NQ NATIONAL QUALIFICATIONS Review

SCOTTISH QUALIFICATIONS AUTHORITY

# National Qualifications Review Report

## *Mathematics*

# Subject Review: Mathematics

# 1 Subject summary

## Summary of Courses

Intermediate 1 Mathematics
Intermediate 2 Mathematics
Higher Mathematics
Advanced Higher Mathematics
Advanced Higher Applied Mathematics

## Recommendations

### Recommendations which can be implemented this session (2001-2002)

**1. Course frameworks for Advanced Higher Mathematics and Applied Mathematics**
Investigate how the Course frameworks at Advanced Higher level should be re-designed to avoid the possibility of two Courses being obtained by undertaking less than six Unit credits. There is potential for a number of targeted, focused and specific Advanced Higher Courses to be created from the Units already available – these possibilities should be investigated in consultation with relevant key stakeholders in the sector.

This would involve reducing the number of optional Units in all Advanced Higher mathematics Courses. As part of this project, Advanced Higher Applied Mathematics would be re-designed to remove alternative mandatory Units and to reduce the number of optional Units in the Course (see section 2.1).

Recommendations from the investigation would be implemented as soon as is practicable for centres and SQA.

### Recommendations which can be implemented for next session (starting August 2002)

**2. Design of questions in examination papers in Mathematics**
Investigate assessment methodologies, techniques, and designs, that facilitate levelling questioning in Mathematics examination papers (at all levels). Practicable models should be developed to support examination-setting teams. This should have the effect of making Mathematics examinations a less stressful experience for candidates (see section 2.4)

Recommendations from the investigation would be implemented as soon as is practicable for centres and SQA

### Mid-long term recommendations

**3. Advances in calculator technologies and capabilities**
The use of advanced calculators in examination question papers should be fully researched, internationally benchmarked and developed into practicable methodologies and techniques to assist setting teams in utilising their full assessment potential (see section 2.4)

# 2 Course report

## 2.1 Structure of Courses

### Component Units

**Int 1 Mathematics**

**Mandatory Units**
Mathematics 1 (40 hours)
Mathematics 2 (40 hours)
**Optional Units, choose one from:**
Mathematics 3 (40 hours)
Applications of Mathematics (40 hours)

Mathematics 1 (Int 1) and 2 (Int 1) taken together with Applications of Mathematics (Int 1) form a Course designed to meet the needs of candidates who require a Mathematics qualification at Intermediate 1, but who do not intend to proceed to a Mathematics Course at Intermediate 2. The Course aims to enhance candidates' skills in applying their mathematics in a range of contexts with the emphasis on real-life applications.

**Int 2 Mathematics**

**Mandatory Units**
Mathematics 1 (40 hours)
Mathematics 2 (40 hours)
**Optional Units, choose one from:**
Mathematics 3 (40 hours)
Applications of Mathematics (40 hours)

The Intermediate 2 Mathematics Course which contains Mathematics 1 (Int 2), 2 (Int 2) and 3 (Int 2) is designed to meet the needs of candidates who wish to progress to Higher Mathematics. In this Course, the emphasis is placed on developing an appreciation of the power of mathematical language and the efficiency of algorithms.

Mathematics 1 (Int 2) and 2 (Int 2) taken together with Applications of Mathematics (Int 2) form a Course designed to meet the needs of candidates who require a Mathematics qualification at Intermediate 2, but who do not intend to proceed to a Mathematics Course at Higher. The Course aims to enhance candidates' skills in applying their mathematics in a range of contexts, with the emphasis on real-life applications.

**Higher Mathematics**

**Mandatory Units**
Mathematics 1 (40 hours)
Mathematics 2 (40 hours)
**Optional Units, choose one from:**
Mathematics 3 (40 hours)
Statistics

The Higher Mathematics Course has the objective of meeting the needs of candidates at a stage of their education where career aspirations are particularly important. The Course has obvious relevance for candidates with interests in fields such as commerce, engineering and science where the Mathematics learned will be put to direct use. For other candidates, the Course can be an important component of a group award or used to gain entry to a Higher Education institution.

**Advanced Higher Mathematics**

**Mandatory Units**
Mathematics 1 (40 hours)
Mathematics 2 (40 hours)

**Optional Units, choose one from:**
Mathematics 3 (40 hours)
Statistics 1 (40 hours)
Mechanics 1 (40 hours)
Numerical Analysis 1(40 hours)

**Advanced Higher Applied Mathematics**

**Mandatory Units**
Statistics 1 (40 hours)
Statistics 2 (40 hours)
or
Mechanics 1 (40 hours)
Mechanics 2 (40 hours)
or
Numerical Analysis 1 (40 hours)
Numerical Analysis 2 (40 hours)
**Optional Units, choose one from**
Mathematics 1 (40 hours)
Statistics 1 (40 hours)
Mechanics 1 (40 hours)
Numerical Analysis 1 (40 hours)

The choice of optional Units at this level is wider than at lower levels. This breadth of choice is provided in response to the variety of candidates' aspirations in Higher Education studies or areas of employment, and should satisfy most needs regardless of whether or not a specialism in Mathematics is the primary intention.

The Advanced Higher Courses offer breadth and depth of mathematical experience and relevance to further study or employment in mathematical and physical sciences, computer science, engineering, biological and social sciences, medicine, accounting, business and management. When the Advanced Higher Applied Mathematics Course is taken in addition to the Advanced Higher Mathematics Course, an opportunity is offered for the candidate to acquire exceptional breadth and depth of mathematical experience.

## Uptake in 2001 (figures for 2000 are in brackets)

| | |
|---|---|
| Intermediate 1 Mathematics | 4 281 (2 971) |
| Intermediate 2 Mathematics | 12 350 (8 829) |
| Higher Mathematics | 20 863 (20 050) |
| Advanced Higher Mathematics | 1 247 |
| Advanced Higher Applied Mathematics | 157 |

## Do the Courses meet the design criteria (1-7)?

Yes — Intermediate 1; Intermediate 2; Higher; and Advanced Higher Mathematics Courses meet the design criteria.

However, the Advanced Higher Applied Mathematics Course does not meet Criterion 5 (mandatory Units) in that there are a total of six alternative mandatory Units (arranged in three sets of two). It only just meets the requirements of Criterion 6 (optional Units) in that, at present, it offers the maximum number of optional Units (four).

It is worth noting that the Advanced Higher Mathematics Course also only just meets the extended requirements of Criterion 6 (optional Units) in that, at present, it offers the maximum number of optional Units (four).

### Issues and recommendations

**Issue: Course frameworks for Advanced Higher Mathematics and Applied Mathematics**

The Course frameworks for Mathematics and Applied Mathematics at Advanced Higher level allow 'double counting' of Units and create the possibility that a candidate could obtain two Course Awards by undertaking less than six Unit credits. This has been the subject of comments submitted to the review.

There are six alternative mandatory Units in Advanced Higher Applied Mathematics — three of these can also be taken as optional Units.

There are four optional Units in Advanced Higher Mathematics, three of which are also in Advanced Higher Applied Mathematics.

A consequence of this overlap, and of the high degree of optionality in both Courses, is that an Advanced Higher Applied Mathematics Course award and Advanced Higher Mathematics Course award can be achieved by a candidate who has undertaken and achieved fewer than six Units.

With careful selection, a candidate who has only achieved four Units can achieve both Advanced Higher Courses. Candidates would, however, still have to pass both external examinations to achieve graded awards in Advanced Higher Mathematics and Advanced Higher Applied Mathematics.

A further potential consequence is that candidates sitting one examination could undertake questions in their chosen component Units from the other Course (which they are barred from doing but, realistically, is very difficult to police).

Comments have been received from Higher Education and other end users (eg the Mathematics Council) that candidates not taking the full six Units across the two Advanced Highers would be disadvantaged in their progression in the subject area. (They could enter HE on the merit of two Advanced Higher Courses without having properly covered the full breadth expected of such achievement.) The design of the Courses could result in the status of both mathematics Advanced Highers being diminished by Higher Education and other end users.

**Recommendation: Course frameworks for Advanced Higher Mathematics and Applied Mathematics**

Investigate how the Course frameworks at Advanced Higher level should be re-designed to avoid the possibility of two Courses being obtained by undertaking less than six Unit credits. There is potential for a number of targeted, focused and specific Advanced Higher Courses to be created from the Units already available — these possibilities should be investigated in consultation with relevant key stakeholders in the sector.

This would involve reducing the number of optional Units in all Advanced Higher mathematics Courses. As part of this project, Advanced Higher Applied Mathematics would be redesigned to remove alternative mandatory Units, and to reduce the number of optional Units in the Course.

Recommendations from the investigation would be implemented as soon as is practicable for centres and SQA.

## 2.2 Assessment rationale

**Internal assessment**

All Units are internally assessed against the requirements of the Outcomes, PCs and Evidence Requirements detailed in the Unit specifications. All Courses are assessed externally against the Grade Descriptions, as detailed in the Course Arrangements.

Intermediate 1, Intermediate 2, Higher and Advanced Higher Mathematics Courses each have three single-credit component Units — two mandatory and one optional. Each Unit has clearly-defined assessment parameters detailed in the Outcomes and PCs and exemplified in the NAB item. NAB items take the form of test papers and marking schemes with given threshold scores. The tests consist of

questions that assess whether or not the Unit Outcomes have been met — there is no 'headroom' (ie the nature of the test questions dictates that only Unit level performance can be demonstrated).

**Relationship between internal and external assessment**

For each Mathematics Course, there is a separation between the design parameters for Unit NAB tests and the design parameters for Course examinations. These ensure that there is clear differentiation between the requirements of the Units and the Course.

**Estimates and appeals**

The Mathematics NAB tests can provide a general indication of how a candidate is likely to perform in the Course. Mathematics NAB tests do not generate sufficient evidence to assess candidates against the Grade Descriptions, particularly at A and B grades. Performance in NAB test items, especially when consistently high scoring, can contribute to evidence for Estimates and Appeals, but only as supporting evidence.

Evidence for Estimates and Appeals must show a sufficient breadth of coverage of the component Units of each Course, and must relate to the Grade Descriptions. This could come, for example, from a prelim fully reflecting the nature, scope and depth of the relevant Question Paper. It is important that evidence for Estimates is based on a sufficient breadth of coverage of at least two component Units, an appropriate balance of routine and non-routine questions, and a appropriate balance of questions assessing grades C, B and A.

### Does the assessment approach meet the assessment criterion (8)?

Yes.

### Issues and recommendations

None.

## 2.3 Internal assessment of Units

### Description of the overall approach to internal assessment

In all but five of the Units at all levels in Mathematics (and Advanced Higher Applied Mathematics), Outcomes are assessed by a holistic end-of-Unit, unseen, closed-book written test.

In the five exceptional Units (*Applications of Mathematics* at Intermediate 1 and 2, Advanced Higher *Statistics 1* and *2*, and Advanced Higher *Numerical Analysis 1*), one Outcome is an assignment, which is a natural part of the learning and teaching of the Unit. There are opportunities for discussion between student and teacher/lecturer, and these consultative activities may take up to 5 or 6 hours, but it should take approximately an hour for each candidate to complete their final report. The other Outcomes in each of these five Units are assessed by the same type of end of Unit test described above.

The duration of all end-of-Unit tests is approximately 45 minutes, with flexibility to suit individual circumstances. Successful achievement of a Unit is demonstrated by the candidate achieving the threshold of attainment (ie the cut-off score) specified for the Unit. The threshold score is set at a level which gives confidence that all the Performance Criteria for each Outcome have been attained, and varies across Units. Each Outcome assessment has its own threshold score — candidates who fail to achieve a threshold score need only be re-tested on the Outcome where it has not been achieved.

Most Units, then, would involve 45 minutes of assessment activity.

Comments submitted to the review reveal a sizeable minority with adverse opinions on internal assessment. However, these responses were not related to the burden or volume of assessment, but to the lack of headroom in the NABs and the part they play in estimates and appeals. It should also be noted that most of these respondents were opposed to internal assessment per se.

### Note on comparability with other Courses in the same broad subject area

The volume and level of demand of internal assessment in Mathematics is appropriate and compares favourably with Physics, Technological Studies and Computing.

### Does the internal assessment of the Units meet the assessment criterion (9)?

Yes.

### Issues and recommendations

None.

## 2.4 External assessment of the Course

### Description of the overall approach to external assessment

**Intermediate 1 Mathematics - 2 component parts**

Paper 1 (non-calculator) – 35 minutes
Paper 2 – 55 minutes

**Intermediate 2 Mathematics - 2 component parts**

Paper 1 (non-calculator) – 45 minutes
Paper 2 – 1 hour 30 minutes

**Higher Mathematics - 2 component parts**

Paper 1(non-calculator) – 1 hour 10 minutes
Paper 2 – 1 hour 30 minutes

**Advanced Higher Mathematics - 1 component**

Examination – 3 hours

**Advanced Higher Applied Mathematics - 1 component**

Examination – 3 hours

Intermediate 1, Intermediate 2 and Higher all have an examination consisting of two question papers, one of which does not allow the use of calculators. The Advanced Highers only have one question paper in their examination.

### Note on comparability with other Courses in the same broad subject area

The volume, nature and weight of external assessment for the Intermediate 1, Intermediate 2, Higher and Advanced Higher mathematics Courses is comparable with Technological Studies (which has slightly longer examinations), Physics and Computing.

At Higher, Mathematics has a negative (ie more difficult) National Rating of -0.26, and is placed towards the top of a broad grouping of Physics, Technological Studies, Mathematics, Information Systems and Computing. A similar picture emerges at other levels.

### Does the external assessment of the Courses meet the assessment criterion (10)?

Yes

## Issues and recommendations

**1. Issue: Design of questions in examination papers in Mathematics**

The setting team has difficulty in producing appropriately levelled and differentiating (particularly between A and B Grades) examination questions papers within the current examination time constraints.

All Mathematics examination Question Papers contain a balance of short questions (designed to mainly test knowledge and understanding) and extended response questions which also test problem solving skills. Some questions are set in more complex contexts to provide evidence for performance at Grades A and B. Currently, approximately 60% of the questions give opportunities at Grade C. This relatively high mark for a Grade C reflects a more general problem of setting Mathematics papers at both an appropriate level of performance (ie 50%, 60%, and 70% cut-off scores corresponding to grades C, B and A) and with adequate opportunities to differentiate between grades.

It could be argued that increasing examination time would allow for appropriate levelling and differentiation of Question Paper responses. However, increasing the duration of the examinations and the number of questions would increase the complexity and burden of assessment for candidates. Other approaches have to be explored — for example, it should be possible to develop an assessment model based on increasing the number of marks available and allowing differentiation within each individual examination question.

**Recommendation: Design of questions in examination papers in Mathematics**

Investigate assessment methodologies, techniques, and designs, that facilitate levelling questioning in Mathematics examination papers (at all levels).[IMG1] Practicable models should be developed to support examination-setting teams. This should have the effect of making Mathematics examinations a less stressful experience for candidates. Recommendations from the investigation should be implemented as soon as is practicable for centres and SQA.

**2. Issue: Advances in calculator technologies and capabilities**

The introduction of non-calculator and calculator examination papers has increased opportunities for differentiating questions in the mathematics examinations. The advances in calculator technologies in recent years (eg in graphical and calculus work) have also increased these opportunities. As with many areas, the rate of technological advance has outstripped the development of assessment methodologies.

Comments have been received that the current calculator question papers can be tackled almost completely without a calculator — where required, its use is relatively basic. Advanced calculators are being used in school examinations in other countries and, in the correct context, they have much to offer the range and scope of examination questioning.

**Recommendation: Advances in calculator technologies and capabilities**

The use of advance calculators in examination question papers should be fully researched and internationally benchmarked, and practicable methodologies should be developed to assist setting teams in utilising their full assessment potential.

# 2.5 Quality Assurance

## Description of the overall approach to quality assurance

SQA's standard setting, vetting and marking procedures apply. Attendance at markers' meetings is mandatory.

## Do the quality assurance arrangements for the Courses meet criterion 11?

Yes

## Issues and recommendations

None

## 2.6 Administration

### Issues and Recommendations

None

# 3 Summary of review process and issues raised

| Who | Mechanism | Feedback received |
|---|---|---|
| Subject specialists | Direct from Mathematics Assessment Panel<br><br>Consultation responses received by post, fax and e-mail from the field (i.e. Mathematics Council, Principal Teachers, Teachers and Lecturers, HE Institutions) | <br><br>A separate collation will be made available |
| SQA co-ordinators | SQA SAMs and CRMs | No issues raised |
| Principal Assessors and Senior Moderators | Direct and via Panel | Increase length of examination, with particular reference to calculator and non-calculator timing<br>Use of advanced calculators<br>The Assessment Panel rejected a proposal for 'graded' NABs |
| Units within SQA | Internal views sought, especially from Question Paper Unit | No issues |
| Candidates and parents | SPTC Survey | Internal Assessment performance poor indicator of the final exam performance.<br>Pupils found the exam 'too hard' - although fewer (28) than in 2000 (109).<br>Maths and English examination too close together. |
| Other surveys and reports | MORI January 2001<br><br>HMI – National Investigation into the Experience of Higher Still Assessment in Schools and Colleges<br><br>HMIE (August 2001) | Teachers of Mathematics (72%) are more inclined to maintain that students are not prepared for the external exam than other subjects<br><br>External examination was considered much more difficult than the Unit assessment<br><br>Teachers criticised SQA for issuing changes to NABs during the course of the session, a particular concern in a number of mathematics departments |

# Appendix 1 – facts and figures

## Approximate total assessment loads

**Mathematics, Int 1**

NAB Test: 3 x 45 min = 2 hr 15 min
External Assessment: Paper 1 (non-calculator) - 35 min; Paper 2 - 55 min = 1 hr 30 min
Total = 3 hr 45 min

**Mathematics, Int 2**

NAB Test: 3 x 45 min = 2 min 15 min
External Assessment: Paper 1 (non-calculator) - 45 min; Paper 2 - 1hr 30 min = 2 hr 15 min
Total = 4 hr 30 min

**Mathematics, H**

NAB Test: 3 x 45 min = 2 hr 15 min
External Assessment: Paper 1 (non-calculator) - 1hr 10 minutes; Paper 2 - 1hr 30 min = 2 hr 40 min
Total = 4 hr 55 min

**Mathematics, AH**

NAB Test: 3 x 1 hr/* + approx 2 hr = 3 hr /*5 hr
External Assessment = 3 hr examination
Total = 6 hr/*8 hr

**Applied Mathematics, AH**

NAB Test: 3 x 1 hr/* + approx 1 hr = 3 hr/*4 hr
External Assessment = 3 hr examination
Total = 6 hr/*7 hr + prelim

*depending on chosen optional Unit