[CO56/SQP027]

Higher Mathematics

Time: 1 hour 10 minutes

NATIONAL QUALIFICATIONS

Units 1, 2 and 3
Paper I
(Non-calculator)
Specimen Question Paper **(Revised)**

Read Carefully

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SCOTTISH QUALIFICATIONS AUTHORITY

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{(g^2 + f^2 - c)}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

Table of standard derivatives:

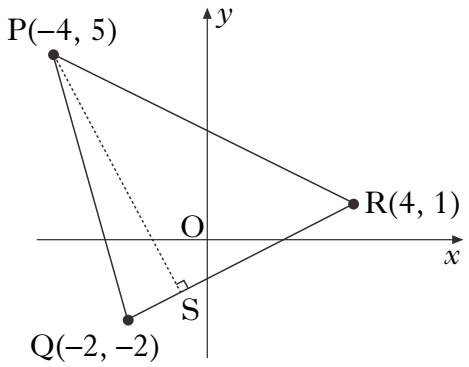
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted. *Marks*

1. P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **(3)**



2. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 3u_n + 5$ with first term u_1.

(*a*) Explain why this sequence has a limit as n tends to infinity. **(1)**

(*b*) Find the **exact** value of this limit. **(2)**

3. (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. **(3)**

(*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. **(1)**

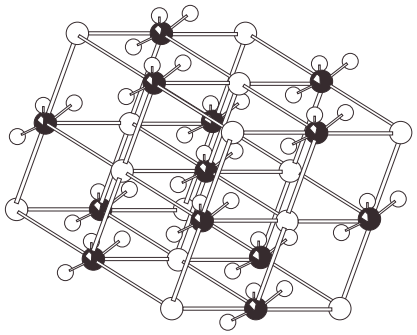
(*c*) Find the stationary points of $y = f(x)$ and determine the nature of each. **(5)**

(*d*) Sketch the graph of $y = f(x)$. **(1)**

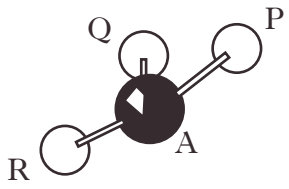
4. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x + 30)°$ is $\frac{4\sqrt{3} + 3}{10}$. **(3)**

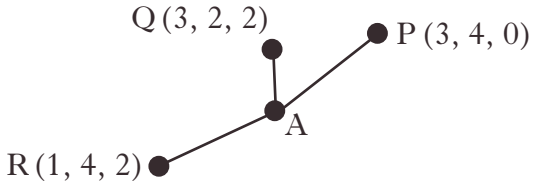
Marks

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.





(*a*) Show that the cosine of angle PQR is $\frac{1}{2}$. **(5)**

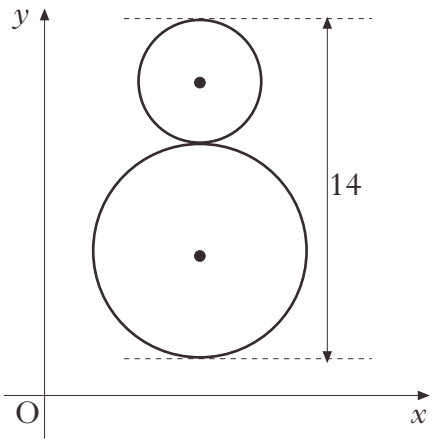
(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2 : 1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

6. A bakery firm makes ginger-bread men each 14 cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis.

Find the equation of the "head". **(5)**



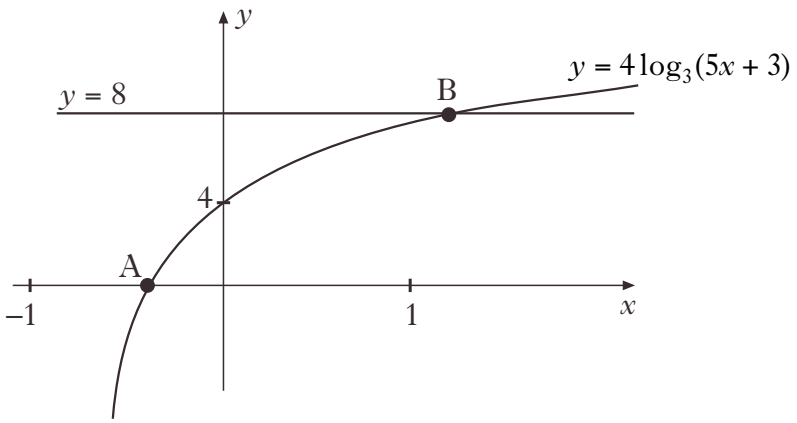
Marks

7. Find the value of $\int_1^2 \frac{u^2+2}{2u^2}\,du$. **(5)**

8. Sketch the graph of $y = 2\sin(x-30)°$ for $0 \le x < 360$. **(4)**

9. Find $\frac{dy}{dx}$ given that $y = \sqrt{1+\cos x}$. **(3)**

10. Part of the graph of $y = 4\log_3(5x + 3)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B.
Find the x-coordinate of B. **(3)**



[*END OF QUESTION PAPER*]

[CO56/SQP027]

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Statistics:

Sample standard deviation

$$s = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\sum x_i^{\,2} - \frac{1}{n}\left(\sum x_i\right)^2\right)}$$

where n is the sample size.

Sums of squares and products:

$$S_{xx} = \sum \left(x_i - \bar{x}\right)^2 = \sum x_i^{\,2} - \frac{1}{n}\left(\sum x_i\right)^2$$

$$S_{yy} = \sum \left(y_i - \bar{y}\right)^2 = \sum y_i^{\,2} - \frac{1}{n}\left(\sum y_i\right)^2$$

$$S_{xy} = \sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = \sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i$$

Linear regression:

The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β, a and b, are given by:

$$a = \bar{y} - b\bar{x}$$
$$b = \frac{\sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum \left(x_i - \bar{x}\right)^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient r:

$$r = \frac{\sum \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum \left(x_i - \bar{x}\right)^2 \sum \left(y_i - \bar{y}\right)^2}} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

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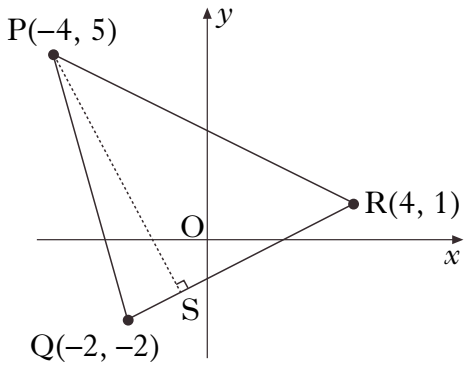


2. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 3u_n + 5$ with first term u_1.

(*a*) Explain why this sequence has a limit as n tends to infinity. **(1)**

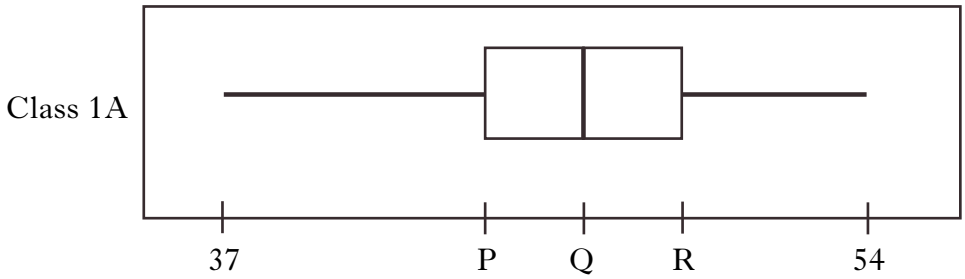
(*b*) Find the **exact** value of this limit. **(2)**

3. Class 1A sat a test out of 60. The marks are shown in the stem-and-leaf diagram below.

3	7											
4	0	1	2	2	4	4						
4	5	5	6	7	7	7	8	8	8	9	9	9
5	0	1	1	2	3	4						

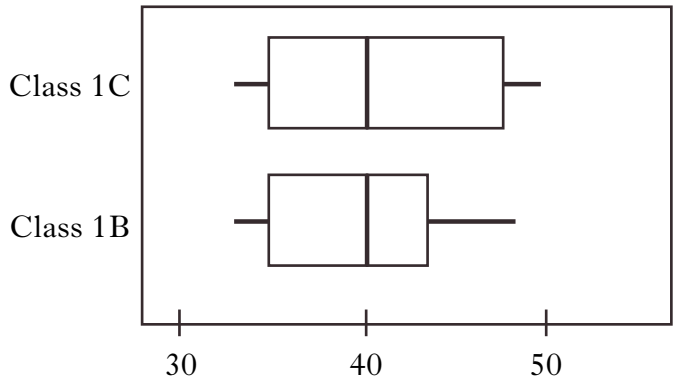
$n = 25$ 3 7 represents 37

The diagram below shows an incomplete boxplot for this data.



(*a*) Find the values associated with the points P, Q and R. **(2)**

(*b*) The boxplot below shows the data for classes 1B and 1C.



Compare the results of these two classes. **(2)**

Marks

4. (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. **(3)**

(*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. **(1)**

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(*d*) Sketch the graph of $y = f(x)$. **(1)**

5. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x + 30)°$ is $\frac{4\sqrt{3} + 3}{10}$. **(3)**

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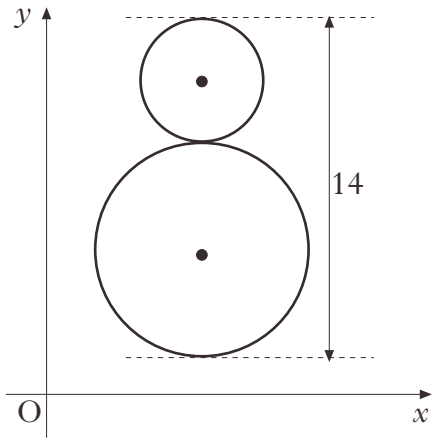

7. Find the value of $\int_1^2 \frac{u^2 + 2}{2u^2}\,du$. **(5)**

8. Sketch the graph of $y = 2\sin(x - 30)°$ for $0 \leq x < 360$. **(4)**

Marks

9. A random device moves one unit to the right with probability 0·3, one unit to the left with probability 0·3 or remains in the same position after each trial.

(*a*) Tabulate the probability distribution of X, the position of the device, after one trial. **(2)**

(*b*) A calculator produces the following random numbers.

0·764	0·380	0·410	0·175	0·458
0·552	0·709	0·935	0·451	0·854

(i) Explain how you would use these numbers to simulate ten trials of this random experiment. **(2)**

(ii) List the results of your simulation. **(1)**

10. The total lifetime (in years) of 5 year old washing machines of a certain make is a random variable whose cumulative distribution function F is given by

$$F(x) = \begin{cases} 0 & \text{for } x \leq 5 \\ 1 - \dfrac{25}{x^2} & \text{for } x > 5 \end{cases}$$

(*a*) Find the probability that such a washing machine will be in service for:

(i) less than 8 years; **(1)**

(ii) more than 10 years. **(2)**

(*b*) Find the probability density function $f(x)$. **(2)**

(*c*) Calculate the exact value of the median lifetime of these washing machines. **(3)**

[*END OF QUESTION PAPER*]

[CO56/SQP027]

Higher Mathematics

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Table of standard derivatives:

$f(x)$	$f'(x)$
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All questions should be attempted. *Marks*

1. ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC. **(3)**

2. Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees.

A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees.

Which pesticide will be more effective in the long term? **(5)**

3. (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants. **(3)**

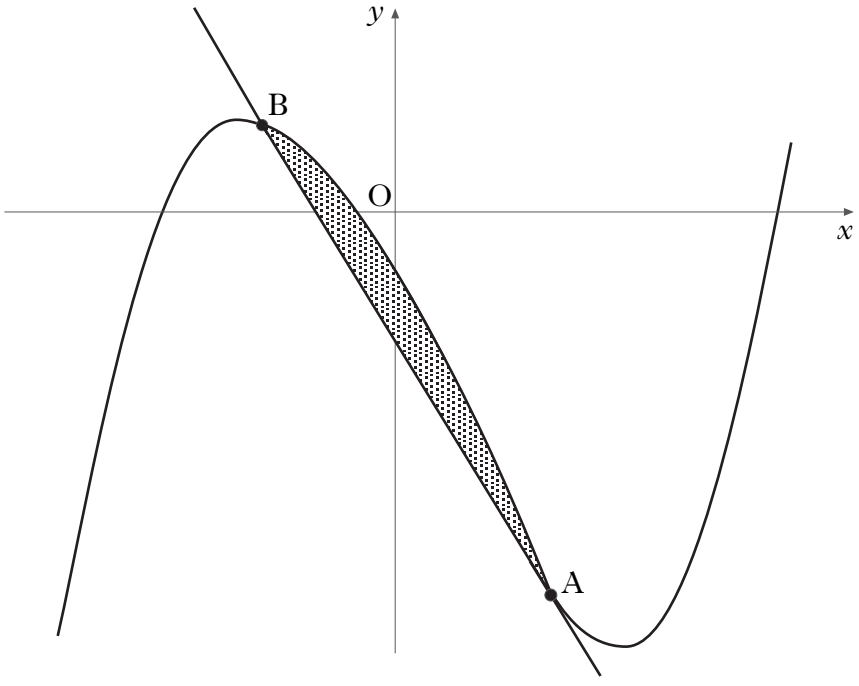
(*b*) Hence, or otherwise, find the coordinates of the turning point of the function f. **(1)**

4. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

(*a*) Find the equation of the tangent at A. **(3)**

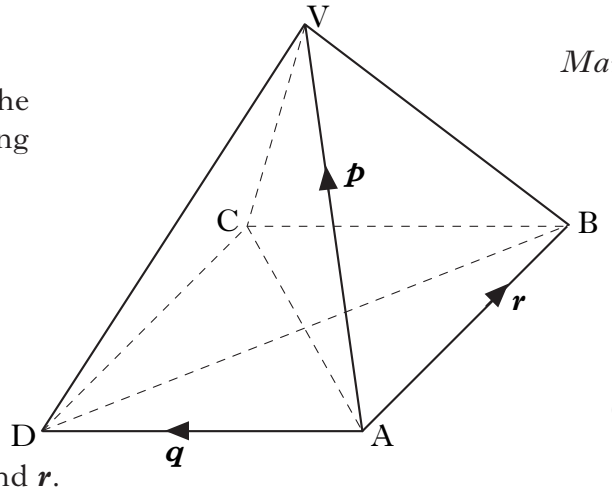
(*b*) Hence find the coordinates of B. **(4)**

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



Marks

5. VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.



$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$.

(*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$.

(ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. **(3)**

(*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$.

(ii) Hence show that angle CVA is 90°. **(4)**

6. $f(x) = 2\cos x° + 3\sin x°$.

(*a*) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$. **(4)**

(*b*) Hence solve $f(x) = 0{\cdot}5$ for $0 \leq x < 360$. **(3)**

(*c*) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the x-axis for $0 \leq x < 360$. **(2)**

7. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **(2)**

(*b*) **Hence**

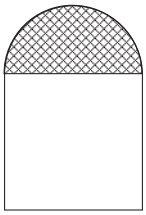
(i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$

(ii) solve this equation in the interval $0 \leq x < 90$. **(3)**

8. The roots of the equation $(x - 1)(x + k) = -4$ are equal.
Find the values of k. **(5)**

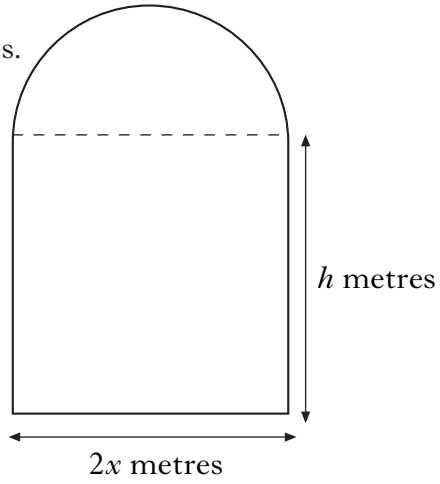
Marks

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.



The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.

The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

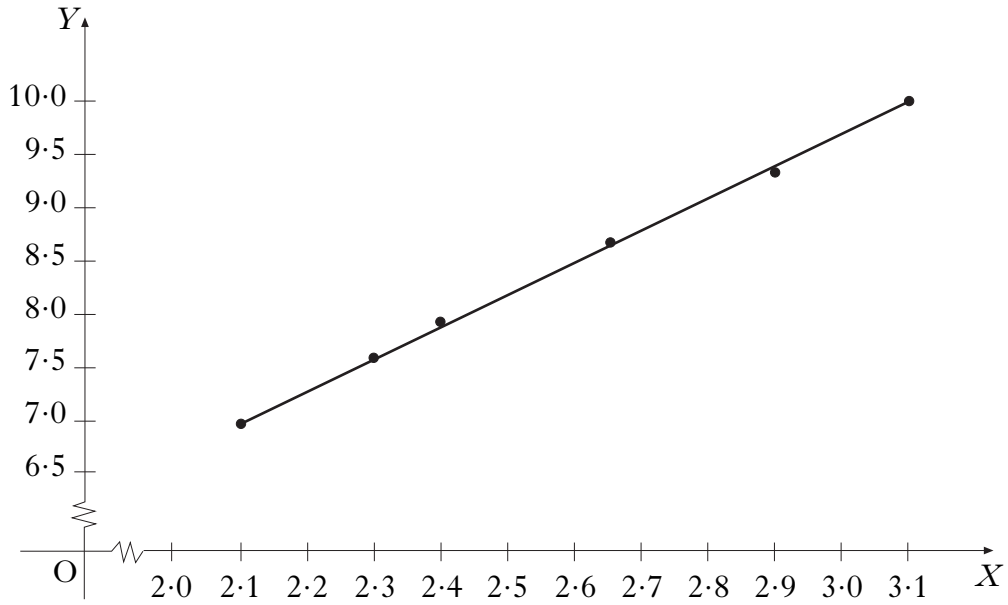
Marks

10. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X (= $\log_e x$)	2·10	2·31	2·40	2·65	2·90	3·10
Y (= $\log_e y$)	7·00	7·60	7·92	8·70	9·38	10·00

A graph was drawn and is shown below.



Find the equation of the line in the form $Y = mX + c$. **(3)**

[*END OF QUESTION PAPER*]

[C056/SQP027]

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Sample standard deviation $s = \sqrt{\frac{1}{n-1}\sum\left(x_i - \bar{x}\right)^2} = \sqrt{\frac{1}{n-1}\left(\sum x_i^2 - \frac{1}{n}(\sum x_i)^2\right)}$ where n is the sample size.

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Product moment correlation coefficient r:

$$r = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum\left(x_i - \bar{x}\right)^2 \sum\left(y_i - \bar{y}\right)^2}} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

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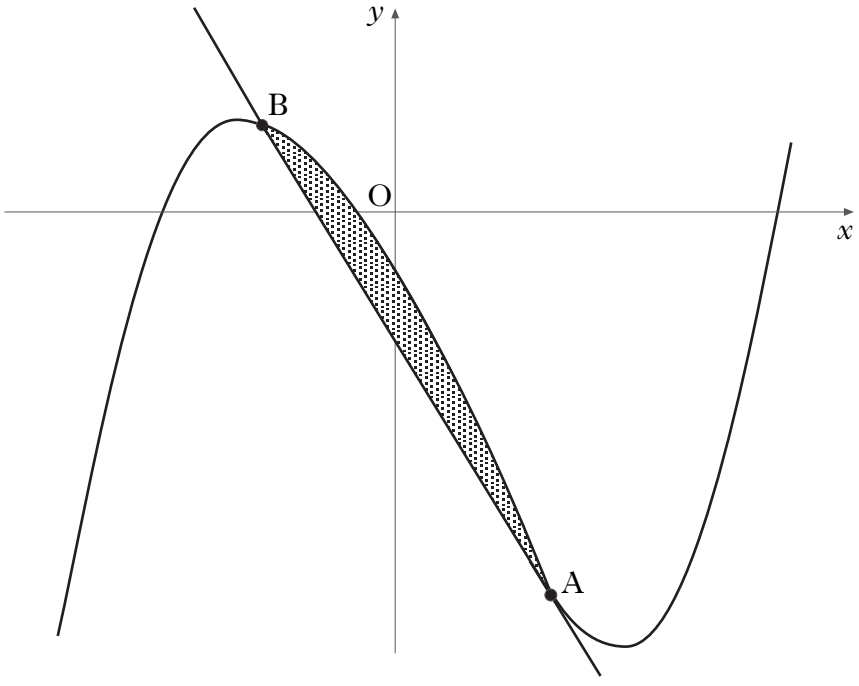
(*b*) Hence find the coordinates of B. **(4)**

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



Marks

5. In an archery competition, the probability that a particular competitor hits the target with any shot is $\frac{3}{4}$. In the competition, she is allowed three shots.

(*a*) Find the probability that she hits the target:

(i) exactly twice; **(2)**

(ii) at least once. **(2)**

(*b*) State a statistical assumption that you have made. **(1)**

6. A market gardener wishes to investigate the relationship between the total weight of tomatoes produced by a tomato plant and the amount of fertiliser used. An experiment was carried out where known amounts of fertiliser were applied to 8 similar plants. The results are shown in the table.

Weight of fertiliser (g)	x	0	2	4	6	8	10	12	14
Tomato yield (kg)	y	4·44	5·13	5·45	5·27	5·81	6·04	5·90	6.23

A scatter diagram shows that a linear model is appropriate.

You may assume that $\sum y = 44 \cdot 27$, $\sum y^2 = 247 \cdot 3665$ and $\sum xy = 328 \cdot 58$.

(*a*) (i) Determine the equation of the least squares regression line of y on x. **(6)**

(ii) Use the regression equation to predict the tomato yield for 9 g of fertiliser. **(1)**

(*b*) Calculate the product moment correlation coefficient and comment on your answer. **(3)**

7. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **(2)**

(*b*) **Hence**

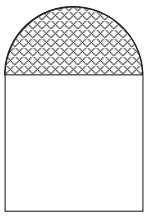
(i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$

(ii) solve this equation in the interval $0 \leq x < 90$. **(3)**

8. The roots of the equation $(x - 1)(x + k) = -4$ are equal.
Find the values of k. **(5)**

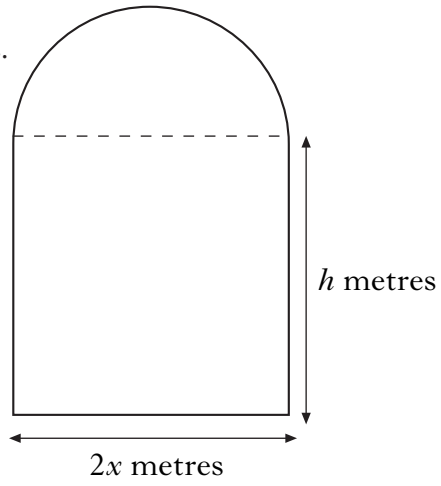
Marks

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.



The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.

The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

10. The random variable X has a probability density function

$$f(x) = \begin{cases} kx^2(1-x) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(*a*) Find the value of k. **(2)**

(*b*) Find the probability that X lies in the range $0 \leq X \leq \frac{2}{3}$. **(2)**

[*END OF QUESTION PAPER*]

[CO56/SQP027]

Higher Mathematics

NATIONAL QUALIFICATIONS

Paper I

Specimen Marking Instructions

SCOTTISH QUALIFICATIONS AUTHORITY

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
1	**ans: $y = -2x - 3$** •[1] process: use gradient formula •[2] strategy: know how to find perp. gradient •[3] communicate: state equation of st. line 3 marks	•[1] $m_{QR} = \frac{1}{2}$ •[2] $m_{PS} = -2$ •[3] PS: $y - 5 = -2(x + 4)$
2	**ans: $-1 < 0 \cdot 3 < 1$, $\frac{50}{7}$** •[1] communicate: state condition for limit •[2] strategy: know how to find limit •[3] process: complete strategy for exact limit 3 marks	•[1] $-1 < 0{\cdot}3 < 1$ •[2] *eg* $L = 0{\cdot}3L + 5$ •[3] $L = \frac{50}{7}$
3a	**ans: $f(1) = 0$, $(x - 4)$, $(x - 1)$** •[1] strategy: know how to find a root of a cubic •[2] interpret: state remaining quadratic factor •[3] process: complete factorisation 3 marks	•[1] *eg* $1 \mid 1 \quad -6 \quad 9 \quad -4$ $\quad\quad\quad\quad 1 \quad -5 \quad 4$ $\quad\quad 1 \quad -5 \quad 4 \quad 0$ •[2] $x^2 - 5x + 4$ •[3] $(x - 4)(x - 1)$
3b	**ans: (1,0), (4,0), (0, –4)** •[4] communicate: state axes intersections 1 mark	•[4] (1,0), (4,0), (0,–4)
3c	**ans: max at (1,0), min at (3, –4)** •[5] strategy: know that derivative = 0 •[6] process: differentiate •[7] process: find stationary points •[8] strategy: know how to test nature •[9] communicate: complete nature test 5 marks	•[5] $\frac{dy}{dx} = 0$ •[6] $3x^2 - 12x + 9$ •[7] (1,0), (3,–4) •[8] *eg* x | . . . | 1 | . . . | 3 | . . . y' | + | 0 | – | 0 | + •[9] max at (1,0), min at (3,–4)
3d	**ans: sketch** •[10] communicate: draw sketch 1 mark	•[10] sketch

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
4	**ans:** $\dfrac{4\sqrt{3}+3}{10}$ •[1] strategy: know to expand •[2] strategy: know to use right-angled trig. •[3] process: complete proof 3 marks	•[1] $\sin x° \cos 30° + \cos x° \sin 30°$ •[2] $\sin x° = \frac{4}{5}$ ***and*** $\cos x° = \frac{3}{5}$ •[3] $\frac{4}{5} \cdot \frac{\sqrt{3}}{2} + \frac{3}{5} \cdot \frac{1}{2}$
5a	**ans: proof** •[1] strategy: eg know to use scalar product •[2] interpret: find scalar product •[3] process: find magnitude •[4] process: find magnitude •[5] process: complete proof 5 marks	•[1] $\cos PQR = \dfrac{\overrightarrow{PQ}.\overrightarrow{RQ}}{\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{RQ}\rvert}$ •[2] $\overrightarrow{PQ}.\overrightarrow{RQ} = 4$ •[3] $PQ = \sqrt{8}$ •[4] $RQ = \sqrt{8}$ •[5] substitution leading to $\frac{1}{2}$
5b	**ans: M(2,3,2) T $\left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$; proof** •[6] interpret: find coordinates of M •[7] strategy: know how to find T •[8] process: find appropriate vector eg $\overrightarrow{PM}$ •[9] process: complete calc. of coordinates of T •[10] strategy: know how to find distance in 3D •[11] process: complete proof 6 marks	•[6] $M = (2,3,2)$ •[7] $eg\ \overrightarrow{PT} = \frac{2}{3}\overrightarrow{PM}$ •[8] $\overrightarrow{PM} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ •[9] $T = \left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$ •[10] $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ •[11] PT $= 2\sqrt{\frac{2}{3}}$, QT $= 2\sqrt{\frac{2}{3}}$, RT $= 2\sqrt{\frac{2}{3}}$
6	**ans:** $(x-5)^2 + (y-13)^3 = 9$ •[1] interpret: find radius from equ. of circle •[2] interpret: interpret diagram •[3] interpret: state centre from equ. of circle •[4] interpret: interpret diagram •[5] interpret: state equation of circle 5 marks	•[1] $radius_{body} = 4$ •[2] $radius_{head} = 3$ •[3] $centre_{body} = (5,6)$ •[4] $centre_{head} = (5,13)$ •[5] $(x-5)^2 + (y-13)^2 = 9$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
7	**ans: 1** •[1] strategy: know to split into sums/differences •[2] process: write in integrable form •[3] process: write in integrable form •[4] process: integrate •[5] process: evaluate limits of integration 5 marks	•[1] strategy: separate into sums/differences •[2] $\frac{1}{2}$ •[3] u^{-2} stated or implied by the answer to •[4] •[4] $\frac{1}{2}u - u^{-1}$ •[5] 1
8	**ans: sketch** for $y = a\sin(x + b)$ •[1] interpret: know "b" represents translation •[2] interpret: know "a" represents vert. scaling •[3] strategy: know the order of transformations •[4] communicate: complete sketch 4 marks	•[1] "–30°" means move $y = \sin x° : \begin{pmatrix} +30° \\ 0 \end{pmatrix}$ •[2] "2" means stretch two-fold parallel to y-axis •[3] order is "–30°" then "2" •[4] sketch evidence for •[1], •[2] and •[3] may be either stated, or implied by a correct sketch at the •[4] stage.
9	**ans: $-\frac{1}{2}\sin x(1+\cos x)^{-\frac{1}{2}}$** •[1] strategy: know how to deal with √ •[2] process: start differentiation •[3] process: apply chain rule 3 marks	•[1] $(1+\cos x)^{\frac{1}{2}}$ •[2] $\frac{1}{2}(1+\cos x)^{-\frac{1}{2}}$ •[3] $\times -\sin x$
10	**ans: $\frac{6}{5}$** •[1] interpret: interpret diagram •[2] strategy: know how to solve log equation •[3] process: complete solving process 3 marks	•[1] $\log_3(5x+3) = 2$ •[2] $5x + 3 = 3^2$ •[3] $x = \frac{6}{5}$

[*END OF MARKING INSTRUCTIONS*]

[CO56/SQP027]

Higher Mathematics

NATIONAL QUALIFICATIONS

Paper II

Specimen Marking Instructions

SCOTTISH QUALIFICATIONS AUTHORITY

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
1	**ans: $y = 2x - 5$** •^{1} process: use gradient formula •^{2} interpret: know parallel lines have equal gradients •^{3} communicate: state equation of st. line 3 marks	•^{1} $m_{AB} = 2$ •^{2} $m_{DC} = 2$ •^{3} $y - 11 = 2(x - 8)$
2	**ans: Pestkill** •^{1} interpret: state recurrence relation •^{2} interpret: state recurrence relation •^{3} strategy: know how to find limit •^{4} process: complete strategy for limits •^{5} communicate: limit condition & conclusion 5 marks	•^{1} $0{\cdot}4u_n + 300$ •^{2} $0{\cdot}2u_n + 360$ •^{3} *eg* $L = aL + b$ •^{4} 500 *and* 450 •^{5} limits are valid since $\lvert a \rvert < 1$ in both cases and Pestkill more effective
3a	**ans: $2(x + 2)^2 - 11$** •^{1} process: start with a eg $2(x^2 + 4x)$ •^{2} process: continue for b eg $2(x + 2)^2$ •^{3} process: complete by finding c 3 marks	•^{1} $a = 2$ •^{2} $b = 2$ •^{3} $c = -11$
3b	**ans: (–2, 11)** •^{4} interpret: state turning point of $a(x + b)^2 + c$ 1 mark	•^{4} (–2,11)

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
4a	**ans:** $y = -5x - 3$	
	•[1] strategy: know to differentiate •[2] process: evaluate gradient •[3] communicate: state equation of tangent 3 marks	•[1] $\frac{dy}{dx} = 3x^2 - 2x - 6$ •[2] $m_{tgt} = -5$ •[3] $y + 8 = -5(x - 1)$
4b	**ans:** **B = (–1,2)**	
	•[4] strategy: know how to find intersection •[5] process: carry out strategy •[6] strategy: know how to solve cubic •[7] process: carry out strategy 4 marks	•[4] attempt to simplify and equate y's •[5] $x^3 - x^2 - x + 1 = 0$ •[6] *eg* reduce to linear & quadratic •[7] B = (–1,2)
4c	**ans:** **area** = $1\frac{1}{3}$	
	•[8] strategy: know how to find area •[9] process: integrate •[10] process: evaluate limits 3 marks	•[8] *eg* $\int (x^3 - x^2 - 6x - 2) - (-5x - 3)\, dx$ •[9] $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$ •[10] $1\frac{1}{3}$
5a	**ans:** $(x - 4)^2 + (y + 1)^2 = 25$	
	•[1] interpret: state centre from equ. of circle •[2] interpret: interpret information/ diagram •[3] communicate: state equation of circle 3 marks	•[1] centre = (4,–1) •[2] $r = 5$ •[3] $(x - 4)^2 + (y + 1)^2 = 25$
5b	**ans:** **c < –19**	
	•[4] interpret: crucial aspect : $r^2 > 36$ •[5] strategy: know to use $r^2 = g^2 + f^2 - c$ •[6] process: solve inequality 3 marks	•[4] $r^2 > 36$ •[5] $g^2 + f^2 - c = 16 + 1 - c$ •[6] $c < -19$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
6a	**ans: 9**	
	•[1] process: evaluate scalar product •[2] strategy: know to use distributive law •[3] process: evaluate scalar product & complete 3 marks	•[1] $\frac{9}{2}$ •[2] $\boldsymbol{p}.\boldsymbol{q} + \boldsymbol{p}.\boldsymbol{r}$ •[3] 9
6b	**ans: proof**	
	•[4] interpret: interpret 3D representation •[5] strategy: know to use approp. scalar product •[6] strategy: know to use distributive law •[7] process: evaluate scalar product & complete 4 marks	•[4] $\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r}$ •[5] $\boldsymbol{p}.(\boldsymbol{p} - \boldsymbol{q} - \boldsymbol{r})$ •[6] $\boldsymbol{p}.\boldsymbol{p} - \boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ •[7] 0
7a	**ans: $\sqrt{13}\cos(x - 56\cdot3)°$**	
	•[1] communicate: state expansion •[2] interpret: compare & equate coefficients •[3] process: solve for k •[4] process: solve for α 4 marks	•[1] $k\cos x\cos\alpha + k\sin x\sin\alpha$ explicitly stated •[2] $k\cos\alpha = 2$ ***and*** $k\sin\alpha = 3$ explicitly stated •[3] $k = \sqrt{13}$ •[4] $\alpha = 56\cdot3$
7b	**ans: 138·8, 334·3**	
	•[5] strategy: know how to solve trig. equation •[6] process: solve for $x - \alpha$ •[7] process: complete solving process 3 marks	•[5] $\cos(x - 56\cdot3)° = \frac{0\cdot5}{\sqrt{13}}$ •[6] $x - 56\cdot3 = 82\cdot0, 278\cdot0$ •[7] $x = 138\cdot3, 334\cdot3$
7c	**ans: 146·3°**	
	•[8] strategy: know how to set function = 0 •[9] process: complete solving process 2 marks	•[8] $\sqrt{13}\cos(x - 56\cdot3)° = 0$ •[9] $x = 146\cdot3$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
8a	**ans: proof**	
	•[1] strategy: know to use approp. trig. rule •[2] strategy: know to use approp. trig. rule 2 marks	•[1] substitute $1 - 2\sin^2 x°$ for $\cos 2x°$ •[2] substitute $1 - \sin^2 x°$ for $\cos^2 x°$ and complete
8b	**ans: 19·5**	
	•[3] strategy: know to express in standard form •[4] process: factorise quadratic •[5] process: complete solving process 3 marks	•[3] $3\sin^2 x° + 2\sin x° - 1 = 0$ •[4] $(3\sin x° - 1)(\sin x° + 1) = 0$ •[5] $x = 19{\cdot}5$ and no other answers
9	**ans: $k = -5$ or 3**	
	•[1] strategy: know to express in standard form •[2] strategy: know condition for equal roots •[3] process: apply the strategy •[4] process: start the solving process •[5] process: complete the solving process 5 marks	•[1] $x^2 + kx - x + 4 - k = 0$ •[2] $b^2 - 4ac = 0$ •[3] $(k-1)^2 - 4(4-k)$ •[4] $k^2 + 2k - 15 = 0$ •[5] $k = -5,\ k = 3$

Qu	Marking Scheme Give 1 mark for each •	Illustrations of evidence for awarding a mark at each •
10ai	**ans:** $\boldsymbol{h = \frac{1}{2}(10 - \pi x - 2x)}$ •[1] strategy: know to form equ. for perimeter •[2] process: make h the subject 2 marks	•[1] *eg* $2h + 2x +$ semicircle $= 10$ •[2] $h = \frac{1}{2}(10 - \pi x - 2x)$
10aii	**ans: proof** •[3] strategy: know how to set up equ. for L •[4] communicate: complete proof 2 marks	•[3] $L = 2 \times 2xh + \frac{1}{2}\pi x^2$ •[4] $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$ $L = 20x - 2\pi x^2 - 4x^2 + \frac{1}{2}\pi x^2$
10b	**ans:** $\boldsymbol{x = \frac{20}{3\pi + 8}}$, $\boldsymbol{h = \frac{5(\pi + 4)}{3\pi + 8}}$ •[5] strategy: know that max. means $L' = 0$ •[6] process: differentiate •[7] process: solve $L' = 0$ •[8] strategy: know to check nature of max/min •[9] communicate: complete evaluation 5 marks	•[5] $L' = 0$ •[6] $L' = 20 - 8x - 3\pi x$ •[7] $x = \frac{20}{3\pi + 8} = x_0 (= 1{\cdot}148)$ •[8] *eg* x | x_0^- | x_0 | x_0^+ L' | + | 0 | − maximum at x_0 •[9] $h = \frac{5\pi + 20}{3\pi + 8} (= 2{\cdot}049)$
11a	**ans:** $\mathbf{Y = 3X + 0{\cdot}7}$ •[1] interpret: interpret gradient from graph •[2] strategy: know how to find "c" •[3] process: complete evaluation 3 marks	•[1] $m = 3$ •[2] *eg* $7{\cdot}00 = 3 \times 2{\cdot}10 + c$ •[3] *eg* $Y = 3X + 0{\cdot}7$
11b	**ans:** $\boldsymbol{a = 2{\cdot}01}$, $\boldsymbol{b = 3}$ •[4] strategy: know to express in suitable form •[5] process: start solving process •[6] process: complete solving process •[7] communicate: express answer in required form 4 marks	•[4] $\ln y = 3 \ln x + 0{\cdot}7$ •[5] *eg* $\ln \frac{y}{x^3} = 0{\cdot}7$ •[6] *eg* $\frac{y}{x^3} = e^{0{\cdot}7}$ •[7] $a = 2{\cdot}01$, $b = 3$

[*END OF MARKING INSTRUCTIONS*]