

# Comparison document

(Version 2.2 September 2015 compared to previous version)

## Advanced Higher Mathematics Course Assessment Specification (C747 77)

**Valid from August 2015**

This edition: ~~June-September~~ 2015, version 2.42

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Please refer to the note of changes at the end of this Course Assessment Specification for details of changes from previous version (where applicable).

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# Assessment

To gain the award of the Course, the learner must pass all the Units as well as the Course assessment. Course assessment will provide the basis for grading attainment in the Course award.

## Course assessment

SQA will produce and give instructions for the production and conduct of Course assessments based on the information provided in this document.

## Added value

The purpose of the Course assessment is to assess added value of the Course as well as confirming attainment in the Course and providing a grade. The added value for the Course will address the key purposes and aims of the Course, as defined in the Course Rationale. It will do this by addressing one or more of breadth, challenge or application.

In this Course assessment, added value will focus on the following:

- ◆ breadth — drawing on knowledge and skills from across the Course
- ◆ challenge — requiring greater depth or extension of knowledge and skills
- ◆ application — requiring application of knowledge and skills in practical or theoretical contexts as appropriate

This added value consists of:

- ◆ using mathematical reasoning skills to think logically, provide justification and solve problems
- ◆ using a range of complex concepts
- ◆ selecting and applying complex operational skills
- ◆ using reasoning skills to interpret information and to use complex mathematical models
- ◆ effectively communicating solutions in a variety of mathematical contexts
- ◆ explaining and justifying concepts through the idea of rigorous proof
- ◆ thinking creatively

To achieve success in the Course, learners must show that they can apply knowledge and skills acquired across the Course to unseen situations.

The question paper requires learners to demonstrate aspects of breadth, challenge and application in mathematical contexts. The use of a calculator will be permitted.

## **Grading**

Course assessment will provide the basis for grading attainment in the Course award.

The Course assessment is graded A–D. The grade is determined on the basis of the total mark for the Course assessment.

A learner's overall grade will be determined by their performance across the Course assessment.

### **Grade description for C**

For the award of Grade C, learners will have demonstrated successful performance in all of the Units of the Course. In the Course assessment, learners will typically have demonstrated successful performance in relation to the mandatory skills, knowledge and understanding for the Course.

### **Grade description for A**

For the award of Grade A, learners will have demonstrated successful performance in all of the Units of the Course. In the Course assessment, learners will typically have demonstrated a consistently high level of performance in relation to the mandatory skills, knowledge and understanding for the Course.

### **Credit**

To take account of the extended range of learning and teaching approaches, remediation, consolidation of learning and integration needed for preparation for external assessment, six SCQF credit points are available in Courses at National 5 and Higher, and eight SCQF credit points in Courses at Advanced Higher. These points will be awarded when a Grade D or better is achieved.

## **Structure and coverage of the Course assessment**

The Course assessment will consist of one Component: a question paper.

### **Component 1 — question paper**

The purpose of the question paper is to assess mathematical skills. A calculator may be used.

The question paper will sample the skills, knowledge and understanding that are contained in the 'Further mandatory information on Course coverage' section at the end of this Course Assessment Specification.

The question paper will consist of a series of short and extended response questions (some of which may be set in contexts) that require the application of skills developed in the Course. Learners will be expected to communicate responses clearly and to justify solutions. The paper will have 100 marks.

Assessors can give learners access to the formulae contained in the formulae sheet accompanying the Advanced Higher Mathematics Course assessment. Assessors can also give learners access to any other derivative or formula which does not form part of this Course.

## **Setting, conducting and marking of assessment**

### **Question paper**

The question paper will be set and marked by SQA and conducted in centres under conditions specified for external examinations by SQA. Learners will complete this in 3 hours.

## Further mandatory information on Course coverage

The following gives details of mandatory skills, knowledge and understanding for the Advanced Higher Mathematics Course. Course assessment will involve sampling the skills, knowledge and understanding. This list of skills, knowledge and understanding also provides the basis for the assessment of Units of the Course.

This includes:

- the ability to use mathematical reasoning skills to think logically, provide justification and solve problems
- knowledge and understanding of a range of complex concepts
- the ability to select and apply complex operational skills
- the ability to use reasoning skills to interpret information and to use complex mathematical models
- the ability to effectively communicate solutions in a variety of contexts
- the ability to explain and justify concepts through the idea of rigorous proof
- the ability to think creatively

These skills will be assessed across the Course, in the context of the mandatory knowledge.

<b>Mathematics: Methods in Algebra and Calculus (Advanced Higher)</b>	
<b>1.1 Applying algebraic skills to partial fractions</b>	
<b>Sub-skill</b>	<b>Description</b>
Expressing rational functions as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	Express a proper rational function as a sum of partial fractions where the denominator may contain: distinct linear factors, an irreducible quadratic factor, a repeated linear factor  Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise
<b>1.2 Applying calculus skills through techniques of differentiation</b>	
<b>Sub-skill</b>	<b>Description</b>
Differentiating exponential and logarithmic functions	Differentiate functions involving $e^x$ , $\ln x$
Differentiating functions using the chain rule	Apply the chain rule to differentiate the composition of at most 3 functions

<p>Differentiating functions given in the form of a product and in the form of a quotient</p>	<p>Differentiate functions of the form <math>(f(x)g(x))</math> and <math>\left(\frac{f(x)}{g(x)}\right)</math></p> <p>Use the derivative of <math>\tan x</math>.</p> <p>Know the definitions of <math>\cot x</math>, <math>\sec x</math>, <math>\operatorname{cosec} x</math>. Learners should be able to derive and use derivatives of <math>\tan x</math>, <math>\cot x</math>, <math>\sec x</math>, <math>\operatorname{cosec} x</math>.</p> <p>Differentiating functions which require more than one application or combination of applications of chain rule, product rule and quotient rule</p> <p>Know and use <math>\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}</math></p> <p>Use logarithmic differentiation; recognise when it is appropriate in extended products, quotients, and in functions where the variable occurs in an index</p>
<p>Differentiating inverse trigonometric functions</p>	<p>Differentiating expressions of the form eg <math>\sin^{-1} kx</math>, <math>\tan^{-1}[f(x)]</math></p>
<p>Finding the derivative of functions defined implicitly</p>	<p>Use differentiation to find the first derivative of a function defined implicitly including in context</p> <p>Use differentiation to find the second derivative of a function defined implicitly</p>
<p>Finding the derivative of functions defined parametrically</p>	<p>Use differentiation to find the first derivative of a function defined parametrically including in context</p> <p>Use differentiation to find the second derivative of a function defined parametrically</p> <p>Solve practical related rates by first establishing a functional relationship between appropriate variables</p>

1.3 Applying calculus skills through techniques of integration	
Sub-skill	Description
Integrating expressions using standard results	<p>Use <math>\int e^x dx, \int \frac{dx}{x}, \int \sec^2 x dx</math></p> <p>Use the integrals of</p> $\frac{1}{\sqrt{a^2 - x^2}}, \frac{1}{a^2 + x^2}$ <p>Recognise and integrate expressions of the form</p> $\int g(f(x))f'(x)dx \text{ and } \int \frac{f'(x)}{f(x)} dx$ <p>Use partial fractions to integrate proper rational functions where the denominator may have:</p> <ul style="list-style-type: none"> <li>i) two separate or repeated linear factors</li> <li>ii) three linear factors with constant numerator</li> <li>iii) three linear factors with non-constant numerator</li> <li>iv) a linear factor and an irreducible quadratic factor of the form <math>ax^2 + bx + c</math></li> </ul>
Integrating by substitution	Integrate where the substitution is given
Integrating by parts	<p>Use integration by parts with one application</p> <p>Use integration by parts involving repeated applications</p>

1.4 Applying calculus skills to solving differential equations	
Sub-skill	Description
Solving first order differential equations with variables separable	<p>Solve equations that can be written in the form</p> $\frac{dy}{dx} = g(x)h(y) \text{ or } \frac{dy}{dx} = \frac{g(x)}{h(y)}$ <p>Find general and particular solutions given suitable information</p>
Solving first order linear differential equations using an integrating factor	<p>Solve equations by first writing linear equations in the standard form</p> $\frac{dy}{dx} + P(x)y = f(x)$ <p>Find general and particular solutions given suitable information</p>
Solving second order differential equations	<p>Find the general solution and particular solution of second order homogeneous ordinary differential equations of the form</p> $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$ <p>with constant coefficients where the roots of the auxiliary equation are real or complex conjugates</p> <p>Solve second order non-homogeneous ordinary differential equations of the form</p> $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ <p>with constant coefficients using the auxiliary equation and particular integral method</p>

## Mathematics: Applications in Algebra and Calculus (Advanced Higher)

### 1.1 Applying algebraic skills to the binomial theorem and to complex numbers

Sub-skill	Description
Expanding expressions using the binomial theorem	<p>Use the binomial theorem</p> $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r, \text{ for } r, n \in \mathbf{N}$ <p>Expand an expression of the form <math>(ax^p + by^q)^n</math>, where <math>a, b \in \mathbf{Q}; p, q \in \mathbf{Z}; n \leq 7</math>.</p> <p>Using the general term for a binomial expansion, find a specific term in an expression</p>
Performing algebraic operations on complex numbers	<p>Perform all of the operations of addition, subtraction, multiplication and division</p> <p>Finding the square root</p> <p>Find the roots of a quartic with real coefficients when one complex root is given</p> <p>Solve equations involving complex numbers</p>

### 1.2 Applying algebraic skills to sequences and series

Sub-skill	Description
Finding the general term and summing arithmetic and geometric progressions	<p>Apply the rules on sequences and series to find the <math>n</math>th term, sum to <math>n</math> terms, sum to infinity, common difference, and common ratio of arithmetic and geometric sequences respectively</p> <p>Determine the condition for a particular geometric series to converge</p>
Using the Maclaurin expansion to find a stated number of terms of the power series <a href="#">for a simple function</a>	<p>Use the Maclaurin expansion to find a power series for a simple non-standard function</p> <p>Use the Maclaurin expansion to find a power series</p>

1.3 Applying algebraic skills to summation and mathematical proof	
Sub-skill	Description
Applying summation formulae	Know and use sums of certain series and other straightforward results and combinations thereof (2, 4 and 5 appear on formulae sheet and are therefore 'use' only)
Using proof by induction	Use mathematical induction to prove summation formulae  Use proof by induction
1.4 Applying algebraic and calculus skills to properties of functions	
Sub-skill	Description
Finding the asymptotes to the graphs of rational functions	Find the vertical asymptote to the graph of a rational function  Find the non-vertical asymptote to the graph of a rational function
Investigating features of graphs and sketching graphs of functions	Investigate points of inflection  Investigate other features: stationary points, domain and range, symmetry (odd/even), continuous/discontinuous, extrema of functions: the maximum and minimum values of a continuous function $f$ defined on a closed interval $[a,b]$ can occur at stationary points, end points or points where $f'$ is not defined  Sketch graphs using features given or obtained  Sketch related functions:  <ul style="list-style-type: none"> <li>modulus functions</li> <li>inverse functions</li> <li>differentiated functions</li> <li>translations and reflections</li> </ul>
1.5 Applying algebraic and calculus skills to problems	
Sub-skill	Description
Applying differentiation to problems, in context	Apply differentiation to problems in context  Apply differentiation to optimisation
Applying integration to problems, in context	Apply integration to volumes of revolution <u>where the volume generated is by the rotation of the area under a single curve about the <math>x</math> and <math>y</math> axes</u>  Use calculus to determine corresponding connected integrals  Apply integration to the evaluation of areas including integration with respect to $y$

## Mathematics : Geometry, Proof and Systems of Equations (Advanced Higher)

### 1.1 Applying algebraic skills to matrices and systems of equations

Sub-skill	Description
Using Gaussian elimination to solve a $3 \times 3$ system of linear equations	<p>Find the solution to a system of equations <math>\mathbf{Ax} = \mathbf{b}</math>, where <math>\mathbf{A}</math> is a <math>3 \times 3</math> matrix and where the solution is unique</p> <p>Candidates should understand the term augmented matrix</p> <p>Show that a system of equations has no solutions (inconsistency)</p> <p>Show that a system of equations has an infinite number of solutions (redundancy)</p> <p>Compare the solutions of related systems of two equations in two unknowns and recognise ill-conditioning</p>
Understanding and using matrix algebra	<p>Perform matrix operations (at most order 3): addition, subtraction, multiplication by a scalar, multiplication of matrices</p> <p>Know and apply the properties of matrix addition and multiplication:</p> <ol style="list-style-type: none"> <li><math>A + B = B + A</math></li> <li><math>AB \neq BA</math> (in general)</li> <li><math>(A + B) + C = A + (B + C)</math> (associativity)</li> <li><math>(AB)C = A(BC)</math> (associativity)</li> <li><math>A(B + C) = AB + AC</math> (addition is distributive over multiplication)</li> </ol> <p>Know and apply key properties of the transpose, identity matrix and the inverse:</p> <ol style="list-style-type: none"> <li><math>(a_{ij})'_{m \times n} = (a_{ji})_{n \times m}</math> ie rows and columns interchange</li> <li><math>(A')' = A</math></li> <li><math>(A + B)' = A' + B'</math></li> <li><math>(AB)' = B'A'</math></li> <li>A matrix <math>A</math> is orthogonal if <math>A'A = I</math></li> <li>The <math>n \times n</math> identity matrix <math>I_n</math> : for any square matrix <math>A</math>, <math>AI_n = I_nA = A</math></li> </ol>

	<p>7. <math>B = A^{-1}</math> if <math>AB = BA = I</math></p> <p>8. <math>(AB)^{-1} = B^{-1}A^{-1}</math></p>
Calculating the determinant of a matrix	<p>Find the determinant of a <math>2 \times 2</math> matrix and a <math>3 \times 3</math> matrix</p> <p>Determine whether a matrix is singular</p> <p>Know and apply <math>\det(AB) = \det A \det B</math></p>
Finding the inverse of a matrix	<p>Know and use the inverse of a <math>2 \times 2</math> matrix</p> <p>Find the inverse of a <math>3 \times 3</math> matrix, for example by matrix algebra</p> <p>Use <math>2 \times 2</math> matrices to carry out geometric transformations in the plane</p> <p>The transformations should include rotations, reflections and dilatations</p> <p>Apply combinations of transformations</p>
<b>1.2 Applying algebraic and geometric skills to vectors</b>	
<b>Sub-skill</b>	<b>Description</b>
Calculating a vector product	<p>Use a vector product method in three dimensions to find the vector product</p> <p>Evaluate the scalar triple product <math>\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})</math></p>
Working with lines in three dimensions	<p>Find the equation of a line in parametric, symmetric or vector form, given suitable defining information</p> <p>Find the angle between two lines in three dimensions</p> <p>Determine whether or not two lines intersect and, where possible, find the point of intersection</p>
Working with planes	<p>Find the equation of a plane in vector form, parametric form or Cartesian form, given suitable defining information</p> <p>Find the point of intersection of a plane with a line which is not parallel to the plane</p> <p>Determine the intersection of two or three planes</p> <p>Find the angle between a line and a plane or between two planes</p>

1.3 Applying geometric skills to complex numbers	
Sub-skill	Description
Performing geometric operations on complex numbers	<p>Plot complex numbers in the complex plane (an Argand diagram)</p> <p>Know the definition of modulus and argument of a complex number</p> <p>Convert a given complex number from Cartesian to polar form and vice-versa</p> <p>Use de Moivre's theorem with integer and fractional indices</p> <p>Apply de Moivre's theorem to multiple angle trigonometric formulae</p> <p>Apply de Moivre's theorem to find the nth roots of a complex number</p> <p>Interpret geometrically certain equations or inequalities in the complex plane</p>
1.4 Applying algebraic skills to number theory	
Sub-skill	Description
Using Euclid's algorithm to find the greatest common divisor of two positive integers	<p>Use Euclid's algorithm to find the greatest common divisor of two positive integers, ie use the division algorithm repeatedly</p> <p>Express the greatest common divisor (of two positive integers) as a linear combination of the two</p> <p>Express integers in bases other than ten</p> <p>Know and use the Fundamental Theorem of Arithmetic</p>
1.5 Applying algebraic and geometric skills to methods of proof	
Sub-skill	Description
Disproving a conjecture by providing a counter-example	<p>Disprove a conjecture by providing a counter-example</p> <p>Know and be able to use the symbols <math>\exists</math> and <math>\forall</math></p> <p>Write down the negation of a statement</p>
Using indirect or direct proof in straightforward examples	<p>Prove a statement by contradiction</p> <p>Use further proof by contradiction</p> <p>Use proof by contrapositive</p> <p>Use direct proof in straightforward examples</p>

# Administrative information

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Published: ~~June~~ September 2015 (version 2.~~4~~2)

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## History of changes to Course Assessment Specification

Version	Description of change	Authorised by	Date
2.0	Pages 7–15: extensive changes to 'Further mandatory information on Course coverage' section.	Qualifications Development Manager	April 2015
2.1	Page 5: 'Structure and coverage of the Course assessment' section — reference to the formulae sheet added.  Pages 8, 12, 14 and 15: 'Further mandatory information on Course coverage' section updated to include references to formulae sheet.	Qualifications Development Manager	June 2015
<u>2.2</u>	<u>Pages 11 and 12: 'Further mandatory information on Course coverage' section clarified: amendments to second sub-skill for Assessment Standard 1.2 and description relating to second sub-skill for Assessment Standard 1.5.</u>	<u>Qualifications Development Manager</u>	<u>September 2015</u>

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Note: You are advised to check SQA's website ([www.sqa.org.uk](http://www.sqa.org.uk)) to ensure you are using the most up-to-date version of the *Course Assessment Specification*.

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