SCOTTISH QUALIFICATIONS AUTHORITY

MATHEMATICS
Advanced Higher

Fifth edition – published August 2003

NOTE OF CHANGES TO ARRANGEMENTS FIFTH EDITION PUBLISHED AUGUST 2003

COURSE TITLE: Mathematics (Advanced Higher)

COURSE NUMBERS AND TITLES FOR ENTRY TO COURSES : C100 13 Mathematics: Maths 1, 2 and 3

National Course Specification

Course Details — Core skills details updated.

National Unit Specification:

All Units — No optional units.

Note that the optional component units of the Advanced Higher Mathematics course prior to the 2004 diet of examinations:

D326 13	Statistics 1 (AH)
D327 13	Mechanics 1 (AH)
D328 13	Numerical Analysis 1 (AH)

are still available as free standing units.

These units were designed to provide a rounded experience of each of these applied mathematics topics for candidates not wishing to take the more in depth study offered in the Advanced Higher Applied Mathematics course.

National Course Specification

MATHEMATICS (ADVANCED HIGHER)

COURSE NUMBER C100 13 Mathematics: Maths 1, 2 and 3

COURSE STRUCTURE

C100 13 Mathematics: Maths 1, 2 and 3

This course consists of three mandatory units as follows:

D321 13	*Mathematics 1 (AH)*	*1 credit (40 hours)*
D322 13	*Mathematics 2 (AH)*	*1 credit (40 hours)*
D323 13	*Mathematics 3 (AH)*	*1 credit (40 hours)*

Administrative Information

Publication date: August 2003

Source: Scottish Qualifications Authority

Version: 05

National Course Specification: general information (cont)

COURSE Mathematics (Advanced Higher)

In common with all courses, this course includes 40 hours for induction, extending the range of learning and teaching approaches, additional support, consolidation, integration of learning and preparation for external assessment. This time is an important element of the course and advice on the use of the overall 160 hours is included in the course details.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained a Higher Mathematics course award or its component units or equivalent. *Mathematics 1 (AH)* assumes knowledge of outcomes 2 and 3 of *Mathematics 3 (H)*.

CORE SKILLS

The course gives automatic certification of the following:

Complete core skills for the course	Numeracy	H
Additional core skills components for the course	Critical Thinking	H

For information about the automatic certification of core skills for any individual unit in this course, please refer to the general information section at the beginning of the unit.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifcations 2001/2002 BA0906 August 2001.*

National Course Specification: course details

COURSE Mathematics (Advanced Higher)

RATIONALE

As with all mathematics courses, Advanced Higher Mathematics aims to build upon and extend candidates' mathematical skills, knowledge and understanding in a way that recognises problem solving as an essential skill and enables them to integrate their knowledge of different aspects of the subject. The aim of developing mathematical skills and applying mathematical techniques in context will be furthered by exploiting the power of calculators and computer software where appropriate.

Because of the importance of these features, the grade descriptions for Advanced Higher Mathematics emphasise the need for candidates to undertake extended thinking and decision making to solve problems and integrate mathematical knowledge. The use of coursework tasks, therefore, to practise problem solving as set out in the grade descriptions, is strongly encouraged.

The course offers candidates, in an interesting and enjoyable manner, an enhanced awareness of the range and power of mathematics.

COURSE CONTENT

The syllabus is designed to build upon and extend candidates' mathematical learning in the areas of algebra, geometry, and calculus. The units, *Mathematics 1 (AH)* and *Mathematics 2 (AH), and Mathematics 3 (AH)* are progressive and continue the development of algebra, geometry and calculus from Higher level.

COURSE Mathematics (Advanced Higher)

The outcomes and the performance criteria for each unit are statements of basic competence. Additionally, the course makes demands over and above the requirements of individual units. Within the 40 hours of flexibility time, candidates should be able to integrate their knowledge across the component units *Mathematics 1 (AH), Mathematics 2 (AH)* and *Mathematics 3 (AH).* The experience of extended thinking and decision making is important and will be enhanced when candidates are exposed to coursework tasks which require them to interpret problems, select appropriate strategies, come to conclusions and communicate these intelligibly.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

Detailed content

The content listed below should be covered in teaching the course. All of this content will be subject to sampling in the external assessment. Where comment is offered, this is intended to help in the effective teaching of the course.

References in this style indicate the depth of treatment appropriate to grades A and B.

CONTENT	COMMENT	TEACHING NOTES
Mathematics 1 (AH) **Algebra**		
know and use the notation $n!$, ${}^nC_r$ and $\binom{n}{r}$	eg Calculate	Candidates should be aware of the size of $n!$ for small values of n, and that calculator results consequently are often inaccurate (especially if the
know the results $\binom{n}{r}=\binom{n}{n-r}$	eg Solve, for $n \in \mathbf{N}$,	formula ${}^nC_r = \dfrac{n!}{r!(n-r)!}$ is used). These results
and $\binom{n}{r-1}+\binom{n}{r}=\binom{n+1}{r}$		can be established numerically or from Pascal's triangle. This will be linked with elementary number theory in Mathematics 2 (AH), where they provide an opportunity to introduce the concept of direct proof.
know Pascal's triangle	Pascal's triangle should be extended up to $n = 7$.	For assessment purposes $n \le 5$.
know and use the binomial theorem $(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$, for $r, n \in \mathbf{N}$	eg Expand $(x + 3)^4$ eg Expand $(2u - 3v)^5$ [A/B]	In the work on series and complex numbers (de Moivre's theorem) the binomial theorem will be extended to integer and rational indices.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
evaluate specific terms in a binomial expansion	eg Find the term in x^7 in $\left(x+\frac{2}{x}\right)^9$	
express a proper rational function as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	eg Express $\frac{5-10x}{1-3x-4x^2}$ in partial fractions.	The denominator may include a repeated linear factor or an irreducible quadratic factor. This is also required for integration of rational functions and useful for graph sketching when asymptotes are present.
include cases where an improper rational function is reduced to a polynomial and a proper rational function by division or otherwise [A/B]	eg Express $\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}$ in partial fractions [A/B].	When the degree of the numerator of the rational function exceeds that of the denominator by 1, non-vertical asymptotes occur.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
Differentiation know the meaning of the terms limit, derivative, differentiable at a point, differentiable on an interval, derived function, second derivative use the notation: $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ recall the derivatives of x^{α} (α rational), $\sin x$ and $\cos x$		
know and use the rules for differentiating linear sums, products, quotients and composition of functions: $(f(x)+g(x))' = f'(x)+g'(x)$ $(kf(x))' = kf'(x)$, where k is a constant the chain rule: $(f(g(x)))' = f'(g(x)).g'(x)$ the product rule: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ the quotient rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$		Candidates should be exposed to formal proofs of differentiation, although proofs will not be required for assessment purposes. Once the rules for differentiation have been learned, computer algebra systems (CAS) may be used for consolidation/extension. However, when CAS are being used for difficult/real examples the emphasis should be on the understanding of concepts rather than routine computation. When software is used for differentiation in difficult cases, candidates should be able to say which rules were used.
differentiate given functions which require more than one application of one or more of the chain rule, product rule and the quotient rule [A/B]		

CONTENT	COMMENT	TEACHING NOTES
know the derivative of tan x the definitions and derivatives of sec x, cosec x and cot x the derivatives of e^x (exp x) and ln x		Link with the graphs of these functions. The definitions of e^x and ln x should be revised and examples given of their occurrence.
know the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$		Candidates should be aware that not all functions are differentiable everywhere, eg $f(x) = \|x\|$ at $x = 0$. The use of software allows further exploration here.
know the definition of higher derivatives $f^n(x)$, $\frac{d^n y}{dx^n}$		Candidates should also know that higher derivatives can have discontinuities and be aware of the graphical effects of this, ie lack of smoothness. An example of this is: $f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ for which f and f' are continuous but f'' is not.
apply differentiation to: a) rectilinear motion b) extrema of functions: the maximum and minimum values of a continuous function f defined on a closed interval [a,b] can occur at stationary points, end points or points where f' is not defined [A/B] c) optimisation problems	eg Find the acceleration of a particle whose displacement s metres from a certain point at time t seconds is given by $s = 8 - 75t + t^3$. eg Find the maximum value of the function $f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$ [A/B]	Optimisation problems should be linked with graph sketching.

CONTENT	COMMENT	TEACHING NOTES
Integration know the meaning of the terms integrate, integrable, integral, indefinite integral, definite integral and constant of integration		CAS may be used for consolidation/extension. However, when CAS are being used for difficult/real examples the emphasis should be on understanding of the concepts rather than routine computation. When software is being used for integration in difficult cases, candidates should be able to say which rules were used.
recall standard integrals of x^{α} $(\alpha \in Q, \alpha \neq -1)$, $\sin x$ and $\cos x$		
$\int (af(x) + bg(x))dx = a\int f(x)dx + b\int g(x)dx,\ a, b \in \mathbf{R}$		
$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx\ ,\ a < c < b$		
$\int_b^a f(x)dx = -\int_a^b f(x)dx,\ b \neq a$		
$\int_a^b f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$		
know the integrals of e^x, x^{-1}, $\sec^2 x$		

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
integrate by substitution: expressions requiring a simple substitution	Candidates are expected to integrate simple functions on sight. eg $\int xe^{x^2}\,dx$	Where substitutions are given they will be of the form $x = g(u)$ or $u = g(x)$.
expressions where the substitution will be given	eg $\int \cos^3 x \sin x\,dx$, $u = \cos x$	
the following special cases of substitution		
$\int f(ax+b)dx$	eg $\int \sin(3x+2)dx$	
$\int \frac{f'(x)}{f(x)}dx$	eg $\int \frac{2x}{x^2+3}dx$	
use an elementary treatment of the integral as a limit using rectangles		
apply integration to the evaluation of areas including integration with respect to y [A/B].	Other applications may include (i) volumes of simple solids of revolution (disc/washer method) (ii) speed/time graph [A/B].	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
Properties of functions know the meaning of the terms function, domain, range, inverse function, critical point, stationary point, point of inflexion, concavity, local maxima and minima, global maxima and minima, continuous, discontinuous, asymptote		Candidates are expected to recognise graphs of simple functions, be able to sketch the graphs by hand and know their key features, eg behaviour of trigonometric and exponential functions. The assessment should be structured to ensure that candidates carry out and display the calculations required to identify the important features on the graph. Candidate learning can be enhanced through the use of calculators with a graphic facility and CAS.
determine the domain and the range of a function		
use the derivative tests for locating and identifying stationary points	ie Concave up $\Leftrightarrow f''(x) > 0$; concave down $\Leftrightarrow f''(x) < 0$; a necessary and sufficient condition for a point of inflexion is a change in concavity.	Care should be exercised when using the second derivative test in preference to the first derivative test. The second derivative may not exist, and even when it does and can easily be computed, it may not be helpful, eg the function $f(x) = x^4$ at $x = 0$. Here, $f''(0) = 0$ which is inconclusive. The first derivative test, however, easily shows there is a local minimum at $x = 0$.
sketch the graphs of sin x, cos x, tan x, e^x, ln x and their inverse functions, simple polynomial functions		

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
know and use the relationship between the graph of $y = f(x)$ and the graphs of $y = kf(x)$, $y = f(x) + k$, $y = f(x + k)$, $y = f(kx)$, where k is a constant		
know and use the relationship between the graph of $y = f(x)$ and the graphs of $y= \|f(x)\|$ $y = f^{-1}(x)$	ie Reflection in the line $y = x$ eg $f(x) = e^x,\ -\infty < x < \infty$ $f^{-1}(x) = \ln x,\ x > 0$	Care must be taken over the domain and range when finding inverses.
given the graph of a function f, sketch the graph of a related function		
determine whether a function is even or odd or neither and symmetrical and use these properties in graph sketching		
sketch graphs of real rational functions using available information, derived from calculus and/or algebraic arguments, on zeros, asymptotes (vertical and non-vertical), critical points, symmetry	For rational functions, the degree of the numerator will be less than or equal to three and the denominator will be less than or equal to two.	

CONTENT	NOTES	TEACHING NOTES
Systems of linear equations use the introduction of matrix ideas to organise a system of linear equations know the meaning of the terms matrix, element, row, column, order of a matrix, augmented matrix use elementary row operations (EROs) reduce to upper triangular form using EROs solve a 3 × 3 system of linear equations using Gaussian elimination on an augmented matrix find the solution of a system of linear equations Ax = b, where A is a square matrix, include cases of unique solution, no solution (inconsistency) and an infinite family of solutions [A/B]. know the meaning of the term ill-conditioned [A/B]. compare the solutions of related systems of two equations in two unknowns and recognise	Ill-conditioning can be introduced by comparing the solutions of the following systems: a. $x + 0.99y = 1.99$ $0.99x + 0.98y = 1.97$ b. $x + 0.99y = 2.00$ **$0.99x + 0.98y = 1.97$ [A/B]**	Only 3 × 3 cases are required for assessment purposes, and at grade C they are restricted to those with a unique solution. Larger systems can be tackled using computer packages or advanced calculators.

ill-conditioning [A/B]		

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Mathematics 2 (AH)		
Further differentiation know the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$		Link with the graphs of these functions in Mathematics 1 (AH).
differentiate any inverse function using the technique: $y = f^{-1}(x) \Rightarrow f(y) = x \Rightarrow (f^{-1}(x))' f'(y) = 1$, etc., and know the corresponding result $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$		This technique should be initially used on the inverse trigonometric functions. Calculators with graphic facility and computer packages may be used to investigate these derivatives. Link the last result with the chain rule applied to $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ and with the logarithmic and exponential functions.
understand how an equation $f(x, y) = 0$ defines y implicitly as one (or more) function(s) of x	eg , $x^2 + y^2 = 1 \Leftrightarrow y = \pm\sqrt{1 - x^2}$, ie two functions defined on [–1, 1].	
use implicit differentiation to find first and second derivatives [A/B]		Link with obtaining the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$.
use logarithmic differentiation, recognising when it is appropriate in extended products and quotients and indices involving the variable [A/B].	eg Find the derivatives of $y = \dfrac{x^2\sqrt{7x-3}}{1+x}$, $y = 2^x$ [A/B].	
understand how a function can be defined parametrically		

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
understand simple applications of parametrically defined functions	eg $x^2 + y^2 = r^2$, $x = r\cos\theta$, $y = r\sin\theta$	
use parametric differentiation to find first and second derivatives [A/B], and apply to motion in a plane	eg If the position is given by $x = f(t)$, $y = g(t)$ then the velocity components are given by $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and the speed by $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ The instantaneous direction of motion is given by $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	
apply differentiation to related rates in problems where the functional relationship is given explicitly or implicitly	eg Explicitly: $V = \frac{1}{3}\pi r^2 h$; given $\frac{dh}{dt}$, *find* $\frac{dV}{dt}$. Implicitly: $x^2 + y^2 = r^2$ where x, y are functions of t; given $\frac{dx}{dt}$, find $\frac{dy}{dt}$ using $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.	
solve practical related rates by first establishing a functional relationship between appropriate variables [A/B]		

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Further integration know the integrals of $\frac{1}{\sqrt{1-x^2}}, \frac{1}{1+x^2}$		Link this with the derivatives of inverse trigonometric functions.
use the substitution $x = at$ to integrate functions of the form $\frac{1}{\sqrt{a^2-x^2}}, \frac{1}{a^2+x^2}$		
integrate rational functions, both proper and improper, by means of partial fractions; the degree of the denominator being ≤ 3 the denominator may include: (i) two separate or repeated linear factors (ii) three linear factors [A/B] (iii) a linear factor and an irreducible quadratic factor [A/B]		Candidates' learning can be enhanced by completing the square but this will not be formally assessed.
integrate by parts with one application	eg $\int x \sin x dx$	
integrate by parts involving repeated applications [A/B]	eg $\int x^2 e^{3x} dx$ [A/B].	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
know the definition of differential equation and the meaning of the terms linear, order, general solution, arbitrary constants, particular solution, initial condition solve first order differential equations (variables separable)	 ie, equations that can be written in the form $\frac{dy}{dx} = \frac{g(x)}{h(y)}$	Link with differentiation. Begin by verifying that a particular function satisfies a given differential equation. Candidates should know that differential equations arise in modelling of physical situations, such as electrical circuits and vibrating systems, and that the differential equation describes how the system will change with time so that initial conditions are required to determine the complete solution.
formulate a simple statement involving rate of change as a simple separable first order differential equation, including the finding of a curve in the plane, given the equation of the tangent at (*x,* y), which passes through a given point		Link with motion in a straight line. Further similar contexts can be found in the Mechanics 1 (AH) unit.
know the laws of growth and decay: applications in practical contexts		Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good motivating examples and can build on the knowledge and use of logarithms.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Complex numbers know the definition of i as a solution of $z^2 + 1 = 0$, so that $i = \sqrt{-1}$		A suggested approach is through the solution of quadratics. The introduction of i then allows the provision of two solutions for all quadratics. Thereafter, an essentially geometric approach is recommended.
know the definition of the set of complex numbers as $C = \{a + ib : a, b \in R\}$		
know the definition of real and imaginary parts		
know the terms complex plane, Argand diagram		
plot complex numbers as points in the complex plane		Plotting complex numbers as points can be made easier because some software (and calculators) write complex numbers as ordered pairs.
perform algebraic operations on complex numbers: equality (equating real and imaginary parts), addition, subtraction, multiplication and division		Multiplication by i is equivalent to rotation by 90° and demonstrates a link between operations and transformations in the plane. This could be linked with Matrices in Mathematics 3 (AH).
evaluate the modulus, argument and conjugate of complex numbers		

CONTENT	NOTES	TEACHING NOTES
convert between Cartesian and polar form		$x = r\cos\theta$, $y = r\sin\theta$, $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$. The commonly used practice of writing $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ should be used with reference to quadrants. The exponential form $z = re^{i\theta}$ provides a link with power series in Further sequences and series in Mathematics 3 (AH).
know the fundamental theorem of algebra and the conjugate roots property		The facility in CAS to produce complex factors of polynomials allows candidates to conjecture that a polynomial of degree n has n roots, and that these occur in conjugate pairs when the coefficients are real.
factorise polynomials with real coefficients	eg Find the roots of a quartic when one complex root is given.	
solve simple equations involving a complex variable by equating real and imaginary parts		The triangle inequality $\lvert z + w\rvert \le \lvert z\rvert + \lvert w\rvert$ can be mentioned here.
interpret geometrically certain equations or inequalities in the complex plane	eg $\lvert z\rvert = 1$ $\lvert z - a\rvert = b$ $\lvert z - 1\rvert = \lvert z - i\rvert$ $\lvert z - a\rvert > b$	The proof of de Moivre's theorem for integer powers provides an opportunity to illustrate proof by induction. [Link with Further number theory in Mathematics 3(AH)]. For fractional powers the result can be stated without proof.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
know and use de Moivre's theorem with positive integer indices and fractional indices [A/B]	eg Expand $(\cos\theta + i\sin\theta)^3$ eg Expand $(\cos\theta + i\sin\theta)^{1/2}$ [A/B]	
apply de Moivre's theorem to multiple angle trigonometric formulae [A/B]	eg Express $\sin 5\theta$ in terms of $\sin\theta$ only [A/B] Express $\cos^3\theta$ in terms of $\cos\theta$ and $\cos 3\theta$ [A/B]	For assessment purposes only multiples and powers less than or equal to 5 are required.
apply de Moivre's theorem to find *n*th roots of unity [A/B]		Link with the Argand diagram. An investigative approach to solving $z^n = 1$, $n \geq 2$, may be adopted, leading to discussion of the geometrical significance.
Sequences and series know the meaning of the terms infinite sequence, infinite series, *n*th term, sum to *n* terms (partial sum), limit, sum to infinity (limit to infinity of the sequence of partial sums), common difference, arithmetic sequence, common ratio, geometric sequence, recurrence relation		Higher work on first order linear recurrence relations should be revised. Develop arithmetic and geometric sequences as special cases by considering $x_{n+1} = ax_n + b$ with $a = 1, b \neq 0$ for arithmetic sequences and then $a \neq 0, b = 0$ for geometric sequences.
know and use the formulae $u_n = a + (n-1)d$ and $S_n = \frac{1}{2}n[2a + (n-1)d]$ for the *n*th term and the sum to *n* terms of an arithmetic series, respectively		These results should be explored numerically, conjectures made and proved.

CONTENT	NOTES	TEACHING NOTES
know and use the formulae $u_n = ar^{n-1}$ and $S_n = \frac{a(1-r^n)}{1-r}$, $r \neq 1$, for the *n*th term and the sum to *n* terms of a geometric series, respectively		
know and use the condition on *r* for the sum to infinity to exist and the formula $S_\infty = \frac{a}{1-r}$ for the sum to infinity of a geometric series where $\lvert r \rvert < 1$		
expand $\frac{1}{1-r}$ as a geometric series and extend to $\frac{1}{a+b}$ [A/B]		This result should be linked to the binomial expansion $(1 - r)^{-1}$. In addition, link to Maclaurin expansions in Mathematics 3 (AH).
know the sequence $\left(1+\frac{1}{n}\right)^n$ and its limit		The definition $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ should be explored numerically.
know and use the Σ notation		
know the formula $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ and apply it to simple sums	eg $\sum_{r=1}^{n}(ar+b) = a\sum_{r=1}^{n} r + \sum_{r=1}^{n} b$	Link with proof by mathematical induction in the next section.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Elementary number theory and methods of proof understand the nature of mathematical proof	Simple examples involving the real number line, inequalities and modulus. eg $\lvert x + y\rvert \leq \lvert x\rvert + \lvert y\rvert$	Candidates should appreciate the need for proof in mathematics. A general strategy which could be used would be to explore a situation, make conjectures, verify (perhaps using software) and then finally prove.The triangle inequality can be linked with complex numbers.
understand and make use of the notations $\Rightarrow, \Leftarrow$ and $\Leftrightarrow$		
know the corresponding terminology implies, implied by, equivalence		
know the terms natural number, prime number, rational number, irrational number		
know and use the fundamental theorem of arithmetic		
disprove a conjecture by providing a counter-example		
use proof by contradiction in simple examples	eg The irrationality of $\sqrt{2}$. eg The infinity of primes.	It should be emphasised that examples do not prove a conjecture except in proof by exhaustion.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
use proof by mathematical induction in simple examples		The concept of mathematical induction may be introduced in a familiar context such as demonstrating that 2p coins and 5p coins are sufficient to pay any sum of money greater than 3p.
prove the following results $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$		Candidates should use the application of: $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ to prove results about other sums, such as the sum of the first n odd numbers is a perfect square.
the binomial theorem for positive integers de Moivre's theorem for positive integers	Straightforward proofs could be asked for in assessments without guidance. eg $1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1$, for all $n \in \mathbf{N}$ eg 8^n is a factor of $(4n)!$ for all $n \in \mathrm{N}$ eg $n < 2^n$ for all $n \in \mathrm{N}$	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Mathematics 3 (AH) **Vectors** know the meaning of the terms position vector, unit vector, scalar triple product, vector product, components, direction ratios/cosines		Higher work on vectors should be revised.
calculate scalar and vector products in three dimensions		The triangle inequality can now be completed with the vector version: $\lvert \boldsymbol{a} + \boldsymbol{b} \rvert \leq \lvert \boldsymbol{a} \rvert + \lvert \boldsymbol{b} \rvert$
know that $\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a}$		
find $\boldsymbol{a} \times \boldsymbol{b}$ and $\boldsymbol{a} \cdot \boldsymbol{b} \times \boldsymbol{c}$ in component form		This should be linked with determinants of matrices.
know the equation of a line in vector form, parametric and symmetric form		
know the equation of a plane in vector form, parametric and Cartesian form		
find the equations of lines and planes given suitable defining information	eg Find the equation of a plane passing through a given point and perpendicular to a given direction (the normal).	
find the angles between two lines, two planes [A/B], and between a line and a plane		Link with systems of equations.
find the intersection of two lines, a line and a plane, and two or three planes		

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Matrix algebra know the meaning of the terms matrix, element, row, column, order, identity matrix, inverse, determinant, singular, non-singular, transpose		Link determinant with vector product.
perform matrix operations: addition, subtraction, multiplication by a scalar, multiplication, establish equality of matrices		Link with Systems of linear equations in Mathematics 1 (AH).
know the properties of the operations: A + B = B + A AB $\neq$ BA in general (AB)C = A(BC) A(B + C) = AB + AC (A')' = A (A + B)' = A' + B' (AB)' = B'A' $(AB)^{-1} = B^{-1}A^{-1}$ det (AB) = det A det B		
calculate the determinant of 2 × 2 and 3 × 3 matrices		
know the relationship of the determinant to invertability		
find the inverse of a 2 × 2 matrix		The result $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ should be known.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
find the inverse, where it exists, of a 3 × 3 matrix by elementary row operations		Extension to larger matrices may be considered as a use of technology.
know the role of the inverse matrix in solving linear systems		Reference can also be made to the problem of finding the intersection of three lines or three planes in vector work and link its solubility to the existence of the matrix of coefficients.
use 2 × 2 matrices to represent geometrical transformations in the (x, y) plane		The transformations should include rotations, reflections, dilatations and the role of the transpose in orthogonal cases. Link with complex numbers in Mathematics 2 (AH).
Further sequences and series know the term power series		
understand and use the Maclaurin series: $f(x) = \sum_{r=0}^{\infty} \frac{x^r}{r!} f^{(r)}(0)$ find the Maclaurin series of simple functions: e^x, $\sin x$, $\cos x$, $\tan^{-1} x$, $(1 + x)^{\alpha}$, $\ln(1 + x)$, knowing their range of validity		The approach taken should be through polynomials and their derivatives, linking coefficients to values at zero. Graphics software provides a powerful illustration of the concept of convergence. The difficulties of differentiating or integrating power series term by term should be stressed, pointing out that these processes are only valid within the interval of convergence

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
		Link $(1 + x)^{\alpha}$ to $(1 - r)^{-1}$ from series work and to the extension of the binomial theorem.
find the Maclaurin expansions for simple composites, such as e^{2x}		
use the Maclaurin series expansion to find power series for simple functions to a stated number of terms		
use iterative schemes of the form $x_{n+1} = g(x_n)$, $n = 0, 1, 2, \ldots$ to solve equations where $x = g(x)$ is a rearrangement of the original equation	eg $xe^x = 1 \Rightarrow x = e^{-x}$ to give the scheme $x_{n+1} = e^{-x_n}$ with $x_0 = 0.5$	Candidates should be made aware that the equation $f(x) = 0$ can be rearranged in a variety of ways and that some of these may yield suitable iterative schemes while others may not.
use graphical techniques to locate approximate solution x_0		
know the condition for convergence of the sequence $\{x_n\}$ given by $x_{n+1} = g(x_n)$, $n = 0, 1, 2, \ldots$		Cobweb and staircase diagrams help to demonstrate the test for convergence of an iterative scheme for finding a fixed point α ($\alpha = g(\alpha)$) in a neighbourhood of α, namely $\lvert g'(x)\rvert < 1$.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Further ordinary differential equations solve first order linear differential equations using the integrating factor method	eg Write the linear equation $a(x)\frac{dy}{dx} + b(x)y = g(x)$ in the standard form $\frac{dy}{dx} + P(x)y = f(x)$ and hence as $\frac{d}{dx}\left(e^{\int P(x)dx}y\right) = e^{\int P(x)dx}f(x)$	
find general solutions and solve initial value problems	eg Mixing problems, such as salt water entering a tank of clear water which is then draining at a given rate. eg Growth and decay problems, an alternative method of solution to separation of variables. eg Simple electrical circuits.	
know the meaning of the terms: second order linear differential equation with constant coefficients, homogeneous, non-homogeneous, auxiliary equation, complementary function and particular integral	eg The general solution of the homogeneous equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0.$ If $f(x)$ and $g(x)$ are solutions and $f(x) \neq Cg(x)$, ie linearly independent solutions, then the general solution is $C_1 f(x) + C_2\, g(x)$.	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
solve second order homogeneous ordinary differential equations with constant coefficients $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$		Search for a trial solution using $y=e^{mx}$ and hence derive the auxiliary equation $am^2+bm+c=0$.
find the general solution in the three cases where the roots of the auxiliary equation: (i) are real and distinct (ii) coincide (are equal) [A/B] (iii) are complex conjugates [A/B]		Link with Complex numbers in Mathematics 2 (AH). Context applications could include the motion of a spring, both with and without a damping term.
solve initial value problems	eg $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$ with $y=-1$ and $\frac{dy}{dx}=2$ when $x=0$.	
solve second order non-homogeneous ordinary differential equations with constant coefficients: $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$		The general solution is the sum of the general solution of the corresponding homogeneous equation (complementary function) and a particular solution.
using the auxiliary equation and particular integral method [A/B]		For assessment purposes, only cases where the particular solution can easily be found by inspection will be required, with the right-hand side being a low order polynomial or a constant multiple of sin x, cos x or e^{kx}.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
Further number theory and further methods of proof know the terms necessary condition, sufficient condition, if and only if, converse, negation, contrapositive		Number theory can be drawn upon for likely but unproved conjectures, and also for erroneous conjectures, such as: (i) Are there an infinite number of twin primes, n and $n + 2$? (unproved) (ii) Is $2^{2^n} + 1$ prime for all $n \in$ N? (erroneous, holds for n = 1, 2, 3, 4 but not for all higher values).
use further methods of mathematical proof: some simple examples involving the natural numbers		
direct methods of proof: sums of certain series and other straightforward results	eg $x > 1 \Rightarrow x^2 > 1$, the triangle inequality, the sum to n terms of an arithmetic or geometric series.	
further proof by contradiction	eg If $x, y \in \mathbf{R}$ such that $x + y$ is irrational then at least one of x, y is irrational. eg If m,n are integers and $mn = 100$, then either $m \leq 10$ or $n \leq 10$.	
further proof by mathematical induction		
prove the following result $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1);\ n \in \mathbf{N}$	eg Proof of the result $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2 = \frac{n^2(n+1)^2}{4}$ is a useful extension.	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
know the result $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$		
apply the above results and the one for $\sum_{r=1}^{n} r$ to prove by direct methods results concerning other sums	eg $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. eg $\sum_{r=1}^{n} r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$.	The generalisation of these results provides an interesting extension exercise.
know the division algorithm and proof		
use Euclid's algorithm to find the greatest common divisor (g.c.d.) of two positive integers	ie Use the division algorithm repeatedly.	The special cases of finding the g.c.d. of two Fibonacci numbers are possible extensions.
know how to express the g.c.d. as a linear combination of the two integers [A/B]		
use the division algorithm to write integers in terms of bases other than 10 [A/B]		A possible context for this is binary arithmetic.

COURSE Mathematics (Advanced Higher)

ASSESSMENT

To gain the award of the course, the candidate must pass all unit assessments as well as the external assessment. External assessment will provide the basis for grading attainment in the course award.

Where units are taken as component parts of a course, candidates will have the opportunity to achieve at levels beyond that required to attain each of the unit outcomes. This attainment may, where appropriate, be recorded and used to contribute towards course estimates and to provide evidence for appeals. Additional details are provided, where appropriate, with the exemplar assessment materials. Further information on the key principles of assessment are provided in the paper *Assessment*, published by HSDU in May 1996.

DETAILS OF THE INSTRUMENTS FOR EXTERNAL ASSESSMENT

The external assessment will take the form of an examination of up to three hours' duration. Candidates will sit a paper assessing *Mathematics 1 (AH), 2 (AH)* and *3 (AH).* The examination will consist of a balance of short questions designed mainly to test knowledge and understanding, and extended response questions, which also test problem solving skills. These two styles of questions will include ones which are set in more complex contexts to provide evidence for performance at grades A and B.

GRADE DESCRIPTIONS FOR ADVANCED HIGHER MATHEMATICS

Advanced Higher Mathematics courses should enable candidates to solve problems which integrate mathematical knowledge across performance criteria, outcomes and units, and which require extended thinking and decision making. The award of grades A, B and C is determined by the candidate's demonstration of the ability to apply knowledge and understanding to problem solving. To achieve grades A and B in particular, this demonstration will involve more complex contexts including the depth of treatment indicated in the detailed content tables.

In solving these problems, candidates should be able to:

a) interpret the problem and consider what might be relevant;
b) decide how to proceed by selecting an appropriate strategy;
c) implement the strategy through applying mathematical knowledge and understanding and come to a conclusion;
d) decide on the most appropriate way of communicating the solution to the problem in an intelligible form.

Familiarity and complexity affect the level of difficulty of problems/assignments. It is generally easier to interpret and communicate information in contexts where the relevant variables are obvious and where their inter-relationships are known. It is usually more straightforward to apply a known strategy than to modify one or devise a new one. Some concepts are harder to grasp and some techniques more difficult to apply if they have to be used in combination.

Exemplification at grade C and grade A

a) ***Interpret the problem and consider what might be relevant***

At grade C candidates should be able to interpret and model qualitative and quantitative information as it arises within:

- the description of real-life situations
- the context of other subjects
- the context of familiar areas of mathematics

Grade A performance is demonstrated through coping with the interpretation of more complex contexts requiring a higher degree of reasoning ability in the areas described above.

b) ***Decide how to proceed by selecting an appropriate strategy***

At grade C candidates should be able to tackle problems by selecting algorithms drawn from related areas of mathematics or apply a heuristic strategy.

Grade A performance is demonstrated through an ability to decide on and apply a more extended sequence of algorithms to more complex contexts.

c) ***Implement the strategy through applying mathematical knowledge and understanding, and come to a conclusion***

At grade C candidates should be able to use their knowledge and understanding to carry through their chosen strategies and come to a conclusion. They should be able to process data in numerical and symbolic form with appropriate regard for accuracy, marshal facts, sustain logical reasoning and appreciate the requirements of proof.

Grade A performance is demonstrated through an ability to cope with processing data in more complex situations and sustaining logical reasoning, where the situation is less readily identifiable with a standard form.

d) ***Decide on the most appropriate way of communicating the solution to the problem in an intelligible form***

At grade C candidates should be able to communicate qualitative and quantitative mathematical information intelligibly and to express the solution in language appropriate to the situation.

Grade A performance is demonstrated through an ability to communicate intelligibly in more complex situations and unfamiliar contexts.

APPROACHES TO LEARNING AND TEACHING

The approaches to learning and teaching recommended for Higher level should be continued and reinforced whenever possible. Exposition to a group or class remains an essential technique. However, candidates should be more actively involved in their own learning in preparation for future study in higher education. Opportunities for discussion, problem solving, practical activities and investigation should abound in Advanced Higher Mathematics. There also exists much greater scope to harness the power of technology in the form of mathematical and graphical calculators and computer software packages.

Independent learning is further encouraged in the grade descriptions for the course. Coursework tasks and projects/assignments are recommended as vehicles for the introduction of new topics, the illustration or reinforcement of mathematics in context and for the development of extended problem solving, practical and investigative skills, as well as adding interest to the course.

SPECIAL NEEDS

This course specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessments Arrangements A0645/3 December 2001.*

SUBJECT GUIDES

A Subject Guide to accompany the Arrangements documents has been produced by the Higher Still Development Unit (HSDU) in partnership with the Scottish Consultative Council on the Curriculum (SCCC) and Scottish Further Education Unit (SFEU). The Guide provides further advice and information about:

- support materials for each course
- learning and teaching approaches in addition to the information provided in the Arrangements document
- assessment
- ensuring appropriate access for candidates with special educational needs

The Subject Guide is intended to support the information contained in the Arrangements document. The SQA Arrangements documents contain the standards against which candidates are assessed.

National Unit Specification: general information

UNIT	Mathematics 1 (Advanced Higher)
NUMBER	D321 13
COURSE	Mathematics (Advanced Higher)

SUMMARY

This unit is the first of three units, which comprise the Advanced Higher Mathematics course. This unit extends the calculus and graphicacy work from Higher level and introduces matrices for solving systems of linear equations. It provides a basis for progression to Mathematics 2 (AH).

OUTCOMES

1 Use algebraic skills.
2 Use the rules of differentiation on the elementary functions x^n ($n \in Q$), *sin x*, *cos x*, e^x and ln *x* and their composites.
3 Integrate using standard results and the substitution method.
4 Use properties of functions.
5 Use matrix methods to solve systems of linear equations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained :

- Higher Mathematics award, including Mathematics 3 (H)

Administrative Information

Superclass: RB

Publication date: August 2003

Source: Scottish Qualifications Authority

Version: 04

National Unit Specification: general information (cont)

UNIT Mathematics 1 (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher.

CORE SKILLS

Core skills for Advanced Higher remain subject to confirmation and details will be available at a later date.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications 2001/2002 BA0906 August 2001.*

National Unit Specification: statement of standards

UNIT Mathematics 1 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use algebraic skills.

Performance criteria

(a) Expand an expression of the form $(x + y)^n$, $n \in \boldsymbol{N}$ and $n \leq 5$.
(b) Express a proper rational function as a sum of partial fractions where the denominator is a quadratic in factorised form.

OUTCOME 2

Use the rules of differentiation on the elementary functions x^n, $(n \in \boldsymbol{Q})$, $\sin x$, $\cos x$, e^x and $\ln x$ and their composites.

Performance criteria

(a) Differentiate a product.
(b) Differentiate a quotient.
(c) Differentiate a simple composite function using the chain rule.

OUTCOME 3

Integrate using standard results and the substitution method.

Performance criteria

(a) Integrate an expression requiring a standard result.
(b) Integrate using a substitution method where the substitution is given.
(c) Integrate an expression requiring a simple substitution.

OUTCOME 4

Use properties of functions.

Performance criteria

(a) Find the vertical asymptote of a rational function.
(b) Find the non-vertical asymptote of a rational function.
(c) Sketch the graph of a rational function including appropriate analysis of stationary points.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 1 (Advanced Higher)

OUTCOME 5

Use matrix methods to solve systems of linear equations.

Performance criteria

(a) Use Gaussian elimination to solve a 3×3 system of linear equations.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 1 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated and for checking of accuracy and sensibility of answers to be ever present.

In this unit the algebraic skills learnt at Higher level are extended in Outcome 1 to binomial expansions and partial fractions.

In Outcomes 2 and 3, the elementary calculus studied at Higher level is extended to differentiation of sums, products, quotients and composites of elementary functions and to integration using standard results and substitution methods respectively. In both of Outcomes 2 and 3, computer algebra systems can be used extensively for consolidation and extension.

In Outcome 4 the work at Higher level on using calculus methods to sketch graphs of functions is taken further, with enhancement through the use of graphic calculators recommended.

Outcome 5 is the only outcome which does not build upon Higher content. It provides an introduction to matrix methods leading to the use of Gaussian elimination to solve a 3×3 system of linear equations.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

UNIT Mathematics 1 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

Further advice on learning and teaching approaches is contained within the subject guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessments Arrangements A064513 December 2001*.

National Unit Specification: general information

UNIT Mathematics 2 (Advanced Higher)

NUMBER D322 13

COURSE Mathematics (Advanced Higher)

SUMMARY

This unit is the second of three units, which comprise the Advanced Higher Mathematics course. It extends the calculus in Mathematics 1 (AH), extends the work on recurrence relations at Higher level and introduces complex numbers and mathematical proof.

OUTCOMES

1 Use further differentiation techniques.
2 Use further integration techniques.
3 Understand and use complex numbers.
4 Understand and use sequences and series.
5 Use standard methods to prove results in elementary number theory.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Mathematics 1 (AH)

CREDIT VALUE

1 credit at Advanced Higher.

Administrative Information

Superclass: RB

Publication date: August 2003

Source: Scottish Qualifications Authority

Version: 04

National Unit Specification: general information (cont)

UNIT Mathematics 2 (Advanced Higher)

CORE SKILLS

Core skills for Advanced Higher remain subject to confirmation and details will be available at a later date.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifcations 2001/2002 BA0906 August 2001.*

National Unit Specification: statement of standards

UNIT Mathematics 2 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use further differentiation techniques.

Performance criteria

(a) Differentiate an inverse trigonometric function (involving the chain rule).
(b) Find the derivative of a function defined implicitly.
(c) Find the first derivative of a function defined parametrically.

OUTCOME 2

Use further integration techniques.

Performance criteria

(a) Integrate a proper rational function where the denominator is a factorised quadratic.
(b) Integrate by parts with one application.
(c) Find a general solution of a first order differential equation (variables separable type).

OUTCOME 3

Understand and use complex numbers.

Performance criteria

(a) Perform a simple arithmetic operation on two complex numbers of the form a + bi.
(b) Evaluate the modulus and argument of a complex number.
(c) Convert from cartesian to polar form.
(d) Plot a complex number on an Argand diagram.

OUTCOME 4

Understand and use sequences and series.

Performance criteria

(a) Find the n^{th} term and the sum of the first n terms of an arithmetic sequence.
(b) Find the n^{th} term and the sum of the first n terms of a geometric sequence.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 2 (Advanced Higher)

OUTCOME 5

Use standard methods to prove results in elementary number theory.

Performance criteria

(a) Disprove a conjecture by providing a counter-example.
(b) Use proof by contradiction in a simple example.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessment, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 2 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated and for checking of accuracy and sensibility of answers to be ever present.

In this unit, the second of three progressive Mathematics units, outcome 1 extends the differentiation covered in Mathematics 1 (AH) to inverse functions and introduces implicit and parametric differentiation.

Integration is correspondingly extended in outcome 2 to integration by parts and partial fractions and first order differential equations are introduced.

In Outcome 3, candidates are introduced to the complex number system and are required to demonstrate competence in operations on complex numbers.

Higher level work on recurrence relations is extended to the formal study of arithmetic and geometric sequences in outcome 4 and the groundwork is laid for the study of Maclaurin expansions in Mathematics 3 (AH).

As candidates progress in mathematics they should acquire a growing awareness of the importance of mathematical proof and the need for mathematical rigour. It is for this reason that outcome 5 contains an introduction to elementary number theory and methods of proof.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

UNIT Mathematics 2 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

Further advice on learning and teaching approaches is contained within the subject guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Arrangements A0645/3 December 2001*.

National Unit Specification: general information

UNIT Mathematics 3 (Advanced Higher)

NUMBER D323 13

COURSE Mathematics (Advanced Higher)

SUMMARY

This unit is the third of three units, which comprise the Advanced Higher Mathematics course. It introduces vector equations of lines and planes, matrices and their applications to geometrical transformations, and the Maclaurin series with simple applications, and extends number theory and proof. It also extends the work on differential equations from Mathematics 2 (AH).

OUTCOMES

1 Use vectors in three dimensions.
2 Use matrix algebra.
3 Understand and use further aspects of sequences and series.
4 Solve further ordinary differential equations.
5 Use further number theory and direct methods of proof.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Mathematics 1 (AH) and Mathematics 2 (AH)

CREDIT VALUE

1 credit at Advanced Higher.

Administrative Information

Superclass: RB

Publication date: August 2003

Source: Scottish Qualifications Authority

Version: 04

National Unit Specification: general information (cont)

UNIT Mathematics 3 (Advanced Higher)

CORE SKILLS

Core skills for Advanced Higher remain subject to confirmation and details will be available at a later date.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifcations 2001/2002 BA0906 August 2001.*

National Unit Specification: statement of standards

UNIT Mathematics 3 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use vectors in three dimensions.

Performance criteria

(a) Calculate a vector product.
(b) Find the equation of a line in parametric form.
(c) Find the equation of a plane in Cartesian form given a normal and a point in the plane.

OUTCOME 2

Use matrix algebra.

Performance criteria

(a) Perform matrix operations of addition, subtraction and multiplication.
(b) Calculate the determinant of a 3×3 matrix.
(c) Find the inverse of a 2×2 matrix.

OUTCOME 3

Understand and use further aspects of sequences and series.

Performance criteria

(a) Use the Maclaurin series expansion to find a stated number of terms of the power series for a simple function.
(b) Solve a simple non-linear equation using a simple iteration of the form $x_{n+1} = g(x_n)$ where x_0 is given.

OUTCOME 4

Solve further ordinary differential equations.

Performance criteria

(a) Solve a first order linear differential equation using the integrating factor.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 3 (Advanced Higher)

OUTCOME 5

Use further number theory and direct methods of proof.

Performance criteria

(a) Use proof by mathematical induction.
(b) Use Euclid's algorithm to find the greatest common divisor of two positive integers.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessment candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 3 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated, and for checking of accuracy and sensibility of answers to be ever present.

In this unit, the third of three progressive Mathematics units, the themes of earlier work at Higher and Advanced Higher are further developed. Outcome 1 builds on the vector content of Higher Mathematics and extends to the vector equations of lines and planes.

Matrices are studied in greater depth in Outcome 2 in applications to systems of equations.

The work on sequences and series in the previous unit is now, in Outcome 3, applied to Maclaurin expansions.

In Outcome 4, the study of first order differential equations in Mathematics 2 (AH) is continued.

The important topic of proof, also introduced in Mathematics 2 (AH), is further reinforced and developed in Outcome 5.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

Further advice on learning and teaching approaches is contained within the subject guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements A0645/3 December 2001*.