

MATHEMATICS
Higher

Fourth edition – published December 1999

**NOTE OF CHANGES TO ARRANGEMENTS
FOURTH EDITION ON CD-ROM PUBLISHED DECEMBER 1999**

COURSE TITLE: Mathematics (Higher)

COURSE NUMBER: C056 12

National Course Specification

Course Details: Core skills statements expanded

National Unit Specification:

All Units: Core skills statements expanded

National Course Specification

MATHEMATICS (HIGHER)

COURSE NUMBER C056 12

COURSE STRUCTURE

This course has two mandatory and one optional units as follows:

Mandatory units

<i>D321 12</i>	<i>Mathematics 1 (H)</i>	<i>1 credit (40 hours)</i>
<i>D322 12</i>	<i>Mathematics 2 (H)</i>	<i>1 credit (40 hours)</i>

These units should be taken in the order above, followed by whichever optional unit is chosen.

Optional units – choose one from:

<i>D323 12</i>	<i>Mathematics 3 (H)</i>	<i>1 credit (40 hours)</i>
<i>D325 12</i>	<i>Statistics (H)</i>	<i>1 credit (40 hours)</i>

In common with all courses, this course includes 40 hours over and above the 120 hours for the component units. This may be used for induction, extending the range of learning and teaching approaches, support, consolidation, integration of learning and preparation for external assessment. This time is an important element of the course and advice on its use is included in the course details.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained one of the following:

- Standard Grade Mathematics Credit award
- Intermediate 2 Mathematics or its component units including *Mathematics 3 (Int 2)*
- equivalent.

Administrative Information

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Additional copies of this course specification (including unit specifications) can be purchased from the Scottish Qualifications Authority for £7.50. **Note:** Unit specifications can be purchased individually for £2.50 (minimum order £5).

National Course Specification: general information (cont)

COURSE Mathematics (Higher)

CORE SKILLS

This course gives automatic certification of the following:

Complete core skills for the course	Numeracy	H
Additional core skills components for the unit	Critical Thinking	H

For information about the automatic certification of core skills for any individual unit in this course, please refer to the general information section at the beginning of the unit.

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Course Specification: course details

COURSE Mathematics (Higher)

RATIONALE

As with all mathematics courses, Higher Mathematics aims to build upon and extend candidates' mathematical skills, knowledge and understanding in a way that recognises problem solving as an essential skill and enables them to integrate their knowledge of different aspects of the subject.

Because of the importance of these features, the grade descriptions for mathematics emphasise the need for candidates to undertake extended thinking and decision making to solve problems and integrate mathematical knowledge. The use of coursework tasks and statistical assignments to achieve the course grade descriptions in problem solving is encouraged.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates who complete a Higher Mathematics course successfully are expected to have a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within the course, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

Where appropriate, mathematics should be developed in context and the use of mathematical techniques should be applied in social and vocational contexts related to likely progression routes. In particular, the course which contains *Statistics (H)* aims to provide for candidates whose interest lies in areas of work and study where statistical concepts and techniques are important.

The Higher Mathematics course has the particular objective of meeting the needs of candidates at a stage of their education where career aspirations are particularly important. The course has obvious relevance for candidates with interests in fields such as commerce, engineering and science where the mathematics learned will be put to direct use. For other candidates, the course can be an important component of a group award or used to gain entry to a Higher Education institution. All candidates taking the Higher Mathematics course, whatever their career aspirations, should acquire an enhanced awareness of the importance of mathematics to technology and to society in general.

COURSE CONTENT

The syllabus is designed to build upon prior learning in the areas of algebra, geometry, trigonometry and statistics, and to introduce candidates to elementary calculus.

Mathematics 1 (H), *Mathematics 2 (H)* and *Mathematics 3 (H)* are progressive units, each unit covering the range of mathematical topics described above. *Statistics (H)* content includes the following topics: exploratory data analysis, probability and regression analysis.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

The outcomes and the performance criteria for each unit are statements of basic competence. Additionally, the course makes demands over and above the requirements of individual units. Candidates should be able to integrate their knowledge across the component units of the course. Some of the 40 hours of flexibility time should be used to ensure that candidates satisfy the grade descriptions for mathematics courses which involve solving problems and which require more extended thinking and decision making. Candidates should be exposed to coursework tasks which require them to interpret problems, select appropriate strategies, come to conclusions and communicate intelligibly. Candidates completing the course which contains *Statistics (H)* should be given the opportunity to complete a short statistical assignment which will require them to collect (generate) the relevant data, analyse the data and interpret and communicate the conclusion.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged, throughout the course, to make efficient use of the arithmetical, mathematical, statistical and graphical facilities on calculators. At the same time, candidates should also be aware of the limitations of the technology and of the importance of always applying the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, there should be evidence of a checking procedure throughout the mathematical or statistical process. There are various checking procedures which could be used:

- relating to a context – ‘How sensible is my answer?’
- estimate followed by a repeated calculation
- calculation in a different order.

It is expected that candidates will be able to demonstrate attainment in the algebraic, trigonometric and calculus content of the course without the use of computer software or sophisticated calculators.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

Detailed content

The content listed below should be covered in teaching the course. All of this content will be subject to sampling in the external assessment. Part of this assessment will be carried out in a question paper where a calculator will not be allowed. Any of the topics may be sampled in this part of the assessment. The external assessment will also assess problem solving skills, see the grade descriptions on pages 25 and 26. Where comment is offered, this is intended to help in the effective teaching of the course.

References shown in this style of print indicate the depth of treatment appropriate to Grades A and B.

CONTENT	COMMENT
<p>Symbols, terms and sets the symbols: \in, \notin, $\{ \}$</p> <p>the terms: set, subset, empty set, member, element</p> <p>the conventions for representing sets, namely: N, the set of natural numbers, $\{1, 2, 3, \dots\}$ W, the set of whole numbers, $\{0, 1, 2, 3, \dots\}$ Z, the set of integers Q, the set of rational numbers R, the set of real numbers</p> <p>The content listed above is not examinable but candidates are expected to be able to understand its use.</p>	

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>Mathematics 1 (H)</p> <p>Properties of the straight line</p> <p>know that the gradient of the line determined by the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$ and that the distance between the points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p> <p>know that the gradient of a straight line is the tangent of the angle made by the line and the positive direction of the x-axis (scales being equal)</p> <p>recognise the term locus</p> <p>know that the equation of a straight line is of the form $ax + by + c = 0$, and conversely (a, b not both zero)</p> <p>know that the line through (x_1, y_1) with gradient m has equation $y - y_1 = m(x - x_1)$</p> <p>determine the equation of a straight line given two points on the line or one point and the gradient</p> <p>know that the gradients of parallel lines are equal</p> <p>know that the lines with non-zero gradients m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$</p>	

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>solve mathematical problems involving the above properties of straight lines</p> <p>know the concurrency properties of medians, altitudes, bisectors of the angles and perpendicular bisectors of the sides of a triangle</p> <p>Functions and graphs</p> <p>know the meaning of the terms: domain and range of a function, inverse of a function and composite function</p> <p>know the meaning of the terms amplitude and period</p> <p>be aware of the general features of the graphs of $f: x \rightarrow \sin(ax + b)$, $x \rightarrow \cos(ax + b)$ for suitable constants a, b</p> <p>given the graph of $f(x)$ draw the graphs of related functions, where $f(x)$ is a simple polynomial or trigonometric function</p>	<p>Candidates should meet all of these concurrency facts, but they could be covered by particular numerical cases only, using plane coordinate methods. However, they can also be established by vector methods and it is possible to explore this work through an investigative approach using an interactive geometry package.</p> <p>The main use of ‘inverse’ is to obtain the logarithmic function from the exponential function. An extended treatment is not required.</p> <p>Candidates should be fluent in associating functions with their graphs and vice versa: simple polynomial functions and trigonometric functions.</p> <p>eg $y = -f(x), f(x) + 2, 3f(x), f(x + 3), f(x - 1), f'(x)$.</p> <p>eg $y = 3f(x) + 2, f(3x + 2), -3f(x), 3f(x + \frac{\pi}{2})$. [A/B]</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>know the general features of the graphs of the functions:</p> <p>$f: x \rightarrow a^x$ ($a > 1$ and $0 < a < 1$, $x \in \mathbf{R}$)</p> <p>$f: x \rightarrow \log_a x$ ($a > 1$, $x > 0$)</p> <p>use the notation $f(g(x))$ for a composite function and find composite functions of the form $f(g(x))$, given $f(x)$ and $g(x)$</p> <p>recognise the probable form of a function from its graph</p> <p>complete the square in a quadratic of the form $x^2 + px + q$</p> <p>complete the square in a quadratic of the form $ax^2 + bx + c$ [A/B]</p> <p>interpret formulae and equations</p>	<p>Candidates should be fluent in associating functions with their graphs and vice versa: logarithmic and exponential functions.</p> <p>Candidates should recognise that $(\frac{1}{2})^x = 2^{-x}$.</p> <p>Graphs of $y = a^x$ for $a = 2, 3$, etc., and for $a = \frac{1}{2}, \frac{1}{3}$, etc. should be explored, and the inverse function of $f: x \rightarrow a^x$ ($a > 1$, $x \in \mathbf{R}$) should be investigated.</p> <p>Inverse should be used to obtain the logarithmic function from the exponential function.</p> <p>Note: notation $f \circ g$ is not required.</p> <p>Candidates should be able to interpret a diagram containing the sketches of two (or possibly more) relationships, and sketch and annotate the graphs of functions once the critical features, such as points where the graph cuts the coordinate axes and axes of symmetry, as appropriate, have been identified.</p> <p>The completed form would be $(x + a)^2 + b$ where a is an integer.</p> <p>eg $2x^2 - x + 1$. [A/B]</p> <p>eg $y = 3(x - 1)^2 + 2$ has a minimum value when $x = 1$; $y = 3 \cos x + 2$ has maximum value 5.</p> <p>eg $y = (1 - \sin x)^2 + 2$ has minimum value 2; $y = -2 - 3(2x - 1)^2$ has maximum value -2 when $x = \frac{1}{2}$. [A/B]</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>know that π radians = 180°</p> <p>know the exact values of sin/cos $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ radians and tan $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ radians</p> <p>Basic differentiation know the meaning of the terms limit, differentiable at a point, differentiate, derivative, differentiable over an interval, derived function</p> <p>use the notation $f'(x) \frac{dy}{dx}$ and for a derivative</p> <p>know that $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$</p> <p>know that: if $f(x) = x^n$, then $f'(x) = nx^{n-1}$, $n \in \mathbb{Q}$ if $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$ if $f(x) = kg(x)$, then $f'(x) = kg'(x)$ where k is a constant</p>	<p>Introduce within a context, such as investigating gradients of tangents at points on a curve. Gradients of a sequence of chords could be found numerically using a calculator. Opportunities for a cooperative approach should be taken.</p> <p>This could be deferred until rules have been consolidated in examples and applications. Note: differentiation from first principles is not included.</p> <p>eg Differentiate, with respect to x, expressions such as $2x^3 + 1 + \frac{1}{x^2}$.</p> <p>Candidates would be expected to simplify product and rational expressions prior to differentiation, eg $\frac{x^3 + x - 1}{x^2}$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>know the meaning of the terms rate of change, average gradient, strictly increasing, strictly decreasing, stationary point (value), maximum turning point (value), minimum turning point (value), horizontal point of inflexion</p> <p>know that $f'(a)$ is the rate of change of f at a</p> <p>know that $f'(a)$ is the gradient of the tangent to the curve $y = f(x)$ at $x = a$</p> <p>know that the gradient of the curve $y = f(x)$ at any point on the curve is the gradient of the tangent at that point</p> <p>find the gradient of the tangent to a curve $y = f(x)$ at $x = a$</p> <p>find the points on a curve at which the gradient has a given value</p> <p>know and apply the fact that:</p> <p>if $f'(x) > 0$ in a given interval then the function f is strictly increasing in that interval</p> <p>if $f'(x) < 0$ in a given interval then the function f is strictly decreasing in that interval</p> <p>if $f'(a) = 0$, then the function f has a stationary value at $x = a$</p>	<p>Rates of change (equations of motion, for example) are often expressed with respect to time. Work should include other rates, eg rate of change of volume of a sphere with respect to radius.</p> <p>Many relationships in science are expressed in terms of rates of change. Candidates should be able to translate from words to symbols, and vice versa.</p> <p>It is important that the recognised convention for scientific symbols is followed to avoid confusion when candidates meet symbols in other subject areas, eg u (in lower case) for distance and v for speed and velocity.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>find the stationary point(s) (values) on a curve and determine their nature using differentiation</p> <p>sketch a curve with given equation by finding stationary point(s) and their nature, intersections with the axes, behaviour of y for large positive and negative values of x</p> <p>determine the greatest/least values of a function on a given interval</p> <p>solve optimisation problems using calculus</p> <p>Recurrence relations know the meaning of the terms: sequence, nth term, limit as n tends to infinity</p> <p>use the notation u_n for the nth term of a sequence</p> <p>define and interpret a recurrence relation of the form $u_{n+1} = mu_n + c$ (m, c constants) in a mathematical model</p> <p>know the condition for the limit of the sequence resulting from a recurrence relation to exist</p> <p>find (where possible) and interpret the limit of the sequence resulting from a recurrence relation in a mathematical model</p>	<p>The most common method would be a nature table involving f'.</p> <p>eg Given $y = f(x)$ on an interval, differentiate and find values of end points, hence deduce maximum value.</p> <p>Candidates should experience a variety of mathematical models of situations involving recurrence relations.</p> <p>'Series' is not included, but could be approached using sequence of partial sums.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>Mathematics 2 (H)</p> <p>Factor/Remainder Theorem and quadratic theory use the Remainder Theorem to determine the remainder on dividing a polynomial $f(x)$ by $x - h$</p> <p>determine the roots of a polynomial equation</p> <p>use the Factor Theorem to determine the factors of a polynomial</p> <p>know that the roots of $ax^2 + bx + c = 0$, $a \neq 0$, are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p> <p>know that the discriminant of $ax^2 + bx + c$ is $b^2 - 4ac$</p> <p>use the discriminant to: determine whether or not the roots of a quadratic equation are real, and, if real, whether equal or unequal</p> <p>find the condition that the roots of a quadratic equation are real, and, if real, whether equal or unequal</p>	<p>ie Remainder is $f(h)$.</p> <p>ie If $f(h) = 0$ then $x - h$ is a factor of the polynomial $f(x)$ and conversely.</p> <p>Polynomial $f(x)$ is of the third or higher degree, where $f(x)$ can be expressed as a product of factors of which at most one is quadratic and the remainder are linear.</p> <p>eg $f(x) = (x - 2)(x - 3)(2x + 1)(x - 1)$. ie no need to consider $h \in \mathbb{Q}$ for factor $x - h$.</p> <p>eg $f(x) = (2x - 1)(3x + 2)(2x - 5)$. [A/B]</p> <p>Note: distinction between rational and irrational is not included.</p> <p>This condition is to be applied in algebraic contexts when the equation is presented in a standard form, and in familiar geometric contexts.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>know the condition for tangency, intersection of a straight line and a parabola</p> <p>solve quadratic inequalities, $ax^2 + bx + c \geq 0$ (or ≤ 0)</p> <p>determine a quadratic equation with given roots</p> <p>prove that an equation has a root between two given values and find that root to a required degree of accuracy</p> <p>Basic Integration</p> <p>know the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve</p> <p>know that if $f(x) = F'(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$ and $\int f(x) dx = F(x) + C$</p> <p>where C is the constant of integration</p>	<p>eg For what values of p does the equation $x^2 - 2x + p = 0$ have real roots?</p> <p>eg If $\frac{(x-2)^2}{x^2+2} = k$, $k \in \mathbf{R}$, find values of k such that the given equation has two equal roots. [A/B]</p> <p>The properties of the discriminant will also be required for the intersection of straight lines and circles, tangents to circles.</p> <p>eg From the graph of $y = x^2 + x - 2$, solve $x^2 + x - 2 > 0$.</p> <p>eg Find the real values of x satisfying $x^2 + x - 2 > 0$. [A/B]</p> <p>The formulae for the product and the sum of the roots are not included.</p> <p>Introduce in a context such as investigating areas beneath curves. Calculators could be used to determine approximations to areas. Conjectures made about general results should be investigated. Computer packages are available to assist with this approach.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>integrate functions defined by $f(x) = px^n$ for all rational n, except $n = -1$ and the sum or difference of such functions</p> <p>evaluate definite integrals</p> <p>determine the area bounded by the curve $y = f(x)$, the lines $x = a$, $x = b$ and the x-axis using integration</p> <p>determine the area bounded by two curves</p> <p>solve equations of the form $\frac{dy}{dx} = f(x)$ for suitable $f(x)$</p>	<p>eg integrate, with respect to x, $x^3 + 1 + \frac{1}{x^2}$.</p> <p>Candidates would be expected to simplify product and rational expressions prior to integration, eg $\frac{x^3 + x^2 - 1}{x^2}$.</p> <p>ie Area = $\int_a^b f(x)dx$ if $f(x) \geq 0$ for $a \leq x \leq b$ and area = $-\int_a^b f(x)dx$ if $f(x) \leq 0$ for $a \leq x \leq b$.</p> <p>ie If $f(x) \geq g(x)$ for $a \leq x \leq b$, then the area enclosed by the curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is $\int_a^b (f(x) - g(x))dx$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>Trigonometric formulae solve trigonometric equations in a given interval</p> <p>know and apply the addition formulae and double angle formulae</p> $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\sin 2A = 2 \sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$ <p>apply trigonometric formulae in the solution of geometric problems</p> <p>solve trigonometric equations involving addition formulae and double angle formulae</p>	<p>Solution of trigonometric equations should be introduced graphically. eg $2 \cos 2x = 1, 0 \leq x \leq \pi$. eg $\cos^2 2x = 1, 3 \sin^2 x + 7 \sin x - 6 = 0, 0 \leq x \leq 2\pi$. [A/B]</p> <p>Standard formulae will be given. eg $\sin(P + 2Q) = \sin P \cos 2Q + \cos P \sin 2Q, \sin 2P = 2 \sin P \cos P$. eg $\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$. [A/B]</p> <p>Applications include solution of triangles and three-dimensional situations, such as calculating the size of the angle between a line and a plane or between two planes. Work should extend to problems involving compound angles.</p> <p>Candidates should be given the opportunity to find the general solution to trigonometric equations, although in assessments solutions would be on a given interval, eg $0 \leq \theta \leq 2\pi$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>The equation of the circle know that the equation of the circle centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$</p> <p>know that the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$ provided $g^2 + f^2 - c > 0$</p> <p>determine the equation of a circle</p> <p>solve mathematical problems involving the intersection of a straight line and a circle, a tangent to a circle</p> <p>determine whether two circles touch each other</p>	<p>ie Determine the points at which a given line intersects a given circle, determine whether a given line is a tangent to a given circle, eg Show that the line with equation $y = 2x - 10$ is a tangent to the circle $x^2 + y^2 - 4x + 2y = 0$ and state the coordinates of the point of contact.</p> <p>eg The line with equation $x - 3y = k$ is a tangent to the circle $x^2 + y^2 - 6x + 8y + 15 = 0$. Find the two possible values of k. [A/B]</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>Mathematics 3 (H)</p> <p>Vectors in three dimensions</p> <p>know the terms: vector, magnitude (length), direction, scalar multiple, position vector, unit vector, directed line segment, component, scalar product</p> <p>know the properties of vector addition and multiplication of a vector by a scalar</p> <p>determine the distance between two points in three dimensional space</p> <p>know and apply the equality fact $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \Rightarrow a = d, b = e, c = f^*$</p> <p>know and apply the fact that if \mathbf{u} and \mathbf{v} are vectors that can be represented by parallel lines then $\mathbf{u} = k\mathbf{v}$ where k is a constant and the converse</p> <p>know and apply the fact that if A, P and B are collinear points such that $\frac{AP}{PB} = \frac{m}{n}$ then $\vec{AP} = \frac{m}{n}\vec{PB}$</p> <p>determine whether three points with given coordinates are collinear</p> <p>know and apply the basis vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}^*$</p>	<p>Suitable contexts for vectors could be forces and velocity.</p> <p>Triangular concurrency facts can be established by vector methods. Some candidates should be encouraged to explore this work through an investigative approach, which would lead to simple vector proofs.</p> <p>An understanding of both two and three dimensional vectors is expected. The items marked by an asterisk * are quoted for three dimensions but the two dimensional cases are also included.</p> <p>eg Determine the coordinates or position vector of the point which divides the join of two given points in a given ratio.</p> <p>The section formula may be used to find the position vector of P but is not required.</p> <p>Understanding of the concepts should be reinforced by concentrating on numerical examples.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>know the scalar product facts:</p> $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta$ $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3, \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}^*$ $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ <p>determine whether or not two vectors, in component form, are perpendicular</p> <p>use scalar product to find the angle between two directed line segments</p> <p>Further differentiation and integration know and apply the facts that: if $f(x) = \sin x$, then $f'(x) = \cos x$ and $\int \cos x \, dx = \sin x + C$ if $f(x) = \cos x$, then $f'(x) = -\sin x$ and $\int \sin x \, dx = -\cos x + C$ if $f(x) = g(h(x))$, then $f'(x) = g'(h(x)) \cdot h'(x)$</p>	<p>eg If $\mathbf{a} , \mathbf{b} \neq 0$ then $\mathbf{a} \cdot \mathbf{b} = 0$ if and only if the directions of \mathbf{a} and \mathbf{b} are at right angles.</p> <p>The chain rule could be introduced after candidates are fluent in differentiating simple functions; conjectures about derivatives of $(x + 3)^2$, $(x + 3)^3$, $(2x + 3)^2$, $(2x + 3)^3$, etc, could be made and checked. Candidates should be familiar with other forms of the chain rule such as</p> $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>integrate functions defined by $f(x) = (px + q)^n$ for all rational n, except $n = -1$, $f(x) = p \cos(qx + r)$ and $f(x) = p \sin(qx + r)$ and the sum or difference of such functions, where p, q and r are constants</p> <p>Logarithmic and exponential functions know that $a^y = x \Leftrightarrow \log_a x = y$ ($a > 1, x > 0$)</p> <p>know the laws of logarithms: $\log_a 1 = 0$ $\log_a a = 1$ $\log_a(bc) = \log_a b + \log_a c$ $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$ $\log_a(b^n) = n \log_a b$</p> <p>simplify numerical expressions using the laws of logarithms</p> <p>solve simple logarithmic and exponential equations</p> <p>solve for a and b equations of the following forms, given two pairs of corresponding values of x and y: $\log y = a \log x + b$, $y = ax^b$, $y = ab^x$ [A/B]</p>	<p>eg Differentiate $(x + 4)^{1/2}$, $(x - 2)^{-3}$. eg Differentiate $(2x + 5)^3$, $\sin 3x$, $\cos^3 x$. [A/B]</p> <p>eg Integrate $(x + 4)^{1/2}$, $3 \cos x$. eg Integrate $(3x + 1)^3$, $\sin 2x$. [A/B]</p> <p>Change of base is not included and when base is understood in a particular context, $\log x$ can be used for $\log_a x$. The base a will normally be 10 or e and the notation $\ln x$ should be introduced to candidates.</p> <p>Numerical uses of logarithms and exponential evaluations are best done by calculators.</p> <p>eg $5 \log_8 2 + \log_8 4 - \log_8 16$.</p> <p>eg $\ln x = 2$, $\log_{10} x = 1.7$, $10^x = 2.5$ eg $3.4^x = 5$ [A/B]</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>use a straight line graph to confirm a relationship of the form $y = ax^b$, also $y = ab^x$ [A/B]</p> <p>model mathematically situations involving the logarithmic or exponential function [A/B]</p> <p>Further trigonometric relationships express $a \cos \theta + b \sin \theta$ in the form $r \cos(\theta \pm \alpha)$ or $r \sin(\theta \pm \alpha)$</p> <p>solve, by expressing in one of the forms above, equations of the form $a \cos \theta + b \sin \theta = c$</p> <p>find maximum and minimum values of expressions of the form $a \cos \theta + b \sin \theta$ find corresponding values of θ [A/B]</p>	<p>eg From experimental data draw a graph of $\log y$ against $\log x$ and deduce values of a and b such that $y = ax^b$. [A/B]</p> <p>Candidates should be encouraged to show all intermediate working, eg the calculation of r and α, the expansion of trigonometric formulae and the equating of coefficients.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>Statistics (H) Statistical content from Mathematics 2 (Int 2) is underlined and should be integrated into the teaching of this unit, where necessary. This content is required for progression purposes and will be assessed only as part of the overall statistical content.</p> <p>Exploratory Data Analysis <u>calculate the mean, median, mode, range and standard deviation of a data set</u> <u>construct a boxplot</u> from a stem-and-leaf diagram or dotplot</p> <p>interpret a statistical diagram (boxplot, stem-and-leaf diagram or dotplot) in terms of central location (median, mode, quartiles) and variability (range and semi-interquartile range)</p>	<p>Exploratory Data Analysis (EDA) is a method of achieving a quick and simple analysis of a sample data set before further investigations and possible hypothesis testing. Candidates should understand the inferential nature of EDA. The ideas of population, sample, parameter and estimate should be made clear. An EDA is carried out on a sample from a population. What interpretation can be made in the context of the parent population?</p> <p>Candidates could be given different samples from the same population in order to help to illustrate the nature of the subject. They should be introduced to statistical notation as and when appropriate. It would be desirable at this time to introduce \bar{x}, s, μ and σ.</p> <p>Candidates should be encouraged to gather their own data and to use statistical calculators and software in order to generate boxplots of different samples for discussion and interpretation.</p> <p>Sources of data may include the media, other curricular areas, vocational and social contexts and statistical publications.</p> <p>Candidates should appreciate the distinction between discrete and continuous data and between qualitative and quantitative data.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>identify possible outliers and suggest possible action [A/B]</p> <p>Discrete probability distributions know the meaning of the terms <u>probability</u>, outcome and event</p> <p><u>know that probability is a measure of chance between 0 and 1</u></p> <p>use probability <u>defined</u> as: $\frac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$</p> <p><u>where all the outcomes are equally likely</u> or by the limit of the relative frequency of the event</p> <p>know the meaning of nC_r and its use in counting outcomes including the notation $\binom{n}{r}$ [A/B]</p> <p>know and understand the following:</p> <ul style="list-style-type: none"> the addition law for mutually exclusive events the multiplication law for independent events replacement and non-replacement situations <p>the meaning of the terms discrete random variable, sample space, probability function and probability distribution</p>	<p>The definition of an outlier, in common use, is any value outside the range $Q_1 - 1.5q$ to $Q_3 + 1.5q$, where q is the interquartile range. The normal action would be to go back to the original data source and investigate.</p> <p>Probability theory is introduced together with the idea of randomness and simulation.</p> <p>It is appropriate to introduce nC_r through the knowledge of Pascal's Triangle, as a useful aid in the computation of probability, using the definition involving favourable and total outcomes. The addition, multiplication and complement laws of probability can then be investigated, which would naturally include discussion about mutually exclusive and independent events and whether experiments involve replacement or non-replacement. From here, the notion of a random variable together with its probability distribution, mean and standard deviation, presents a natural progression into the world of simulation.</p> <p>Candidates can now be shown the value of tables of random numbers and the generation of random numbers by machine.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
<p>calculate the probability of an event which involves the laws of probability</p> <p>construct the probability distribution of a given random variable</p> <p>calculate the mean and variance of a random variable, as calculated from its probability distribution</p> <p>use tables of random numbers and generate random numbers by calculator or computer</p> <p>simulate the probability distribution of a random variable, using random numbers</p> <p>Continuous probability distributions know the meaning of the terms continuous random variable and probability density function</p> <p>calculate probability from a probability density function of a continuous random variable by integration</p>	<p>Statistics relies heavily on the theory of probability. The relevant probability theory must be developed as it is required for statistics, which may be thought of as the modeling of variability and the assessment of uncertainty using random numbers.</p> <p>eg The probability that a certain phone number is engaged is 0.2. What is the probability that exactly 2 out of the next 3 people who phone this number will find it engaged?</p> <p>Special distributions, such as binomial, are not required</p> <p>This content serves merely as an introduction to the subject of continuous variables. Probability density functions will be restricted to those types of functions studied in <i>Mathematics 1 (H)</i>, Outcome 3 Basic Differentiation and <i>Mathematics 2 (H)</i>, Outcome 2 Basic Integration.</p> <p>Knowledge of the normal distribution is not required but could well be used for illustrative purposes together with normal tables.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

CONTENT	COMMENT
determine cumulative probability from a probability density function by integration	
determine the mean and variance of a continuous random variable from its probability density function	
determine the median and mode of a continuous random variable from its probability density function	Although the median and mode are not used much in practice they do serve as an illustration of the use of calculus from <i>Mathematics 1 (H)</i> and <i>2 (H)</i> .
<p>The relationship between two variables</p> <p>know the meaning of the terms linear regression, regression of y on x, residual, interpolation, extrapolation and correlation</p>	Note that it is assumed that candidates are familiar with the construction and interpretation of scattergraphs (see Intermediate 1 and 2).
determine least squares regression line of y on x , given by $y = \alpha + \beta x$ where estimates for α and β , ie a and b , are given by	Modern technology commonly uses ‘ a ’ as an estimate of α , the intercept and ‘ b ’ as an estimate of β , the gradient of the regression line.
$a = \bar{y} - b\bar{x} \text{ and } b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2} \text{ respectively}$	
predict values using the regression line	Estimation is appropriate if the points are close to a straight line, as demonstrated by a scattergraph and high correlation showing a strong linear association between the variables.
calculate the product moment correlation coefficient and interpret the result	Candidates should be able to relate correlation coefficients to appropriate scattergraphs, noting that the product moment correlation coefficient is a measure of linear fit.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

ASSESSMENT

To gain the award of the course, the candidate must pass all the unit assessments as well as the external assessment. External assessment will provide the basis for grading attainment in the course award.

When units are taken as component parts of a course, candidates should have the opportunity to achieve at levels beyond that required to attain each of the unit outcomes. This attainment may, where appropriate, be recorded and used to contribute towards course estimates and to provide evidence for appeals. Additional details are provided, where appropriate, with the exemplar assessment materials. Further information on the key principles of assessment are provided in the paper, *Assessment* (HSDU, 1996) and in *Managing Assessment* (HSDU, 1998).

DETAILS OF THE INSTRUMENTS FOR EXTERNAL ASSESSMENT

The external assessment will take the form of an examination of 2 hours and 40 minutes' duration. Candidates will sit either a paper assessing *Mathematics 1 (H)*, *Mathematics 2 (H)* and *Mathematics 3 (H)* or one assessing *Mathematics 1 (H)*, *Mathematics 2 (H)* and *Statistics (H)*. The external examination will test the candidate's ability to retain and integrate mathematical knowledge across the component units of the course. Each examination will consist of two papers, one of which will not allow the use of a calculator. The two papers will contain a balance of short questions designed mainly to test knowledge and understanding and extended response questions which also test problem solving skills. These two styles of questions will include ones which are set in more complex contexts to provide evidence for performance at Grades A and B.

The papers will be designed so that approximately 60% of the marks will be opportunities at Grade C.

GRADE DESCRIPTIONS FOR HIGHER MATHEMATICS

Higher Mathematics courses should enable candidates to solve problems that integrate mathematical knowledge across performance criteria, outcomes and units, and which require extended thinking and decision making. The award of Grades A, B and C is determined by the candidate's demonstration of the ability to apply knowledge and understanding to problem-solving. To achieve Grades A and B in particular, this demonstration will involve more complex contexts including the depth of treatment indicated in the detailed content tables.

In solving problems, candidates should be able to:

- a) interpret the problem and consider what might be relevant;
- b) decide how to proceed by selecting an appropriate strategy;
- c) implement the strategy through applying mathematical knowledge and understanding and come to a conclusion;
- d) decide on the most appropriate way of communicating the solution to the problem in an intelligible form.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

Familiarity and complexity affect the level of difficulty of problems. It is generally easier to interpret and communicate information in contexts where the relevant variables are obvious and where their inter relationships are known. It is usually more straightforward to apply a known strategy than to modify one or devise a new one. Some concepts are harder to grasp and some techniques more difficult to apply, particularly if they have to be used in combination.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

Exemplification of problem solving at Grade C and Grade A

a) **Interpret the problem and consider what might be relevant**

At Grade C candidates should be able to interpret qualitative and quantitative information as it arises within:

- the description of real-life situations
- the context of other subjects
- the context of familiar areas of mathematics

Grade A performance is demonstrated through coping with the interpretation of more complex contexts requiring a higher degree of reasoning ability in the areas described above.

b) **Decide how to proceed by selecting an appropriate strategy**

At Grade C candidates should be able to tackle problems by selecting an algorithm, or sequence of algorithms, drawn from related areas of mathematics, or a heuristic strategy.

Grade A performance is demonstrated through an ability to decide on and apply a more extended sequence of algorithms to more complex contexts.

c) **Implement the strategy through applying mathematical knowledge and understanding and come to a conclusion**

At Grade C candidates should be able to use their knowledge and understanding to carry through their chosen strategy and come to a conclusion. They should be able to process data in numerical and symbolic form with appropriate regard for accuracy, marshal facts, sustain logical reasoning and appreciate the requirements of proof.

Grade A performance is demonstrated through an ability to cope with processing data in more complex situations, and sustaining logical reasoning, where the situation is less readily identifiable with a standard form.

d) **Decide on the most appropriate way of communicating the solution to the problem in an intelligible form**

At Grade C candidates should be able to communicate qualitative and quantitative mathematical information intelligibly and to express the solution in language appropriate to the situation.

Grade A performance is demonstrated through an ability to communicate intelligibly in more complex situations and unfamiliar contexts.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

APPROACHES TO LEARNING AND TEACHING

The learning and teaching process should foster positive attitudes to the subject. Exposition to a group or class remains an essential technique at this level, and active candidate involvement in learning should be encouraged through questioning and discussion. However, investigative approaches to learning should also feature prominently in the delivery of the Higher course. Where appropriate, new skills and concepts should be introduced within a context and, when suitable, through an investigative approach, sometimes giving candidates the opportunity to work co-operatively. Coursework tasks can support these approaches, and simultaneously allow the grade descriptions for extended problem solving to be met.

Opportunities should be taken to justify the need for new skills by reference to real problems. While the contexts may include familiar and everyday ones, they should also make use of situations from other subjects, or from areas of interest to candidates in their future study. Contexts which are purely mathematical will also be appropriate, for example, some aspects of quadratic theory might be illustrated in the context of tangency, *Mathematics 2 (H)*.

Wherever possible, candidates should be encouraged to make use of technology. Calculators have great potential provided they are used appropriately and not allowed to provide unnecessary support, nor substitute for personal proficiency. This balance between ‘basic skills’ and appropriate use of technology is essential. Calculators with mathematical and graphical facilities and those with computer algebra systems (CAS) can be utilised as powerful tools both for processing data, especially in the study of statistics, and for reinforcing mathematical concepts. The use of such calculators should help candidates gain confidence in making conjectures based on numerical or graphical evidence. Candidates should be aware that errors are inevitably introduced in the course of computation or in the limitations of the graphical display.

Computers can also make a significant contribution to learning and teaching. The use of software packages in statistics will enhance the learning and teaching and allow candidates greater flexibility through ease of computation and display.

It is envisaged that increased availability and advances in technology will have a continuing and increasing influence in approaches to learning and teaching at this level.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

SPECIAL NEEDS

This course specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment and Certification Arrangements for Candidates with Special Needs/Candidates whose First Language is not English* (SQA, 1998).

SUBJECT GUIDES

A Subject Guide to accompany the Arrangements documents has been produced by the Higher Still Development Unit (HSDU) in partnership with the Scottish Consultative Council on the Curriculum (SCCC) and Scottish Further Education Unit (SFEU). The Guide provides further advice and information about:

- support materials for each course
- learning and teaching approaches in addition to the information provided in the Arrangements document
- assessment
- ensuring appropriate access for candidates with special educational needs

The Subject Guide is intended to support the information contained in the Arrangements document. The SQA Arrangements documents contain the standards against which candidates are assessed.

National Unit Specification: general information

UNIT	Mathematics 1 (Higher)
NUMBER	D321 12
COURSE	Mathematics (Higher)

SUMMARY

Mathematics 1 (H) comprises outcomes in algebra, geometry, graphicacy and elementary calculus. It is a mandatory unit of the Higher Mathematics course and provides a basis for progression to *Mathematics 2 (H)*.

OUTCOMES

- 1 Use the properties of the straight line.
- 2 Associate functions and graphs.
- 3 Use basic differentiation.
- 4 Design and interpret mathematical models of situations involving recurrence relations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained one of the following:

- Standard Grade Mathematics Credit award
- Intermediate 2 Mathematics or its component units including *Mathematics 3 (Int 2)*
- equivalent.

CREDIT VALUE

1 credit at Higher level.

Administrative Information

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National Unit Specification: general information (cont)

UNIT Mathematics 1 (Higher)

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	Numeracy	H
Additional core skills components for the unit	Critical Thinking	H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT Mathematics 1 (Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use the properties of the straight line.

Performance criteria

- (a) Determine the equation of a straight line given two points on the line or one point and the gradient.
- (b) Find the gradient of a straight line using $m = \tan \theta$.
- (c) Find the equation of a line parallel to and a line perpendicular to a given line.

OUTCOME 2

Associate functions and graphs.

Performance criteria

- (a) Sketch and identify related graphs and functions.
- (b) Identify exponential and logarithmic graphs.
- (c) Find composite functions of the form $f(g(x))$, given $f(x)$ and $g(x)$.

OUTCOME 3

Use basic differentiation.

Performance criteria

- (a) Differentiate a function reducible to a sum of powers of x .
- (b) Determine the gradient of a tangent to a curve by differentiation.
- (c) Determine the coordinates of the stationary points on a curve and justify their nature using differentiation.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 1 (Higher)

OUTCOME 4

Design and interpret mathematical models of situations involving recurrence relations.

Performance criteria

- (a) Define and interpret a recurrence relation in the form $u_{n+1} = mu_n + c$ (m, c constants) in a mathematical model.
- (b) Find and interpret the limit of the sequence generated by a recurrence relation in a mathematical model (where the limit exists).

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessment, candidates are required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 1 (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. Within this unit, study of coordinate geometry of the straight line, algebra and trigonometry, with the emphasis on graphicacy, is taken to a greater depth. Previous experience of number patterns is formalised in the context of recurrence relations and differential calculus is introduced.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates working at this level are expected to acquire a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within this unit, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

National Unit Specification: support notes (cont)

UNIT Mathematics 1 (Higher)

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to achieve the algebraic, trigonometric and calculus performance criteria of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment and Certification Arrangements for Candidates with Special Needs/Candidates whose First Language is not English* (SQA, 1998).

National Unit Specification: general information

UNIT	Mathematics 2 (Higher)
NUMBER	D322 12
COURSE	Mathematics (Higher)

SUMMARY

Mathematics 2 (H) comprises topics in algebra, geometry, trigonometry and elementary calculus. It is a mandatory unit of the Higher Mathematics course and provides a basis for progression to *Mathematics 3 (H)*.

OUTCOMES

- 1 Use the Factor/Remainder Theorem and apply quadratic theory.
- 2 Use basic integration.
- 3 Solve trigonometric equations and apply trigonometric formulae.
- 4 Use the equation of the circle.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained one of the following:

- *Mathematics 1 (H)*
- equivalent

CREDIT VALUE

1 credit at Higher level.

Administrative Information

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National Unit Specification: general information (cont)

UNIT Mathematics 2 (Higher)

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	None	
Additional core skills components for the unit	Critical Thinking	H
	Using Numbers	H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT Mathematics 2 (Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use the Factor/Remainder Theorem and apply quadratic theory.

Performance criteria

- (a) Apply the Factor/Remainder Theorem to a polynomial function.
- (b) Determine the nature of the roots of a quadratic equation using the discriminant.

OUTCOME 2

Use basic integration.

Performance criteria

- (a) Integrate functions reducible to the sums of powers of x (definite and indefinite).
- (b) Find the area between a curve and the x -axis using integration.
- (c) Find the area between two curves using integration.

OUTCOME 3

Solve trigonometric equations and apply trigonometric formulae.

Performance criteria

- (a) Solve a trigonometric equation in a given interval.
- (b) Apply a trigonometric formula (addition formula) in the solution of a geometric problem.
- (c) Solve a trigonometric equation involving an addition formula in a given interval.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 2 (Higher)

OUTCOME 4

Use the equation of the circle.

Performance criteria

- (a) Given the centre (a, b) and radius r , find the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.
- (b) Find the radius and centre of a circle given the equation in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.
- (c) Determine whether a given line is a tangent to a given circle.
- (d) Determine the equation of the tangent to a given circle given the point of contact.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 2 (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. Within this unit, the coordinate geometry of *Mathematics 1(H)* is extended to include the circle. The Factor/Remainder Theorem and quadratic theory and addition formulae are added to previous experience in algebra and trigonometry respectively. Basic integration is introduced to extend the calculus of *Mathematics 1(H)*.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates working at this level are expected to acquire a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within this unit, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

National Unit Specification: support notes (cont)

UNIT Mathematics 2 (Higher)

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to achieve the algebraic, trigonometric and calculus performance criteria of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment and Certification Arrangements for Candidates with Special Needs/Candidates whose First Language is not English* (SQA, 1998).

National Unit Specification: general information

UNIT	Mathematics 3 (Higher)
NUMBER	D323 12
COURSE	Mathematics (Higher)

SUMMARY

This optional unit is the third of three progressive units which comprise the Higher Mathematics course. *Mathematics 3 (H)* comprises outcomes in algebra, geometry, trigonometry and elementary calculus at Higher level.

OUTCOMES

- 1 Use vectors in three dimensions.
- 2 Use further differentiation and integration.
- 3 Use properties of logarithmic and exponential functions.
- 4 Apply further trigonometric relationships.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained:

- *Mathematics 2 (H)*
- equivalent

CREDIT VALUE

1 credit at Higher level.

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National Unit Specification: general information (cont)

UNIT Mathematics 3 (Higher)

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	None	
Additional core skills components for the unit	Critical Thinking	H
	Using Numbers	H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT Mathematics 3 (Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use vectors in three dimensions.

Performance criteria

- (a) Determine whether three points with given coordinates are collinear.
- (b) Determine the coordinates of the point which divides the join of two given points internally in a given numerical ratio.
- (c) Use the scalar product.

OUTCOME 2

Use further differentiation and integration.

Performance criteria

- (a) Differentiate $k \sin x$, $k \cos x$.
- (b) Differentiate using the function of a function rule.
- (c) Integrate functions of the form $f(x) = (x + q)^n$, n rational except for -1 , $f(x) = p \cos x$ and $f(x) = p \sin x$.

OUTCOME 3

Use properties of logarithmic and exponential functions.

Performance criteria

- (a) Simplify a numerical expression using the laws of logarithms.
- (b) Solve simple logarithmic and exponential equations.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 3 (Higher)

OUTCOME 4

Apply further trigonometric relationships.

Performance criteria

(a) Express $a \cos \theta + b \sin \theta$ in the form $r \cos (\theta \pm \alpha)$ or $r \sin (\theta \pm \alpha)$.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 3 (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. This optional unit is designed to continue the study of mathematics in the areas of algebra, geometry, trigonometry and calculus, and to provide a basis for the study of more advanced mathematics and applied mathematics.

Within this unit, the coordinate geometry of *Mathematics 1(H)* and *Mathematics 2(H)* is broadened into a three dimensional context with the introduction of vector notation and applications. Calculus is further extended to include differentiation and integration of the sine and cosine functions and the rules for integrating and differentiating composite functions. The exponential and logarithmic functions introduced graphically in *Mathematics 1(H)* are used in applications, and further trigonometry is studied in the context of the wave function.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates working at this level are expected to acquire a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within this unit, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Mathematics 3 (Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – ‘How sensible is my answer?’
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to achieve the algebraic, trigonometric and calculus performance criteria of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates are required to show their working in carrying out these algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment and Certification Arrangements for Candidates with Special Needs/Candidates whose First Language is not English* (SQA, 1998).

National Unit Specification: general information

UNIT	Statistics (Higher)
NUMBER	D326 12
COURSE	Mathematics (Higher)

SUMMARY

Statistics (H) is an optional unit of the Higher Mathematics course which is designed to provide a statistics specialism, and to form the basis from which further study of statistics can be developed. Outcome 2 is necessary for the work in Advanced Higher *Statistics 1*, while experience of Outcomes 1 and 4 is advantageous for that work.

OUTCOMES

- 1 Interpret an Exploratory Data Analysis (EDA) on a data set.
- 2 Work with discrete probability distributions.
- 3 Work with continuous probability distributions.
- 4 Analyse the relationship between two variables.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained one of the following:

- Standard Grade Mathematics Credit award
- Intermediate 2 Mathematics award or its component units
- equivalent

In particular the statistical content of *Mathematics 2 (Int 2)* provides an appropriate basis for *Statistics (H)*.

Administrative Information

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National Unit Specification: general information (cont)

UNIT Statistics (Higher)

CREDIT VALUE

1 credit at Higher level.

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	Numeracy	H
Additional core skills components for the unit	Critical Thinking	H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT **Statistics (Higher)**

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Interpret an Exploratory Data Analysis (EDA) on a data set.

Performance criteria

- (a) Interpret the results of an EDA.

OUTCOME 2

Work with discrete probability distributions.

Performance criteria

- (a) Calculate the probability of an event which involves the laws of probability.
- (b) Construct the probability distribution of a given random variable.
- (c) Calculate the mean and variance of a random variable.
- (d) Simulate the behaviour of a random variable.

OUTCOME 3

Work with continuous probability distributions.

Performance criteria

- (a) Calculate probability from the density function of a continuous random variable.
- (b) Obtain the mean and variance of a continuous random variable.

OUTCOME 4

Analyse the relationship between two variables.

Performance criteria

- (a) Determine the equation of linear regression and use it for prediction.
- (b) Calculate the correlation coefficient and interpret the result.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

National Unit Specification: support notes

UNIT Statistics (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. This optional unit extends the statistical content of Intermediate 1 and 2, and is designed mainly to provide a statistics specialism at Higher level in acknowledgment of the extensive use of statistical processes in many occupations and other areas of study.

Exploratory Data Analysis (a method of achieving a quick and simple analysis of a sample data set) unifies the inferential statistics contained in *Mathematics 2 (Int 1)* and *Mathematics 2 (Int 2)* in Outcome 1.

In Outcomes 2 and 3, discrete and continuous probability distributions are studied in some depth, allowing for discussion of randomness and simulation. The mathematics of continuous probability functions provide a link with the calculus contained in *Mathematics 2 (H)*.

Outcome 4 formalises and extends the work on scattergraphs at Intermediate 2 level to regression analysis and the calculation of the correlation coefficient.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

Where possible, candidates should be given the opportunity to apply the statistics of this unit through a short statistical assignment which will require them to collect (generate) the relevant data, analyse the data and interpret and communicate the conclusion. This assignment does not require to be formally assessed.

National Unit Specification: support notes (cont)

UNIT Statistics (Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, statistical topics should be taught and skills in applying statistics developed through real-life contexts. Candidates should be encouraged, throughout this unit, to make efficient use of the arithmetical, mathematical, statistical and graphical features of calculators as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – ‘How sensible is my answer?’
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

SPECIAL NEEDS

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