducing figures	halving or doubling		
• using scale drawings to solve problems		work out scale factor from a diagram	

Similarity

	Foundation Level	General Level	Credit Level
• ratio of sides of similar figures	rectangles	*right-angled triangles*	*triangles* (emphasis on calculating, using ratio or scale factor; formal proof of similar triangles not required)
• ratio of areas of similar figures	* * *	* * *	
• ratio of surface areas of similar solids	* * *	* * *	
• ratio of volumes of similar solids	* * *	* * *	

Rectangular Cartesian Coordinates

	Foundation Level	General Level	Credit Level
• plotting points and determining coordinates of points	in the first quadrant; in the context of maps and plans locating "objects" on a grid	in all quadrants	
• the equation y = a x + b	* * *	construct a table of values and draw the line	sketch without drawing accurately; know that 'a' represents the gradient and 'b' the intercept on the y-axis

5 2 3 Content Checklist *(continued)*

Three-dimensional Shapes	Foundation Level	General Level	Credit Level
• recognising and naming shapes	cube, pyramid, cylinder, cuboid, cone, sphere	triangular prism	
• recognising shapes from their: two-dimensional representations, nets	familiar shapes; cube and cuboid	complex shapes; pyramid, cylinder, triangular prism	
• drawing nets of shapes	cube and cuboid		

Perimeter

• calculating perimeter of figures	rectilinear figures	circumference of a circle	length of arc of a circle

Area and Volume

• finding areas of figures	rectangle, square and right-angled triangle; irregular figures by counting squares	any triangle (given base and height), circle, *kite, rhombus, parallelogram, composite figures*	sector of a circle
• finding surface areas of solids	* * *	cube, cuboid, *cylinder, triangular prism*	composite solid
• finding volumes of solids	cube, cuboid; other solids by counting cubes	*cylinder, triangular prism*	composite solid

5 2 3 **Content Checklist** *(continued)*

Angle	Foundation Level	General Level	Credit Level
• measuring and drawing angles			
• number of degrees in a right angle, straight angle, half and full turn			
• the relationships between angles made by parallel lines and a transversal	* * *		
• vertically opposite angles the properties of shapes; triangles	* * *	sum of angles of a triangle; isosceles and equilateral triangles; the Theorem of Pythagoras	*the converse of the Theorem of Pythagoras*
quadrilaterals	side, angle and diagonal properties of square, rectangle	*kite, rhombus, parallelogram*	
circles	relationship between radius and diameter	*relationship between tangent and radius; angle in a semi-circle*	*the interdependence of the centre, bisector of a chord and a perpendicular to a chord*
• using above to calculate angles and lengths of sides of figures			

Symmetry

• recognising and drawing symmetrical figures	simple line symmetry	line symmetry; *rotational symmetry*	

5 2 3 Content Checklist *(continued)*

Trigonometry	Foundation Level	General Level	Credit Level
• 8 main compass points			
• 3-figure bearings	from a diagram read off a bearing eg a radar screen	measuring the bearing of B from A; plotting B given the bearing and distance from A; *plotting B and C given the bearing and distance of B from A and C from B;* *plotting C given its bearing from A and B*	calculating the bearing of A from B, given the bearing of B from A
• solving triangles	* * *	*right angled triangles using sine, cosine and tangent (excluding cases where the unknown in the ratio is in the denominator)*	scalene triangles; area of a triangle = $\frac{1}{2}$ab sin C; Sine Rule; Cosine Rule
• constructing geometric models of simple physical situations		gradient of a slope; *angles of elevation/depression*	
• relationships among sine, cosine and tangent functions	* * *	* * *	*know that* $\tan x° = \frac{\sin x°}{\cos x°}$ $\sin^2 x° + \cos^2 x° = 1$
• graphs of sine, cosine and tangent	* * *	* * *	periodicity, symmetry $a \sin bx° + c$, $a \cos bx° + c$, $\tan x°$ eg $3 \sin 2x°$, $2 \cos \frac{1}{2}x°$, $\sin x° + 2$
• solving trigonometric equations	* * *	* * *	eg 2 sin x° = 1 (0 ≤ × ≤ 180) 3 cos x° + 1 = 0 (0 ≤ × ≤ 360)

5 2 3 Content Checklist *(continued)*

Relationships	Foundation Level	General Level	Credit Level
• extending number patterns	simple ones eg 1, 6, 11, 16, … 1, 2, 4, 8, …	eg 1, 4, 9, 16, 25, … 1, 3, 6, 10, …	eg 1, 8, 27, 64, … 2, 5, 10, 17, …
• generalising number patterns	simple rule in words	simple rule in symbols	
• collecting like terms	* * *		
• multiplying expressions	* * *	eg $3 \times 2a = 6a$ $3(x + 2) = 3x + 6$ $5(a - 3) = 5a - 15$ $2(m + 2k) = 2m + 4k$	eg $x(x + 2y) = x^2 + 2xy$ $(a + b)(c + d) = ac + ad + bc + bd$ $(x + 2)(x^2 - 3x + 1) = x^3 - x^2 - 5x + 2$
• factorising expressions: common factor	* * *	eg $3x - 6 = 3(x - 2)$	eg $x^2 + 3x = x(x + 3)$;
difference of two squares	* * *	* * *	$x^2 - y^2 = (x - y)(x + y)$ $4x^2 - 9y^2 = (2x - 3y)(2x + 3y)$;
trinomial expression	* * *	* * *	$x^2 + x - 6 = (x + 3)(x - 2)$ $2x^2 - 5x - 3 = (2x + 1)(x - 3)$ $3x^4 + 5x^2 - 2 = (3x^2 - 1)(x^2 + 2)$
• reducing an algebraic fraction to its simplest form	* * *	* * *	eg $\frac{8a^2}{2a} = 4a$ $\frac{4a^3b^2}{2ab} = 2a^2b$ $\frac{(x+1)^2}{(x+1)^3} = \frac{1}{x+1}, x \neq -1$
• applying the four rules to algebraic fractions	* * *	* * *	eg $\frac{x}{3} + \frac{x}{4} = \frac{7x}{12}$

5 2 3 Content Checklist *(continued)*

Relationships *(continued)*	Foundation Level	General Level	Credit Level
• simplifying expressions using the laws of indices	* * *	* * *	eg $x^4 \times x^3 = x^7$ $\frac{x^7}{x^4} = x^3,\ x \neq 0$ *$\frac{x^2}{x^6} = x^{-4},\ x \neq 0$* *$(x^{\frac{1}{2}})^3 = x^{\frac{3}{2}}$*

Formulae

	Foundation Level	General Level	Credit Level
• evaluating formulae	formula expressed in words	formula expressed in symbols	
• constructing formulae to describe a relationship between/ among values in a table	in words	in symbols	
• constructing formulae to describe a relationship expressed graphically	* * *	* * *	linear relationship; *quadratic relationship;* *a^x, $a \in N$;* *trigonometric relationship eg 3sin 2x*
• changing the subject of a formula	* * *	* * *	
• describing the effect on the subject of a change in a variable	* * *	* * *	

Proportion and Variation

	Foundation Level	General Level	Credit Level
• proportion	direct	inverse	
• variation	* * *	*direct*	inverse, *joint*
• graphs of variation	* * *	*direct*	inverse

5 2 3 Content Checklist *(continued)*

Graphs and Tables	Foundation Level	General Level	Credit Level
• extracting data from pictograms, bar charts, line graphs, pie charts and scattergraphs	straightforward scales (in the case of piecharts pupils would only be expected to pick out the largest/ smallest contributor)	interpretation and use of proportionality between the size of the sector and the angle at the centre of piecharts	possibly involving misleading graphs
• interpreting data from stem-and-leaf diagrams (charts)	* * *		
• extracting data from boxplot and dotplot	* * *	* * *	
• codes	simple ones		
• constructing pictograms, bar charts, scattergraphs and line graphs from given data	given the scale and structure		
• constructing stem-and-leaf diagrams (charts)	* * *		
• constructing a pie chart, boxplot and dotplot	* * *	* * *	
• constructing a frequency table from data without class intervals	given a partially completed table		
• constructing a cumulative frequency column for an ungrouped frequency table	* * *	* * *	
• drawing a best-fitting line by eye on a scattergraph and using it to estimate the value of one variable given the other	* * *	the scattergraph should show high positive or negative correlation, ie indicate the connection between the variables	

5 2 3 **Content Checklist** *(continued)*

Graphs and Tables *(continued)*	Foundation Level	General Level	Credit Level
• significance of the point of intersection of two graphs	* * *	*graphs in context, eg speed-time graphs, car hire problem*	*determination graphically of an approximate solution of an equation; iteration to improve solution of equation (trial and improvement)*
• trends in graphs	one main trend, eg increasing or decreasing		
• communicating qualitative information graphically	* * *	* * *	
• criteria for deciding which graph form to use	* * *		
• extracting data from tables	clearly presenting 2 categories of data, eg timetables, ready reckoners, holiday brochures	up to five categories	
• constructing tables	with guidance		
• trends in tables			
• reading of structured diagrams such as flowcharts	possibly with 2 or 3 decision boxes		

5 2 3 Content Checklist *(continued)*

Use of simple statistics	Foundation Level	General Level	Credit Level
• calculating mean and mode of a data set			
• calculating median and range of a data set	* * *		
• calculating mode of data presented in an ungrouped frequency table			
• calculating mean, median and range of data presented in an ungrouped frequency table	* * *		
• interpreting calculated statistics		examples: • compare mean or range for two sets of data • compare individual data points with the mean/mode (commenting 'well above average' or 'about average')	
• calculating the quartiles and semi-interquartile range from a data set or ungrouped frequency table	* * *	* * *	
• calculating, given the formula, the standard deviation of a data set	* * *	* * *	

5 2 3 Content Checklist *(continued)*

Use of simple statistics *(continued)*	Foundation Level	General Level	Credit Level
• determining the equation of a best-fitting straight line (drawn by eye) on a scattergraph and using it to estimate a y-value given the x-value	* * *	* * *	correlation should be strong positive or negative
• knowing that probability is a measure of chance between 0 and 1	* * *	* * *	
• stating the probability of an outcome	* * *	simple outcomes such as: • a 5 from rolling a die numbered 1 to 6 • one value from a given frequency table	
• finding probability defined as: $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ where all the outcomes are equally likely	* * *	* * *	probability should be found from (experimental data) or from knowledge of the situation (eg probability: drawing an ace from a pack of cards). Students should be aware that probability from experimental data is an estimate.

5 2 3 Content Checklist *(continued)*

Equations	Foundation Level	General Level	Credit Level
• solving simple questions	* * *	eg $x + 3 = 5$ $2x = 3$ *$3x - 5 = x + 11$*	eg $\frac{2}{3}x = 7$ $x + 7 = 4x - 5$ $8x - 2(x - 3) = 5$ $\frac{5(x-2)}{3} - \frac{2(2x+3)}{4} = 1$
• solving quadratic equations	* * *	* * *	real roots eg $x^2 - x - 6 = 0$ *$6x^2 - 5x - 6 = 0$, ie non integral roots*
• solving inequalities	* * *	eg *$2x < 7$* *$2x + 1 < 10$*	eg $x + 7 < 4x - 5$
• solving simultaneous equations	* * *	* * *	

5 2 3 Content Checklist *(continued)*

Functions

Foundation Level	General Level	Credit Level
* * *	* * *	notation f(x) = a x + b, f: x → a x + b;
* * *	* * *	quadratic function; max/min turning point; axis of symmetry; sketching quadratic
* * *	* * *	*exponential function* a^x, $a \in N$
* * *	* * *	*trigonometric functions eg evaluate* $d(h) = 6 - 2\sin(30h)°$
* * *	* * *	$f(x) = \frac{a}{x}$.

5 2 4 Non-calculator numerical skills

The following numerical skills may be assessed in the non-calculator paper. These skills may be assessed within a context that requires the knowledge of facts, rules or routine algorithms. For example:

(i) the ability to multiply two whole numbers may be required as part of an area of a rectangle question;
(ii) the ability to multiply/divide decimal numbers may be required as part of a scientific notation question;
(iii) the ability to multiply/divide whole numbers may be required as part of a surd question.

	Foundation Level	General Level	Credit Level
Whole numbers	addition, subtraction (up to 4 digits, eg 382 + 739 + 115; 8370 – 592); multiplication, division (for 4 digit numbers by single digit numbers, eg 1025 × 8)	multiplication (2 digit by 2 digit numbers for simple cases, eg 38 × 11)	
Decimals	1 add or subtract numbers given to at most 2 decimal places 2 multiply or divide a number given to at most 2 decimal places by a single digit whole number 3 multiply or divide numbers given to at most 2 decimal places by 10 or 100	1 add or subtract numbers given to at most 3 decimal places 2 multiply or divide a number given to at most 3 decimal places by single digit whole number 3 multiply or divide numbers given to at most 3 decimal places by multiples of 10, 100, 1000	

5 2 4 Non-calculator numerical skills *(continued)*

<table>
<tr><th></th><th>Foundation Level</th><th>General Level</th><th>Credit Level</th></tr>
<tr><td>Fractions</td><td>unitary fraction of a quantity,
eg $\frac{1}{6}$ of £24</td><td>simple fractions of a quantity
eg $\frac{2}{3}$ of 18;
addition, subtraction, eg $\frac{1}{2} + \frac{1}{4}$; $\frac{5}{8} - \frac{1}{8}$;
multiplication of simple commonly used fractions and mixed numbers
eg $\frac{1}{2} + \frac{1}{3}$, $4 \times 8\frac{1}{2}$</td><td>four operators applied to fractions including mixed numbers, eg<table>
<tr><td>a</td><td>$\frac{1}{7}$</td><td>$\frac{4}{5}$</td><td>$2\frac{1}{5}$</td><td></td></tr>
<tr><td>b</td><td>$\frac{3}{7}$</td><td>$\frac{1}{5}$</td><td>$1\frac{3}{7}$</td><td></td></tr>
<tr><td>a ± b</td><td></td><td></td><td></td><td></td></tr>
<tr><td>a × b</td><td></td><td></td><td></td><td></td></tr>
<tr><td>a ÷ b</td><td></td><td></td><td></td><td></td></tr>
</table></td></tr>
<tr><td>Percentages</td><td>find simple percentages of quantities,
eg 10% of £140
other simple percentages are:
20%, 25%, $33\frac{1}{3}$%, 50%</td><td>find commonly used whole number percentages of numbers and quantities,
eg 70% of 8 kg; 5% of £3.20</td><td></td></tr>
<tr><td>Integers</td><td>use of negative numbers in context eg temperatures</td><td>add or subtract integers;
multiply a single digit integer by a single digit whole number eg 3 × (–4)</td><td>multiply or divide integers</td></tr>
</table>

5 3 Mathematical Vocabulary

The following terms may be used in external examinations without explanation.

(NB Adjectives and/or plurals associated with any of the above will be assumed, eg cylinder, cylindrical; index, indices.)

Number

Foundation Level	General Level	Credit Level
add	calculate	cube root
annual	consecutive	decimal fraction
approximate	decimal places, correct to _	denominator
average	difference	exact value
correct to the nearest	digit	inverse
decimal	factor	irrational (number)
divide	index	iteration
dozen	integer	non-zero
equals, equal to	inverse	nth root
estimate	multiple	numerator
even (number)	negative integer	numerically
figures, write in _	number line	rational (number)
fraction	per annum	real (number)
greater than	positive integer	significant figures, correct to _
less than	power, raised to the _	surd
more than	prime number	vulgar fraction
multiply	product	
odd (number)	quotient	
per	ratio	
percentage	scientific notation	
rate	square root	
remainder	sum	
rounding off		
square		
subtract		
total		
whole number		

5 3 Mathematical Vocabulary *(continued)*

Money

Foundation Level	General Level	Credit Level
basic (rate/wage)	exchange rates	amount
benefit	expenditure	appreciation
bill	premium (insurance)	compound interest
bonus	principal	depreciation
commission	simple interest	
credit card		
deductions (eg from income)		
deposit		
discount		
double time		
down payment		
gross wage		
hire purchase		
income tax		
instalment		
interest		
invoice		
loss		
mail order		
monthly payment		
national insurance		
net wage		
overtime		
pension		
profit		
receipt		
rent		
salary		
savings		
take home pay		
time and a half		
union dues		
unit (eg electricity)		
VAT		

5 3 **Mathematical Vocabulary** *(continued)*

Shape

Foundation Level	General Level	Credit Level
acute	adjacent	arc
angle	alternate (angles)	chord
area	altitude	collinear
arm of an angle	angle of depression	concurrent
axis	angle of elevation	cross-sectional area
axis of symmetry	base	ellipse
bearing	bilateral symmetry	intercept
breadth	bisect	median
centre	centre of symmetry	parabola
circle	complement	perpendicular bisector
circumference	complementary	quadrant
complete turn	congruent	scalene
cone	coordinates	sector
corner	corresponding angles	segment (of a circle)
cube, cuboid	cosine	subtend
cylinder	equilateral	three-dimensional
degree	gradient	touch
depth	half-turn	two-dimensional
diagonal	hexagon	
diameter	hypotenuse	
direction	intersect	
East	isosceles	
edge	kite	
enlarge	obtuse	
face	opposite angles	
height	origin	
horizontal	parallelogram	
length	pentagon	
line	perpendicular	
net	plane	
North	(regular) polygon	
parallel	prism	
perimeter	quadrilateral	
plan	reflection	
plot	reflex angle	
point	representative fraction	
pyramid	rhombus	
radius	rotation	
rectangle	rotational symmetry	
reduce	scale factor	
revolution	segment (of a line)	
right angle	semicircle	
scale (drawing)	similar	
side	sine	
sketch	supplementary	
solid	surface area	

5 3 Mathematical Vocabulary *(continued)*

Shape *(continued)*

Foundation Level	General Level	Credit Level
South	tangent (to a circle)	
sphere	tangent (trigonometric ratio)	
square	trapezium	
straight	triangular prism	
symmetry	vertically opposite angles	
tiling		
triangle		
vertical		
volume		
West		
width		

5 3 Mathematical Vocabulary *(continued)*

Relationships

Foundation Level	General Level	Credit Level
bar chart/graph	average speed	array
colde	direct variation	boxplot
column	varies (directly) as	conclusion
direct proportion	equation	conjecture
find the value of	generalise	constant
flowchart	inequality	converse
formula	inverse proportion	counter-example
frequency table	law (of variation)	deduce, deduction
graph	linear	disprove
key	median	domain
line graph	nth term	dotplot
mean	probability	evaluate
mode	range	exponent
pictogram	relationship	exponential
pictograph	sequence	express
pie chart	simplify	expression
ready reckoner	solution	factor
row	solve	factorise
rule	stem-and-leaf diagram	formulate
scale	(chart)	function
scattergraph	survey (statistical)	identity
trend		inverse variation,
		varies inversely as
		joint variation,
		varies jointly as
		maximum
		minimum
		period
		polynomial
		proof, prove
		quadratic
		quartiles
		range
		reduce (to simplest form)
		root
		semi-quartile range
		simultaneous
		subject (of a formula)
		substitute
		standard deviation
		theorem
		trigonometric function
		variable

5 3 Mathematical Vocabulary *(continued)*

The following units of measurement and their abbreviations are assumed. Familiarity with the **language** of the more common Imperial measurements is assumed.

	Foundation Level	General Level
length:	millimetre centimetre metre kilometre	
area:	square millimetre square centimetre square metre square kilometre	hectare
volume:	cubic centimere cubic metre millilitre litre	centilitre
mass:	gram kilogram tonne	
time:	second minute hour day week month year leap year	
speed:	metres per second kilometres per hour miles per hour	
temperature:	Celsius Fahrenheit	