NATIONAL QUALIFICATIONS

Time: 1 hour 10 minutes

Specimen Question Paper
(based on the 2000 Question Paper)

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

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2 Full credit will be given only where the solution contains appropriate working.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a.b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(\mathrm{A} \pm \mathrm{B}) &= \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B} \\ \cos(\mathrm{A} \pm \mathrm{B}) &= \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B} \\ \sin 2\mathrm{A} &= 2 \sin \mathrm{A} \cos \mathrm{A} \\ \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

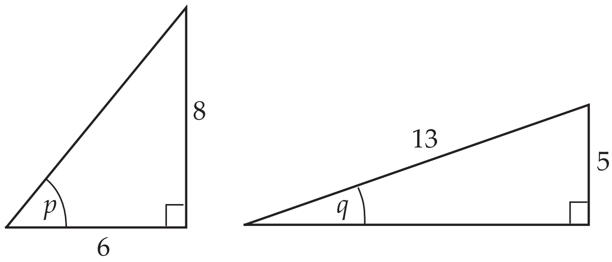
Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

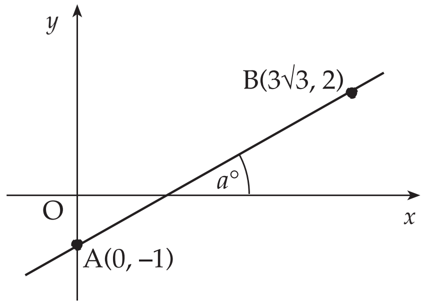
All questions should be attempted *Marks*

1. Find $f'(-2)$ when $f(x) = x^4 - \dfrac{4}{x}$. **2**

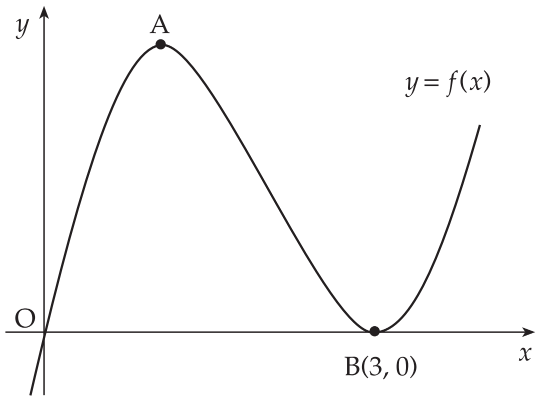
2. The diagram shows two right-angled triangles with sides and angles p and q as shown.
Find the exact value of $\sin(p + q)$. **4**



3. Find the size of the angle $a°$ that the line joining the points A(0, –1) and B(3√3, 2) makes with the positive direction of the x-axis. **3**



4. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B (3, 0).



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) The graphs of $y = g(x)$ and the line $y = k$ intersect in three distinct points.
Write down the range of values of k. **1**

5. (*a*) Express $\cos x - \sin x$ in the form $k\cos(x + a)$ where $k > 0$ and $0 \le a \le 2\pi$. **4**

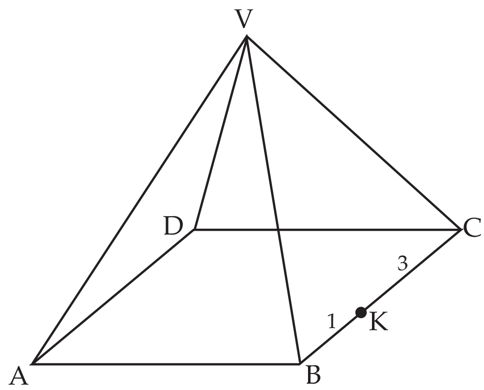
(*b*) Write down the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$. **2**

6. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

7. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.
If $f'(x) = \sin(3x)$ express y in terms of x. **4**

8. For what range of values of k does the equation $x^2 + y^2 + 4x - 2y + k = 0$ represent a circle? **3**

9. V,ABCD is a pyramid with a rectangular base ABCD.



Relative to some appropriate axes

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$.

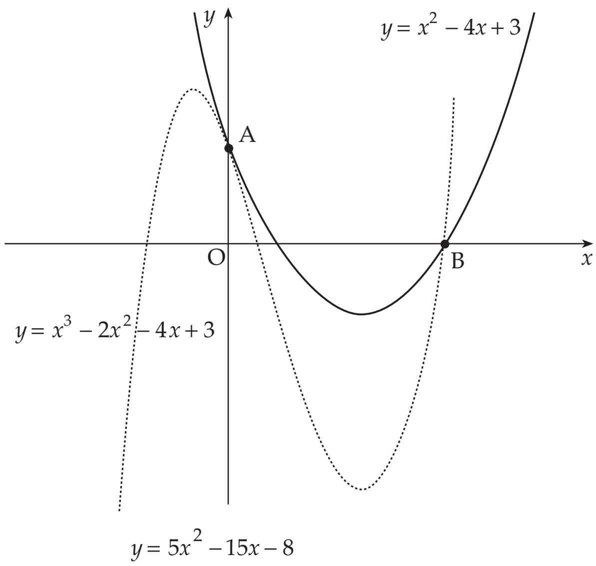
$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$

K divides BC in the ratio 1 : 3.

Find $\overrightarrow{VK}$ in component form. **3**

10. The diagram shows a sketch of the graphs of $y = x^2 - 4x + 3$ and $y = x^3 - 2x^2 - 4x + 3$. The two curves intersect at B and touch at A i.e. at A the curves have a common tangent.



(*a*) (i) The point (a, b) lies on the curve $y = x^2 - 4x + 3$. Find an expression for the gradient of the tangent at (a, b). **1**

(ii) The point (a, c) lies on the curve $y = x^3 - 2x^2 - 4x + 3$. Find an expression for the gradient of the tangent at (a, c). **1**

(iii) Determine the values of a such that these gradients are equal. **2**

(iv) With reference to the diagram identify the points corresponding to the values of a found in (iii). **1**

(*b*) The point A is (0, 3) and B is (3, 0).
Find the area enclosed between the two curves. **5**

11. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2 v_n + 16$.
The two sequences approach the same limit as $n \to \infty$.
Determine the value of a and evaluate the limit. **5**

[*END OF QUESTION PAPER*]

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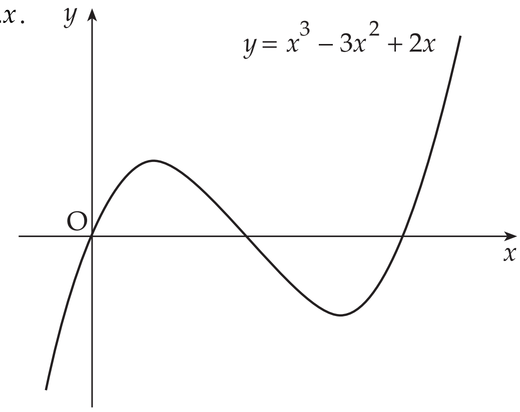
1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.

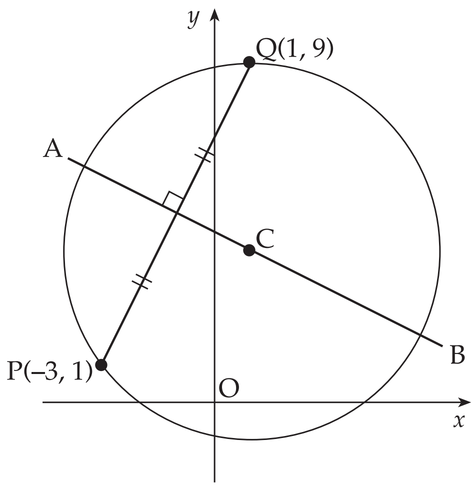


(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

2. For what value of t are the vectors $\boldsymbol{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

3. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). **4**



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.
Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**

4. $f(x) = 3 - x$ and $g(x) = \dfrac{3}{x},\ x \neq 0$.

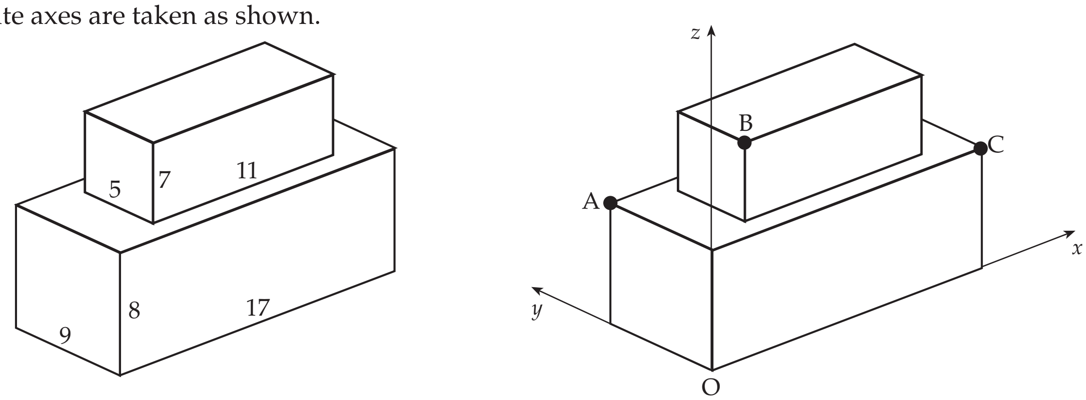
(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \dfrac{3}{3 - x},\ x \neq 3$, find $p(q(x))$ in its simplest form. **3**

5. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

6. A cuboid measuring 11cm by 5cm by 7cm is placed centrally on top of another cuboid measuring 17cm by 9cm by 8cm.

Coordinate axes are taken as shown.



(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8).
Write down the coordinates of B. **1**

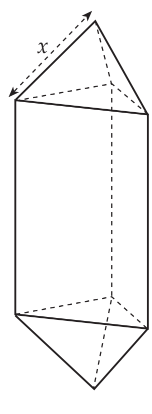
(*b*) Calculate the size of angle ABC. **6**

7. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

8. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.
The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

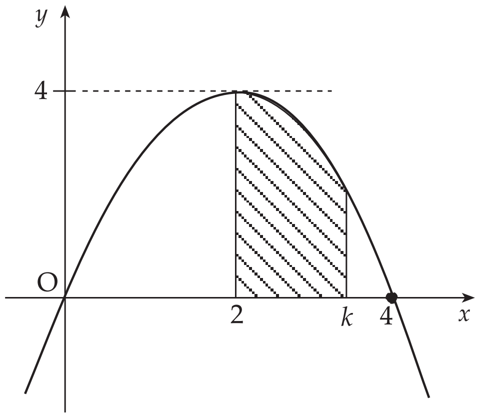
where x is the length of each edge of the tetrahedron.



Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**

Marks

9. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).
The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



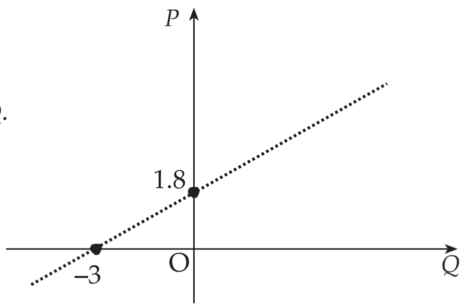
(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by **3**

$$A = -\tfrac{1}{3}k^3 + 2k^2 - \tfrac{16}{3}.$$

10. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$. **5**

11. The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. **2**

It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

[*END OF QUESTION PAPER*]

[CO56/SQP202]

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Sample standard deviation: $s = \sqrt{\frac{1}{n-1}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2\right)}$ where n is the sample size

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$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient: $r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

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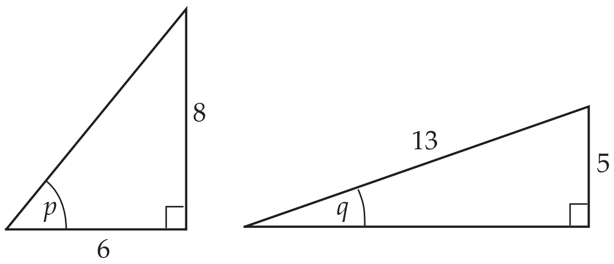
Find the exact value of $\sin(p + q)$. **4**



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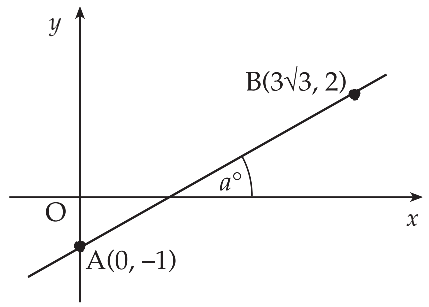

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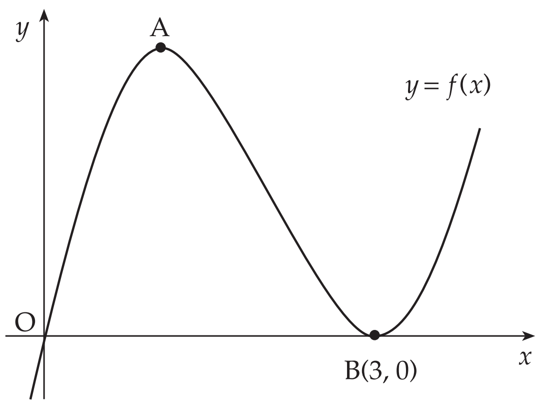

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(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x+2) + 4$.
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) The graphs of $y = g(x)$ and the line $y = k$ intersect in three distinct points.
Write down the range of values of k. **1**

5. The random variable X represents the number of faulty components in a circuit board. X has the following probability distribution:

$$P(X = x) = \begin{cases} \frac{1}{4}k(4-x) & \text{for } x = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(*a*) Find the value of k. **2**

(*b*) Find the expected value and variance of X. **3**

6. A class of 31 students estimated the length of a line which was 88 millimetres long. The distribution of male and female estimates is recorded in the back-to-back stem-and-leaf diagram below.

Male (n = 13)						Stem	Female (n = 18)							
						5	6							
8	8	6	4	4	4	6	1	3	4	4	7	9		
7	7	3	2	0	0	7	0	2	2	3	3	3	7	7
					0	8	1	1						
						9	7							

9 | 7 means 97 millimetres

(*a*) Write down the median value and the semi-interquartile range for the distribution of female estimates. **2**

(*b*) (i) Determine any possible outliers within the female distribution. **2**

(ii) Draw a boxplot to illustrate the distribution of the female estimates. **1**

(*c*) The distribution of the male estimates has a median value of 70 mm and a semi-interquartile range of 5 mm. Compare the distributions of the male and female estimates. **2**

7. For what range of values of k does the equation $x^2 + y^2 + 4x - 2y + k = 0$ represent a circle? **3**

Marks

8. A trial consists of tossing two unbiased coins. Let the random variable X represent the number of heads obtained.

(*a*) Tabulate the probability distribution of X after one trial. **2**

(*b*) A calculator was used to produce the list of random numbers below.

0.667	0.013	0.600	0.277	0.011
0.921	0.836	0.255	0.726	0.247
0.101	0.731	0.222	0.594	0.820
0.934	0.492	0.095	0.402	0.646

Use these random numbers to simulate 10 trials of this random experiment.

Explain your working. **2**

9. The diagram shows a sketch of the graphs of $y = x^2 - 4x + 3$ and $y = x^3 - 2x^2 - 4x + 3$.
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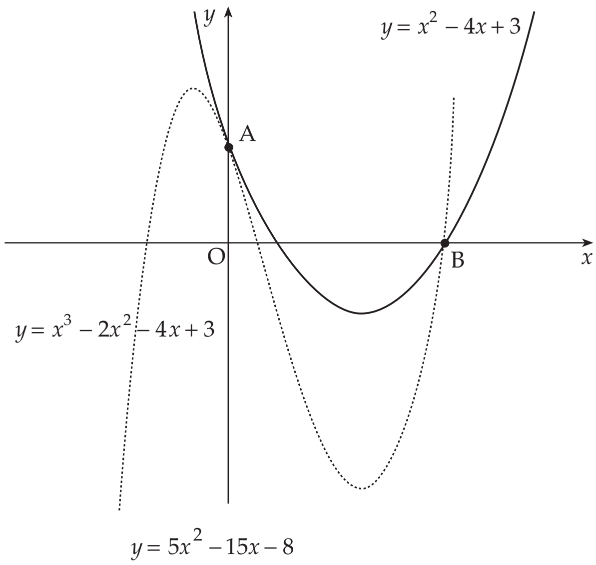

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(*b*) The point A is (0, 3) and B is (3, 0).
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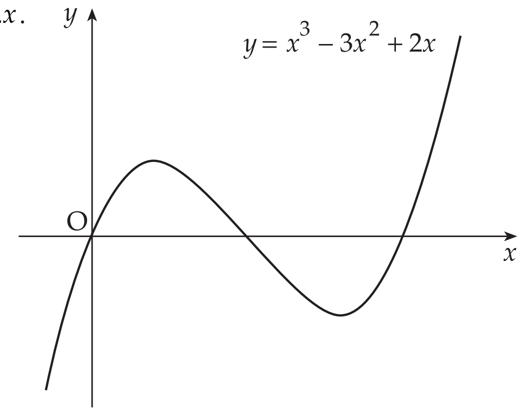

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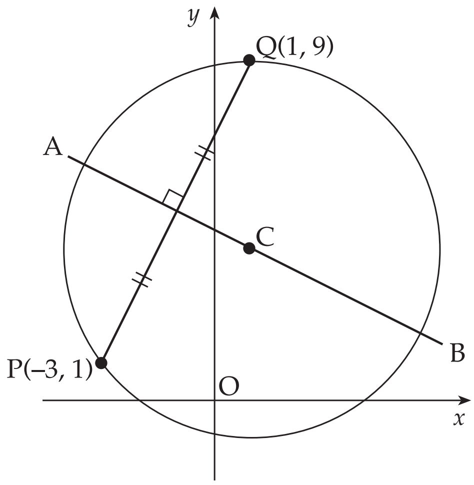

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3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Marks

4. A scientific researcher wishes to investigate the relationship between the amount of time that a rat remains conscious and the amount of anaesthetic administered. An experiment was carried out on 10 rats, similar in size and weight, using known amounts of anaesthetic. The results are shown in the table below.

Dose (ml)	x	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
Time remaining conscious (s)	y	10.9	11.5	9.9	9.3	7.4	6.9	7.1	4.1	4.5	2.3

A scattergraph shows that a linear model is appropriate.

The following summary statistics were calculated:

$$\Sigma y = 73.9, \quad \Sigma y^2 = 630.69 \quad \text{and} \quad \Sigma xy = 34.735$$

(*a*) (i) Determine the equation of the least squares regression line of y on x. **5**

(ii) Use the regression equation to predict the time a rat will remain conscious if it is given a dose of 0.62 ml of anaesthetic. **1**

(*b*) Calculate the product moment correlation coefficient and comment on your answer. **3**

5. A recent survey was carried out on the amount of time, measured in hours, that a typical Scottish family spend watching TV on any given day of the year. The amount of time was found to be modelled by a continous random variable X with the following probability density function:

$$f(x) = \begin{cases} \frac{1}{50}(10 - x) & 0 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

(*a*) Find $P(5 < X < 10)$. **2**

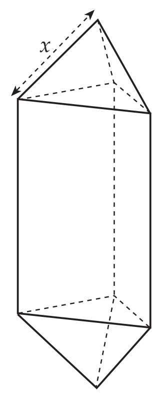
(*b*) Calculate the median value m. **4**

6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

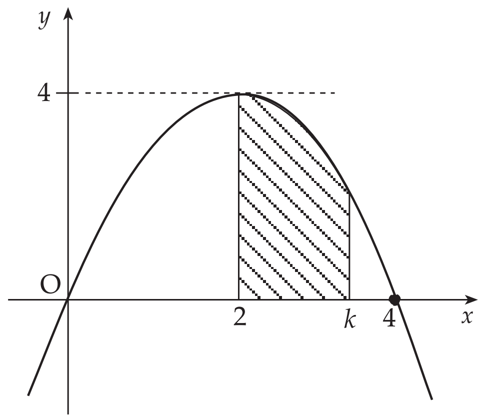
where x is the length of each edge of the tetrahedron.



Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**

7. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$. **3**

8. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$. **5**

9. A Parent-Teacher Association has 12 members consisting of 9 parents and 3 teachers. A group of 5 office bearers is to be selected. Each member of the Association has an equal chance of being chosen as an office bearer.

(*a*) How many different ways are there of choosing 5 office bearers from 12 members? **2**

(*b*) How many different ways are there of choosing 5 office bearers, 4 of whom must be parents and the other a teacher? **2**

(*c*) Hence calculate the probability that the 5 office bearers will consist of 4 parents and 1 teacher. **1**

[*END OF QUESTION PAPER*]

Specimen Paper 1 (Revised)

1 hour 10 min

Mathematics
Higher
Paper I
(Non-calculator)

Read Carefully

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Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Table of standard derivatives

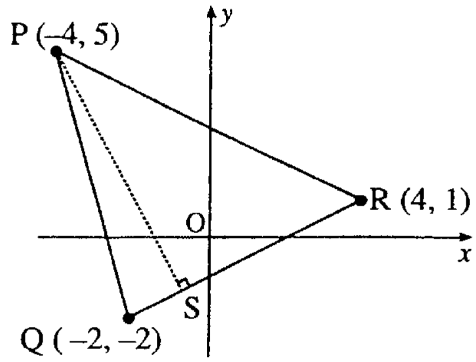
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Marks

1. P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **3**



2. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 3u_n + 5$ with first term u_1.

(*a*) Explain why this sequence has a limit as n tends to infinity. **1**

(*b*) Find the **exact** value of this limit. **2**

3. (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. **3**

(*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. **1**

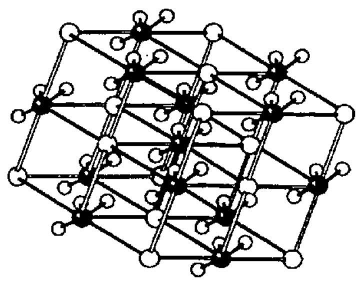
(*c*) Find the stationary points of $y = f(x)$ and determine the nature of each. **5**

(*d*) Sketch the graph of $y = f(x)$. **1**

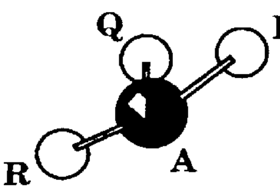
4. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of

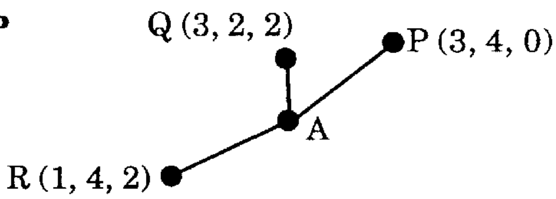
$\sin(x + 30)°$ is $\dfrac{4\sqrt{3} + 3}{10}$. **3**

5. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown.





(*a*) Show that the cosine of angle PQR is $\frac{1}{2}$. **5**

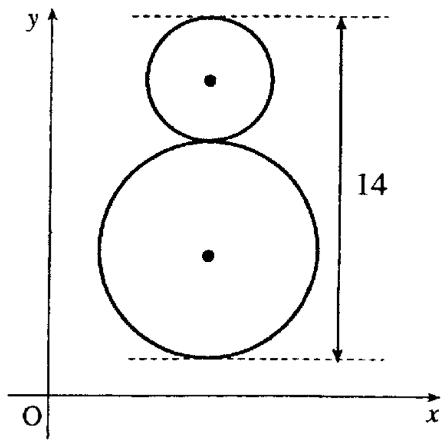
(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **6**

Marks

6. A bakery firm makes ginger-bread men each 14cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis. Find the equation of the "head". **5**

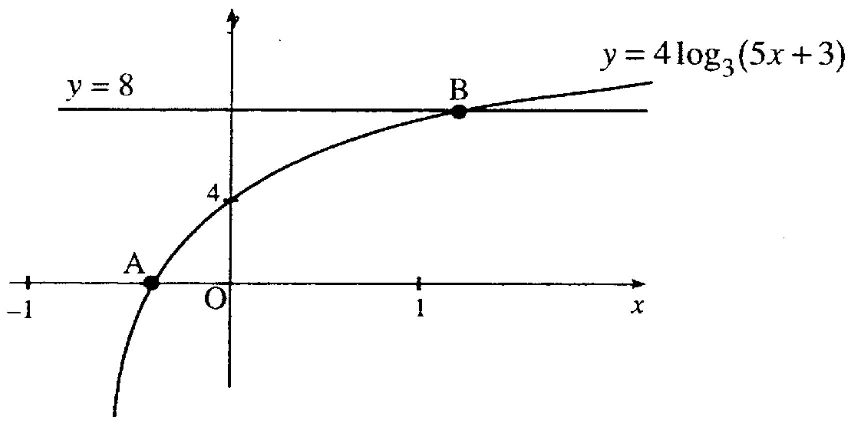


7. Find the value of $\int_1^2 \frac{u^2+2}{2u^2}\,du$. **5**

8. Sketch the graph of $y = 2\sin(x-30)^\circ$ for $0 \le x < 360$. **4**

9. Find $\frac{dy}{dx}$ given that $y = \sqrt{1+\cos x}$. **3**

10. Part of the graph of $y = 4\log_3(5x+3)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B. Find the x-coordinate of B. **3**



[END OF QUESTION PAPER]

Specimen Paper 2 (Revised)

1 hour 30 min

Mathematics
Higher
Paper 2

Read Carefully

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Circle:

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Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
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Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
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Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Marks

1. ABCD is a parallelogram. A,B and C have coordinates (2, 3), (4, 7) and (8, 11).
Find the equation of DC. **3**

2. Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees.
A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees.
Which pesticide will be more effective in the long term? **5**

3. (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x+b)^2 + c$ where a, b and c are constants. **3**

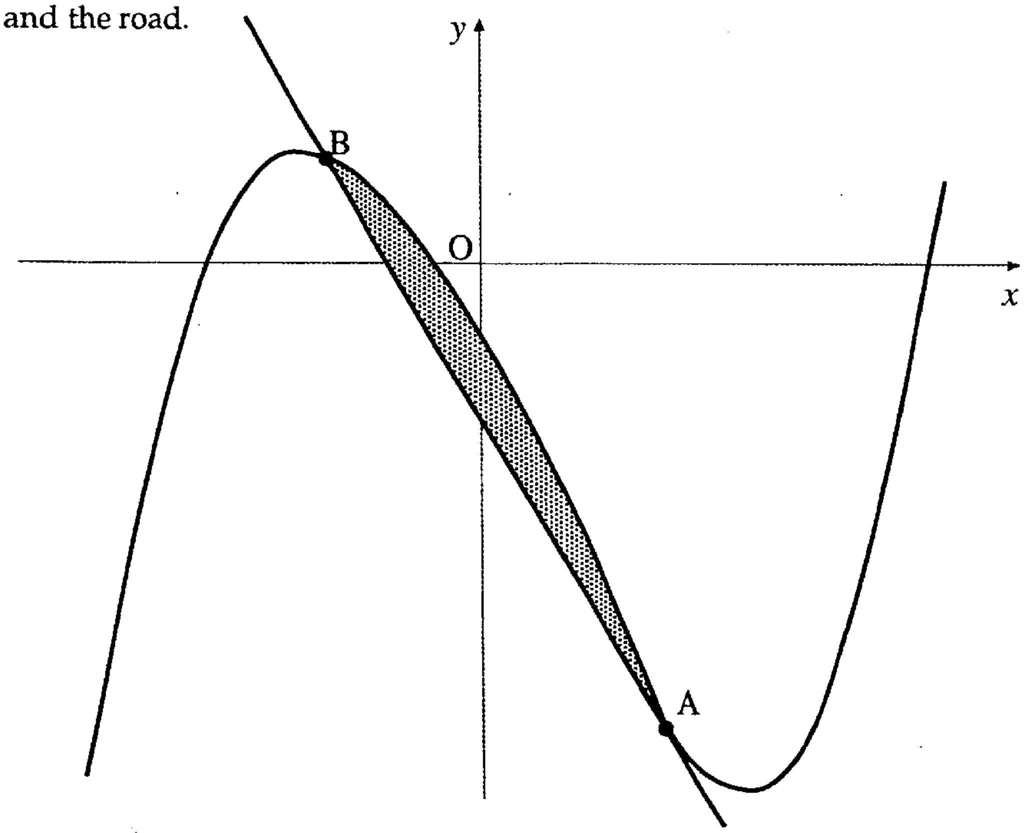
   (*b*) Hence, or otherwise, find the coordinates of the turning point of the function f. **1**

4. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

   (*a*) Find the equation of the tangent at A. **3**

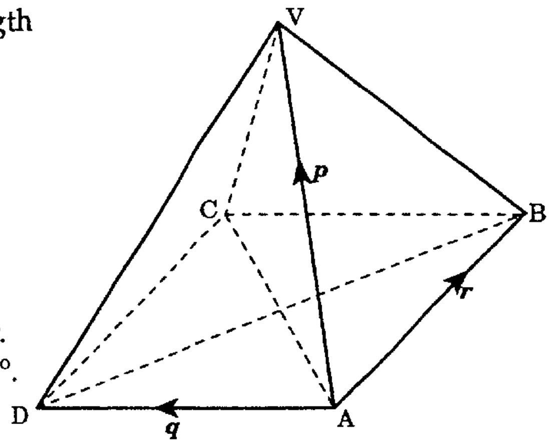
   (*b*) Hence find the coordinates of B. **4**

   (*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **3**



Marks

5. VABCD is a square-based pyramid. The length of AD is 3 units and each sloping face is an equilateral triangle.



$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$ and $\overrightarrow{AB} = \boldsymbol{r}$.

(*a*) (i) Evaluate $\boldsymbol{p}.\boldsymbol{q}$.

(ii) Hence evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$. 3

(*b*) (i) Express $\overrightarrow{CV}$ in terms of $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$.

(ii) Hence show that angle CVA is 90°. 4

6. $f(x) = 2\cos x° + 3\sin x°$.

(*a*) Express $f(x)$ in the form $k\cos(x-\alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. 4

(*b*) Hence solve $f(x) = 0.5$ for $0 \le x < 360$. 3

(c) Find the x-coordinate of the point nearest to the origin where the graph of $f(x) = 2\cos x° + 3\sin x°$ cuts the x-axis for $0 \le x < 360$. 2

7. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. 2

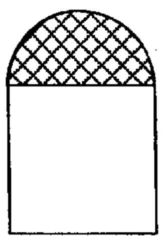
(*b*) **Hence**

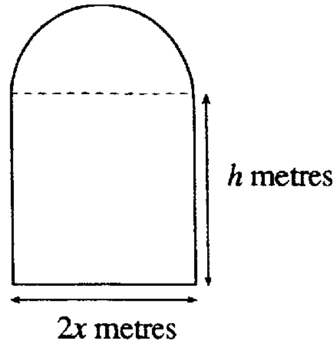
(i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$

(ii) solve this equation in the interval $0 \le x < 90$. 3

8. The roots of the equation $(x-1)(x+k) = -4$ are equal.
Find the values of k. 5

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.
The rectangle measures $2x$ metres by h metres.





(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. 2

(ii) Hence show that the amount of light, L, let in by the window is given by
$L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. 2

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. 5

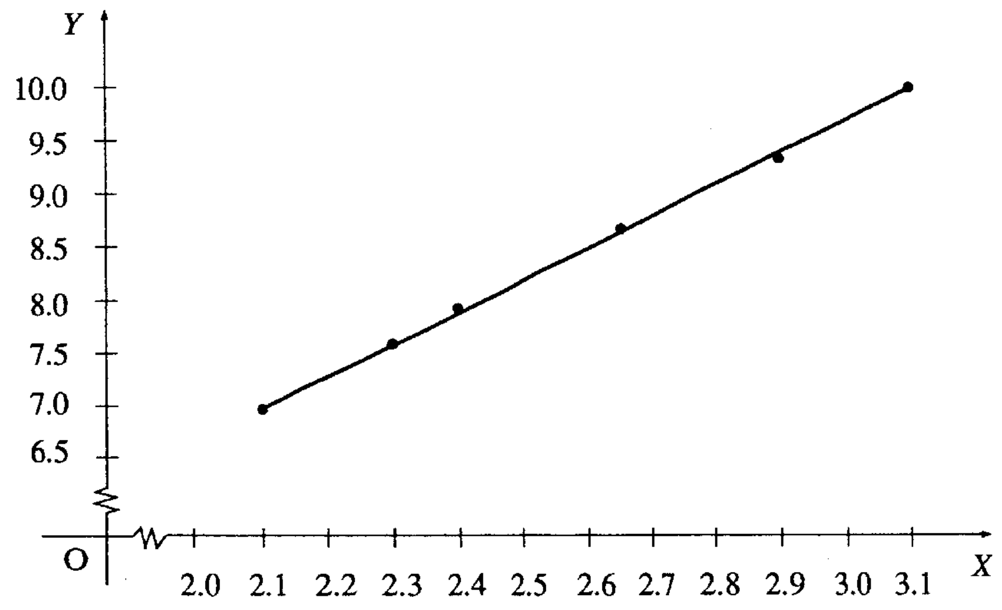
Marks

10. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

$X\ (=\log_e x)$	2·10	2·31	2·40	2·65	2·90	3·10
$Y\ (=\log_e y)$	7·00	7·60	7·92	8·70	9·38	10·00

A graph was drawn and is shown below.



Find the equation of the line in the form $Y = mX + c$. **3**

[*END OF QUESTION PAPER*]

Specimen Paper 1 (Revised)

1 hour 10 min

Mathematics
Higher
Paper I
(Non-calculator)

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$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Statistics

Sample standard deviation: $s = \sqrt{\frac{1}{n-1}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2\right)}$ where n is the sample size.

Sums of squares and products: $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{1}{n}(\Sigma y_i)^2$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i$$

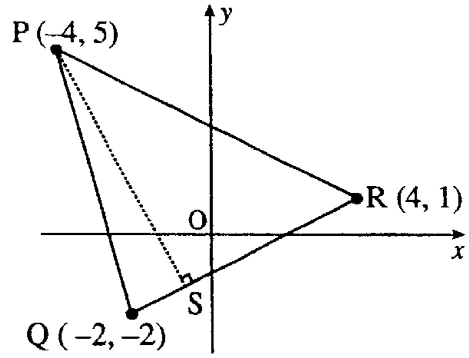
Linear regression: The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β, a and b, are given by:

$$a = \bar{y} - b\bar{x}$$
$$b = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient r: $r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

Marks

1. P(–4, 5), Q(–2, –2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **3**



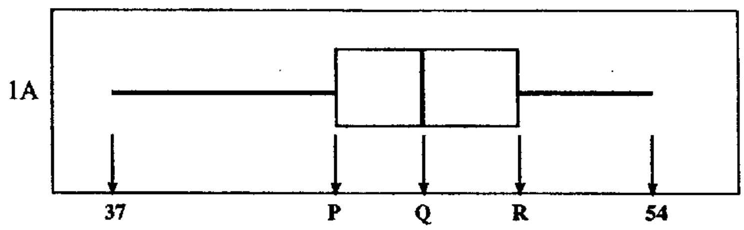
2. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 3u_n + 5$ with first term u_1.
   (*a*) Explain why this sequence has a limit as n tends to infinity. **1**
   (*b*) Find the **exact** value of this limit. **2**

3. Class 1A sat a test out of 60. The marks are shown in the stem-and-leaf diagram below:

3	7											
4	0	1	2	2	4	4						
4	5	5	6	7	7	7	8	8	8	9	9	9
5	0	1	1	2	3	4						

n = 25 3 7 represents 37

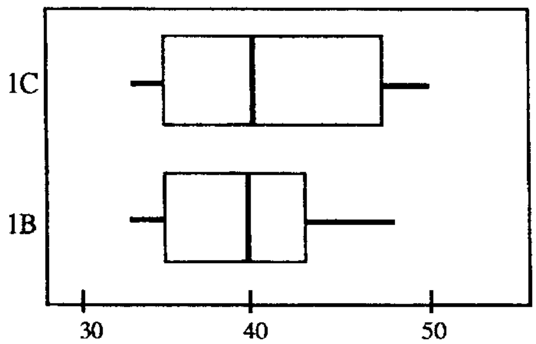
The diagram below shows an incomplete box-plot for this data.



(*a*) Find the values associated with the points P, Q and R. **2**

(*b*) The box-plot below shows the data for classes 1B and 1C.

Compare the results of these two classes. **2**



Marks

4\. (*a*) Show that $(x-1)$ is a factor of $f(x)=x^3-6x^2+9x-4$ and find the other factors. **3**

(*b*) Write down the coordinates of the points at which the graph of $y=f(x)$ meets the axes. **1**

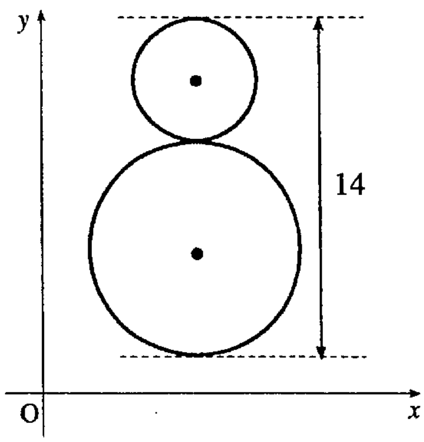
(*c*) Find the stationary points of $y=f(x)$ and determine the nature of each. **5**

(*d*) Sketch the graph of $y=f(x)$. **1**

5\. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of

$\sin(x+30)°$ is $\dfrac{4\sqrt{3}+3}{10}$. **3**

6\. A bakery firm makes ginger-bread men each 14cm high with a circular "head" and "body". The equation of the "body" is
$x^2+y^2-10x-12y+45=0$ and the line of centres is parallel to the y-axis.
Find the equation of the "head". **5**



7\. Find the value of $\displaystyle\int_1^2 \frac{u^2+2}{2u^2}\,du$. **5**

8\. Sketch the graph of $y=2\sin(x-30)°$ for $0 \le x < 360$. **4**

Marks

9. A random device moves one unit to the right with probability 0·3, one unit to the left with probability 0·3 or remains in the same position after each trial.

(*a*) Tabulate the probability distribution of X, the position of the device, after one trial. **2**

(*b*) A calculator produces the following random numbers:

0·764	0·380	0·410	0·175	0·458
0·552	0·709	0·935	0·451	0·854

(i) Explain how you would use these numbers to simulate ten trials of this random experiment. **2**

(ii) List the results of your simulation. **1**

10. The total lifetime (in years) of 5 year old washing machines of a certain make is a random variable whose cumulative distribution function F is given by

$$F(x) = \begin{cases} 0 & \text{for } x \le 5 \\ 1 - \dfrac{25}{x^2} & \text{for } x > 5 \end{cases}$$

(*a*) Find the probability that such a washing machine will be in service for

(i) less than 8 years **1**

(ii) more than 10 years. **2**

(*b*) Find the probability density function $f(x)$. **2**

(c) Calculate the exact value of the median lifetime of these washing machines. **3**

[END OF QUESTION PAPER]

Specimen Paper 2 (Revised)

1 hour 30 min

Mathematics
Higher
Paper 2

Read Carefully

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$$a = \bar{y} - b\bar{x}$$
$$b = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient r: $r = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}} = \dfrac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

Marks

1. ABCD is a parallelogram. A,B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC. **3**

2. Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees.
A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees.
Which pesticide will be more effective in the long term? **5**

3. (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x+b)^2 + c$ where *a*, *b* and *c* are constants. **3**

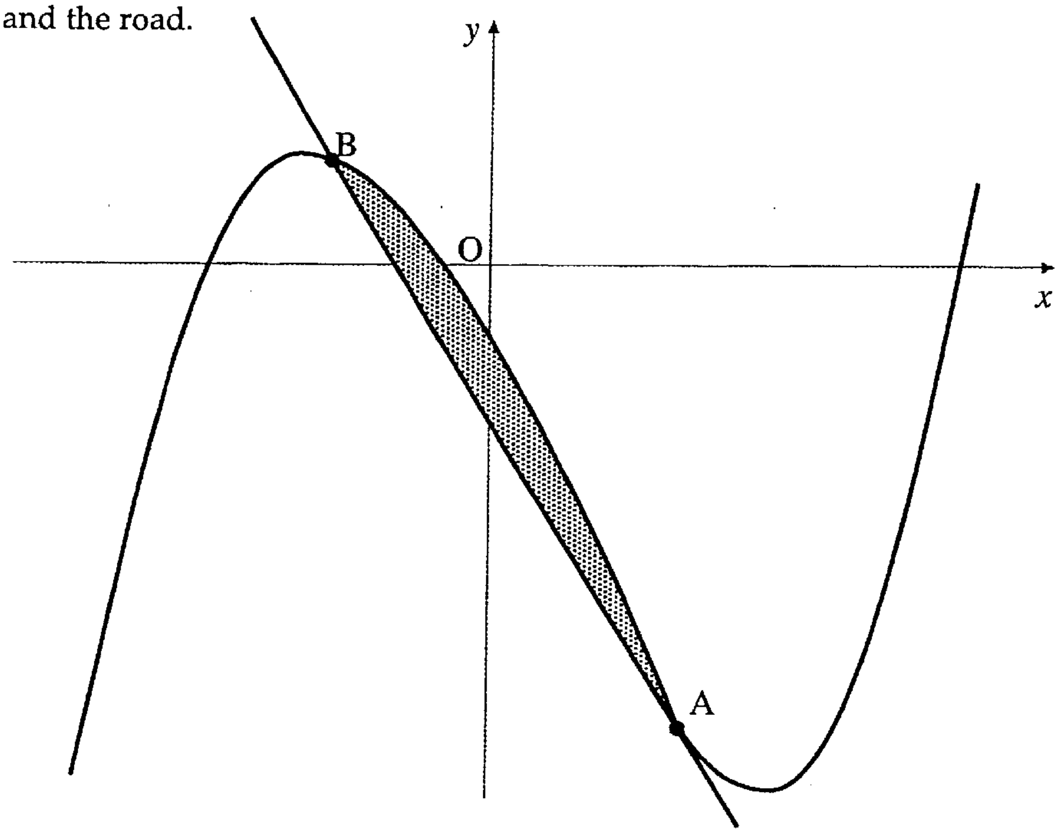
(*b*) Hence, or otherwise, find the coordinates of the turning point of the function *f*. **1**

4. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

(*a*) Find the equation of the tangent at A. **3**

(*b*) Hence find the coordinates of B. **4**

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **3**



Marks

5. In an archery competition, the probability that a particular competitor hits the target with any shot is $\frac{3}{4}$. In the competition, she is allowed three shots.
   (*a*) Find the probability that she hits the target:
      (i) exactly twice. **2**
      (ii) at least once. **2**
   (*b*) State a statistical asumption that you have made. **1**

6. A market gardener wishes to investigate the relationship between the total weight of tomatoes produced by a tomato plant and the amount of fertiliser used. An experiment was carried out where known amounts of fertiliser were applied to 8 similar plants. The results are shown in the table:

Weight of fertiliser (g)	x	0	2	4	6	8	10	12	14
Tomato yield (kg)	y	4.44	5.13	5.45	5.27	5.81	6.04	5.90	6.23

A scatter diagram shows that a linear model is appropriate.

You may assume that $\Sigma y = 44\cdot27$, $\Sigma y^2 = 247\cdot3665$ and $\Sigma xy = 328\cdot58$.

   (*a*) (i) Determine the equation of the least squares regression line of y on x. **6**
      (ii) Use the regression equation to predict the tomato yield for 9g of fertiliser. **1**

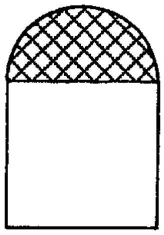
   (*b*) Calculate the product moment correlation coefficient and comment on your answer. **3**

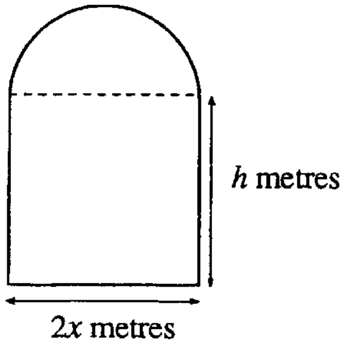
7. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **2**
   (*b*) **Hence**
      (i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$
      (ii) solve this equation in the interval $0 \leq x < 90$. **3**

8. The roots of the equation $(x-1)(x+k) = -4$ are equal.
   Find the values of k. **5**

Marks

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light. The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.





The rectangle measures $2x$ metres by h metres.

(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **2**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **2**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **5**

10. The random variable X has a probability density function

$$f(x) = \begin{cases} kx^2(1-x) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(*a*) Find the value of k. **2**

(*b*) Find the probability that X lies in the range $0 \le X \le \frac{2}{3}$. **2**

[*END OF QUESTION PAPER*]