SCOTTISH QUALIFICATIONS AUTHORITY

Principal Assessor Report 2002

Assessment Panel:	**Mathematics**

Qualification area

Subject(s) and Level(s) included in this report	**Applied Mathematics (Advanced Higher)**

Statistical information: update

Number of entries in 2001	
Pre appeal	142
Post appeal	152

Number of entries in 2002	
Pre appeal	409
Post appeal	

General comments re entry numbers

The huge increase is pleasing. This is due in part to the demise of CSYS but is slightly above the figure expected for this course.

General comments

As with AH Mathematics, the 2002 question paper was designed to be more of a challenge. The influx of new centres and level of preparation of candidates rather exaggerated the effect.

Comments on any significant changes in percentages or distribution of awards

It is only the second presentation and 2001 was an atypical year. However, the proportions getting the awards are broadly in line with CSYS papers II, III, IV, V.

Grade boundaries at C, B and A for each subject area included in the report

Advanced Higher Applied Mathematics: Pass mark stage

Maximum mark 100		
A	**B**	**C**
67%	57%	47%

General commentary on grade boundaries

Notional percentage cut-offs for each grade

Question papers and their associated marking schemes are designed to be of the required standard and to meet the assessment specification for the subject/level concerned.
For National courses the examination paper(s) are set in order that a score of approximately 50% of the total marks for all components merits a grade C (based on the grade descriptions for that grade), and similarly a score of 70 % for a grade A. The lowest mark for a grade B is set by the computer software as half way between the C and A grade boundaries.

Comments on grade boundaries for each subject area

They are reasonable well separated. They may be *slightly* on the low side.

Comments on candidate performance

General comments

The general performance can be described as adequate. The mean mark was 55. 30% got a mark of 67 or over with 30% getting 43 or less. Thus a good spread of marks was evident. The approximate uptake and means were as shown below.

Number
Core mean
Option mean

A & D
185
34.7
17.8

A & F
45
34.7
17.3

A & G
17
30.4
12.9

B & D
21
39.4
18.6

B & E
7
48.3
20.3

B & G
1
59.0
31.0

C & D
74
36.1
18.3

C & E
38
42.1
15.8

C & F
25
46.3

21.2   The mean mark for each core was A 34.0 B 42.2 C 39.6

Areas of external assessment in which candidates performed well

The markers' reports included the following: Statistics Poisson distribution Numerical Analysis Questions B1 to B5 Mechanics Inclined plane question Circular motion question Straight line motion Projectiles

Areas of external assessment in which candidates had difficulty

Writing clear, legible mathematics. Also from markers Statistics Distribution of samples Providing justifications and identifying assumptions Probability questions Providing accurate and precise conclusions Numerical Analysis Bookwork Mechanics Vectors – applied to Work and Power S. H. M. Distinguishing between motion involving uniform and non-uniform acceleration

Areas of common misunderstanding

None apparent

Recommendations

Feedback to centres

Exemplification to be provided early.

It will also be advantageous for candidates to improve their examination technique. Particular point are

- Make sure writing is clear.
- Make sure work is clearly set out.
- Take particular care over manipulation.

Look for accessible marks at the start of a question or one of its parts.

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2002 Applied Mathematics

Advanced Higher

Finalised Marking Instructions

SECTION A (Statistics 1 & 2)

A1. $P(R) = P(R \mid B_1)\,P(B_1) + P(R \mid B_2)\,P(B_2) + P(R \mid B_3)\,P(B_3)$ **1**

$= \frac{3}{10} \times \frac{1}{3} + \frac{4}{8} \times \frac{1}{3} + \frac{2}{10} \times \frac{1}{3}$ **1**

$= \frac{1}{3}$ **1**

$P(R \mid B_1) = \frac{P(B_1 \mid R)\,P(B_1)}{P(R)}$ **1**

$= \frac{\frac{3}{10} \times \frac{1}{3}}{\frac{1}{3}} = \frac{3}{10}$ **1**

(formulae need not be quoted, other methods eg Venn or Tree acceptable)

A2. (a) $P(X = 0) = F(0) = 0.0183$ **1**

(b) $P(X = 4) = F(4) - F(3)$ **1**

$= 0.6288 - 0.4335 = 0.1953$ **1**

(c) $P(X > 4) = 1 - F(4)$ **1**

$= 1 - 0.6288 = 0.3712 \approx 0{\cdot}37.$ **1**

$V(X) = \mu = 4$ so $\sigma = \sqrt{4} = 2.$ **1**

A3. A simple *random* sample of farms would be selected from *each* category. **1**

(Both words needed.)

Total number of farms is 500.

Numbers from each category would be 50/500 of number in the category. **1**

Numbers would be 3 large, 32 medium and 15 small. **1**

If the strata are homogeneous then stratified sampling will provide more precise estimates of population parameters than simple random sampling with the same total sample size. **1**

A random sample might exclude one category altogether. **1**

(Not and exhaustive list, many other possible comments acceptable.)

A4. (a) The 95% confidence interval is given by $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$ or $p = \pm 1{\cdot}96\sqrt{\frac{pq}{m}}$. **2**

(1 mark for the 1·96 and 1 mark for the root term.)

(b) Let $f(p) = p(1 - p)$

$\Rightarrow f'(p) = 1 - 2p = 0$ **1**

when $p = 0.5$

$\Rightarrow f_{\max} = 0.5(1 - 0.5) = 0.25$ (graphical method acceptable) **1**

(Completing the square or reference to quadratic also acceptable.)

(c) The greatest width is therefore $2 \times 1.96\sqrt{\frac{0.25}{1000}}$ **1,1**

$= 0.06$

(First mark for the doubling, second for the rest.)

A5.

Manufacturer	A	B	Other
Observed	192	342	66
Expected	180	360	60

1

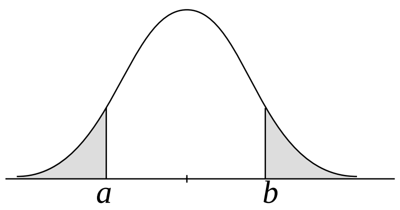
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$= \frac{(192 - 180)^2}{180} + \frac{(342 - 360)^2}{360} + \frac{(66 - 60)^2}{60}$$

$$= 0.8 + 0.9 + 0.6 = 2.3$$ **1**

The critical value of chi-squared with 2 d.f. is 5.991 **1**
at the 5% level of significance. (9·210 for 1% level accepted.)
Since $2.3 < 5.991$, **1**
the null hypothesis (of ratios 30:60:10) cannot be rejected so the data do not provide evidence to cast doubt on the researcher's belief. **1**

A6.



$P(Z > z_p) = (1 - 0.999)/2 = 0.0005$ **1**

$\Rightarrow z_p = 3.29.$ **1**

$$\frac{x - \mu}{\sigma} = \pm 3.29$$ **1**

$x = 140 \pm 3.29 \times 15 (= 140 \pm 49{\cdot}35)$ **1**

$a = 91$ and $b = 189$ to nearest integer (but integers not needed). **1**

A7. The assumption that content weights are normally distributed (or same standard deviation) is required. (Independence not enough.) **1**

The sample has mean 507.38.

$$\left.\begin{array}{l} H_0 : \mu = 505 \\ H_1 : \mu \neq 505 \end{array}\right\}$$ **1**

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{507.38 - 505}{3/\sqrt{10}} = 2.51$$ **1**

The critical values for a two-tailed test at the 5% level of significance are ±1·96. **1**

Since z exceeds 1·96 the null hypothesis would be rejected at the 5% level of significance so the data provide *evidence* that the machine is not operating 'on target'. (Keyword *evidence*.) **1**

(The alternative of setting up a confidence interval and rejecting if outwith, is acceptable.) (A t-test is not appropriate.)

The manufacturer might overlook situations where packs were being overfilled with consequent losses. **1**

A8. $X \sim B(n, p)$ (parameters desirable.) **1**

$E(X) = np$ so that $E(X/n) = np/n (= p)$ **1**

$V(X) = npq$ **1**

so that $V(X/n) = npq/n^2 (= pq/n)$ **1**

(Other methods are, of course, acceptable.)

Use normal approximation to binomial **since *np* and *nq* are both > 5**. **1**

For the binomial distribution with parameters $n = 80$ and $p = 0.20$ the mean and standard deviation are 16 and 3.58. **1,1**

$P(9 \leqslant X \leqslant 23)$ where $X \sim N(16, 3.58^2)$

$\approx P\left(\frac{8.5 - 16}{3.58} \leqslant Z \leqslant \frac{23.5 - 16}{3.58}\right)$ *c.c.* **1**

$= P(-2.09 \leqslant Z \leqslant 2.09)$ **1**

$= \Phi(2.09) - \Phi(-2.09)$

$= 0.9817 - (1 - 0.9817)$

≈ 0.96 **1**

A9. (a) The underlying model is $Y_i = \alpha + \beta x_i + \varepsilon_i$ with $E(\varepsilon_i) = 0$, $V(\varepsilon_i) = \sigma^2$. **1**

Estimate of σ^2 is $s^2 = SSR/(n - 2) = 2.959616/15 = 0.1973$ **1**

(b) Assume that $\varepsilon_i \sim N(0, \sigma^2)$. **1**

$H_0 : \beta = 0$

$H_1 : \beta \neq 0$ (or $\beta > 0$ for a 1-tail test).

$$t = \frac{b}{s/\sqrt{S_{xx}}} = \frac{1.902}{\sqrt{0.1973}/\sqrt{145.94}} = 51.74$$ **1**

with 15 d.f.

With significance level 0.1% the critical value of t with 15 d.f. is 3.733. **1**

(The 0·5% value of 2·131 can be used.)

Reject H_0, since $51.74 > 3.733$ (or equivalent) **1**

the data provide very strong evidence of a non-zero slope parameter. **1**

(c) The 12th residual appears to be an outlier. **1**

The plot has a curvilinear appearance and a non-linear fit (or quadratic fit) may be more appropriate. **1**

Not random is not enough; random scatter about zero is acceptable.

A10. (a) Allocate students to the two groups at random. **1**

Give no indication as to whether students are receiving a dose or not. **1**

Or, nothing else to be consumed.

(b)

Dose	x	Rank
0	242	2
0	242	2
0	242	2
0	244	4.5
0	244	4.5
0	245	6.5
200	245	6.5
0	246	9
200	246	9
200	246	9
0	247	11
0	248	14
0	248	14
200	248	14
200	248	14
200	248	14
200	250	18
200	250	18
200	250	18
200	252	20

1

Rank sum for 0mg dose group is $W = 69.5$. **1**

$\mathrm{E}(W) = \frac{1}{2}n(n + m + 1) = \frac{1}{2}10(10 + 10 + 1) = 105$

$\mathrm{V}(W) = \frac{1}{12}nm(n + m + 1) = \frac{1}{12}10 \times 10(10 + 10 + 1) = 13.23^2 (= 175)$ **1**

$\left.\begin{array}{l} H_0 : \text{Median}_0 = \text{Median}_{200} \\ H_1 : \text{Median}_0 \neq \text{Median}_{200} \end{array}\right\}$ [or 1-tail $M_0 < M_{200}$] **1**

$z = \frac{W - \mathrm{E}(W)}{\sqrt{\mathrm{V}(W)}} = \frac{69.5 - 105}{13.23} = -2.68$ **1**

Since this value is less than -2.58 **1**

the null hypothesis would be rejected at the 1% level of significance so the experiment provides strong evidence that caffeine affects manual dexterity. **1**

[A continuity correction may be used, this gives $z = -2.65$. 5% level acceptable.]

(c) Assess each student twice – once after 0 mg dose and once after a 200mg dose and analyse the differences using a sign test (or t-test). **1** **1**

i.e. from independent to matched pairs sample.

SECTION B (Numerical Analysis 1 & 2)

B1. $$f(x) = \frac{1}{(5x-4)} \qquad f'(x) = \frac{-5}{(5x-4)^2} \qquad f''(x) = \frac{50}{(5x-4)^3}$$

Taylor polynomial is $p(x) = p(1+h) = 1 - 5h + 25h^2$. **3**

For $f(0.99)$, $h = -0.01$; $p(0.99) = 1 + 0.05 + 25 \times (-0.01)^2 = 1.0525$. **2**

Coefficient of h in Taylor polynomial is substantially greater than 1.
Hence $f(x)$ is likely to be sensitive to small changes in x near $x = 1$. **1**

B2. $$L(3) = \frac{(3-2)(3-5)}{(0-2)(0-5)} 1.3271 + \frac{(3-0)(3-5)}{(2-0)(2-5)} 1.5238 + \frac{(3-0)(3-2)}{(5-0)(5-2)} 1.8516$$

$$= \frac{-1.3271}{5} + 1.5238 + \frac{1.8516}{5} = 1.629$$ **3**

B3. $\Delta^2 f_0 = \Delta f_1 - \Delta f_0 = (f_2 - f_1) - (f_1 - f_0) = f_2 - 2f_1 + f_0$
$\Delta^3 f_0 = (f_3 - 2f_2 + f_1) - (f_2 - 2f_1 + f_0) = f_3 - 3f_2 + 3f_1 - f_0$ **2**

Maximum error is $\varepsilon + 3\varepsilon + 3\varepsilon + \varepsilon = 8\varepsilon$. **1**

$\Delta^3 f_0 = 1.658 - 3 \times 1.532 + 3 \times 1.416 - 1.311 = -0.001$
Maximum rounding error in $\Delta^3 f_0 = 8 \times 0.0005 = 0.004$. **2**

$\Delta^3 f_0$ is probably not significantly different from zero. **1**

B4. (a) Difference table is:

i	x	$f(x)$	diff1	diff2	diff3	diff4
0	1.0	1.263	193	47	8	2
1	1.1	1.456	240	55	10	-4
2	1.2	1.696	295	65	6	
3	1.3	1.991	360	71		
4	1.4	2.351	431			
5	1.5	2.782				

3

(b) $p = 0.8$;

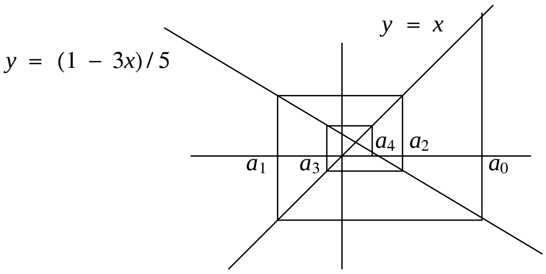
$$f(1.18) = 1.456 + 0.8(0.240) + \frac{(0.8)(-0.2)}{2}(0.055) + \frac{(0.8)(-0.2)(-1.2)}{6}(0.010)$$

$$= 1.456 + 0.192 - 0.004 + 0.0003$$

$$= 1.644$$ **3**

B5. $a_{n+1} = (1 - 3a_n)/5, a_0 = 2.$

$a_1 = -1;\ a_2 = 0.8;\ a_3 = -0.28;\ a_4 = 0.37$ **2**



3

Fixed point has $5a + 3a = 1$, i.e. $a = 1/8$ or 0.125. **1**

B6. $f(x) = ((((3.1x + 2.8)x)x) - 4.2)x + 2.0$ **1**

$f(1.2) = 8.23$ **1**

$f(x)_{max} = ((((3.15x + 2.85)x)x) - 4.15)x + 2.05$ and $f(1.2)_{max} = 8.53$ **1**

B7. In diagonally dominant form:

$$\begin{aligned} 4.24x_1 + 0.27x_2 - 0.46x_3 &= -1.14 \\ 0.71x_1 + 2.78x_2 - 0.08x_3 &= 3.18 \\ 0.46x_1 - 0.34x_2 + 5.17x_3 &= 9.25 \end{aligned}$$ **1**

The aim is to ensure that in each division process the denominator is as large as possible so that the process will converge. **1**

First iterates are $x_1 = -1.14/4.24 = -0.269;\ x_2 = 3.18/2.78 = 1.144$ **1**

B8. (a) $$\int_{x_0}^{x_1} f(x)\,dx = \int_0^1 f(x+ph)h\,dp = h\int_0^1 \left[f(x_0) + f'(x_0)ph + \tfrac{1}{2}f''(x_0)p^2h^2\right]dp$$

$$= h\left[f(x_0)p + f'(x_0)hp^2/2 + f''(x_0)h^2p^3/6\right]_0^1$$

$$= h\left[f(x_0) + \frac{1}{2}f'(x_0)h + \frac{1}{6}f''(x_0)h^2\right]$$

$$= h\left[f(x_0) + \frac{1}{2}f(x_1) - \frac{1}{2}f(x_0) - \frac{1}{4}f''(x_0)h^2 + \frac{1}{6}f''(x_0)h^2\right]$$

$$= \frac{h(f_0 + f_1)}{2} - \frac{h^3 f''(x_0)}{12}$$

(using $f(x_1) = f(x_0) + hf'(x_0) + \frac{1}{2}h^2 f''(x_0) + \ldots$). **5**

(b) Trapezium rule calculation is:

x	$f(x)$	m	$mf(x)$
1	0	1	0
1.1	0.1153	2	0.2307
1.2	0.2625	2	0.5251
1.3	0.4434	2	0.8868
1.4	0.6595	1	0.6595
			2.3020

Hence $I = 2.3020 \times 0.1/2 = 0.1151.$ **2**

B8. c't'd

(c) $f(x) = x^2 \ln x \qquad f'(x) = 2x \ln x + x$

$f''(x) = 2 \ln x + 3$ which has maximum on $[1, 1.4]$ at $x = 1.4$.

$f''(1.4) = 3.673$

$|\text{truncation error}| = 0.1^2 \times 0.4 \times 3.673/12 \approx 0.0012$ **3**

Hence estimate for I is $I = 0.115$ **1**

B9. Gaussian elimination table is:

					sum
	(3.7)	−4.2	2.2	8.4	10.1
	−2.8	1.5	0.7	3.1	2.5
	0	(2.6)	7.1	−2.8	6.9
$R_2 + 2.8R_1/3.7$	0	−1.678	2.365	9.457	10.143
$R_4 + 1.678R_3/2.6$	0	0	6.947	7.650	14.596

$x_3 = 1{\cdot}101 \qquad x_2 = -4{\cdot}084 \qquad x_1 = -3.020$

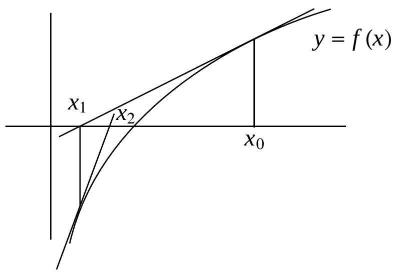
To 1 dec. place $\quad x_1 = 3.0 \qquad x_2 = -4.1 \qquad x_3 = 1.1$

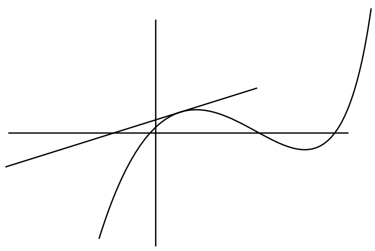
6

B10. Gradient of $y = f(x)$ at x_0 is $f'(x_0) = \dfrac{f(x_0)}{x_0 - x_1}$.

Hence $x_1 - x_0 = -\dfrac{f(x_0)}{f'(x_0)}$, i.e. $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$.

Likewise $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ and in general $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$. **3**





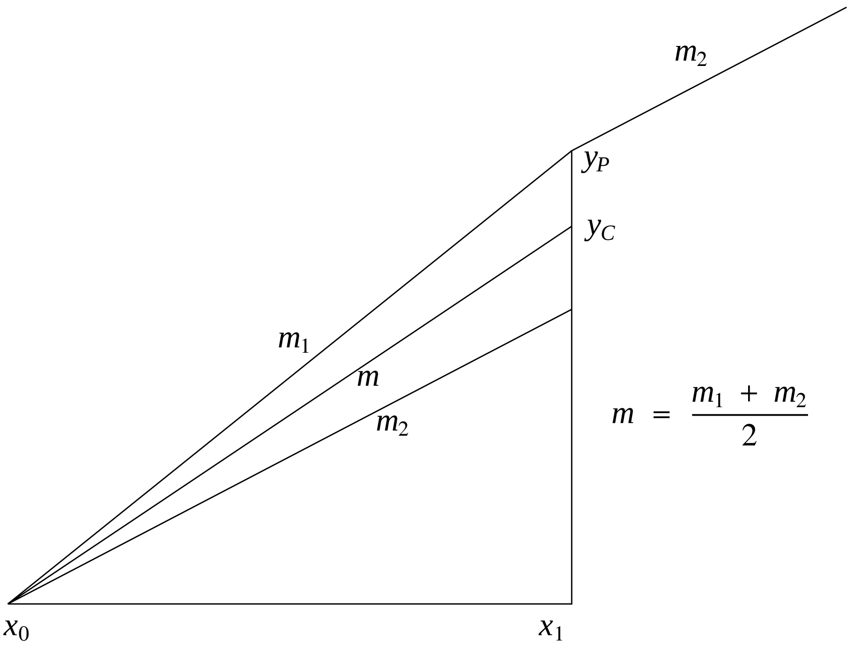
$f(x) = e^x - 3x^2 - 1$ and $f'(x) = e^x - 6x$. $x_0 = 3.5$.

$x_1 = 3.5 - (-4.6345)/12.115 = 3.8825; \quad x_2 = 3.7905; \quad x_3 = 3.7824; \quad x_4 = 3.7823$

Root is 3.782 (to 3 dec. places) **3**

Ill-conditioning of such equations implies that a small change in starting value may lead to very slow convergence or to a different root from that intended. The Newton-Raphson method depends for convergence on the point of intersection of the tangent with x-axis being closer to the root than the initial point. In interval [0, 0.5] there must be a TV of $f(x)$ so that $f'(x) = 0$ and point of intersection may be far from initial point; so iteration may lead to a different root. **3**

B11.



y_c is determined using the average of the gradients at (x_0, y_0) and (x_p, y_p). **3**

x	1	1.1
y	1	1.053
$y' = (x^2 - y + 1)\cos x$	0.5403	
y_p	1.0540	
y'_p	0.5244	
$\frac{1}{2}h\,(y' + y'_p)$	0.0532	
y_p	1.0532	
y'_{p_2}	0.5247	
$\frac{1}{2}h\,(y' + y'_{p_2})$	0.0533	

6

SECTION C (Mechanics 1 & 2)

C1. If $\mathbf{a} = 2t\mathbf{i}$ then, as $\mathbf{v}(0) = \mathbf{0}$,

$$\mathbf{v}(t) = t^2\mathbf{i} \qquad \mathbf{1}$$

and, as $\mathbf{r}(0) = \mathbf{0}$,

$$\mathbf{r}(t) = \frac{1}{3}t^3\mathbf{i}. \qquad \mathbf{1}$$

When $|\mathbf{v}| = 1$, $t = 1$ which gives $\mathbf{r}(1) = \frac{1}{3}\mathbf{i}$.
When $|\mathbf{v}| = 9$, $t = 3$ and hence $\mathbf{r}(3) = 9\mathbf{i}$. **1**
Therefore the distance travelled is $\frac{26}{3}$ m. **1**

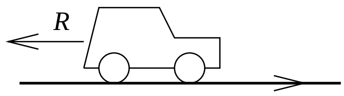
C2. (a) Firstly $\overrightarrow{AB} = 2\mathbf{i} + 10\mathbf{j}$ so **1**

$$\text{Work done} = (2\mathbf{i} + 3\mathbf{j}).(2\mathbf{i} + 10\mathbf{j}) \qquad \mathbf{1}$$
$$= 4 + 30$$
$$= 34 \text{ newton metres}. \qquad \mathbf{1}$$

(b) Rate at which work is done = power

$$= (2\mathbf{i} + 3\mathbf{j}).\tfrac{1}{4}(\mathbf{i} + 5\mathbf{j}) \qquad \mathbf{1}$$
$$= \tfrac{1}{4}(2 + 15)$$
$$= \tfrac{17}{4} \text{ Watts}. \qquad \mathbf{1}$$

C3.



By Newton II, the acceleration is $a = -\dfrac{R}{m}$. **1**

This is constant so use

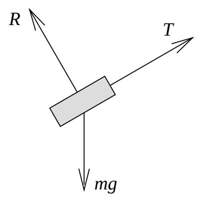
$$v^2 = u^2 + 2ax \qquad \mathbf{1}$$

with $v = 0$ and $u = U$

$$x = -\frac{U^2}{2a} = \frac{mU^2}{2R}. \qquad \mathbf{1}$$

Since U and R are fixed, the stopping distance increases directly as the mass of the car increases. **1**

C4. (a)



The reaction force is

$$R = mg\cos 30° \qquad \mathbf{1}$$

and then resolving forces parallel to the sloping surface when $\theta = 30°$ gives

$$T + \mu R = mg\sin 30° \qquad \mathbf{1}$$

or

$$T = \tfrac{1}{2}mg\left(1 - \sqrt{3}\,\mu\right). \qquad (*) \qquad \mathbf{1}$$

C4. c't'd

(b) Again, resolving forces parallel to the sloping surface

$$2T + \mu mg\cos 45° = mg\sin 45° \qquad \mathbf{1}$$

$$\Rightarrow T = \frac{mg}{2\sqrt{2}}(1 - \mu) \qquad (**) \qquad \mathbf{1}$$

(c) Equating (*) and (**) gives

$$1 - \sqrt{3}\mu = \frac{1}{\sqrt{2}}(1 - \mu) \qquad \mathbf{1}$$

and rearranging

$$\mu = \frac{\sqrt{2} - 1}{\sqrt{6} - 1} \approx 0{\cdot}29. \qquad \mathbf{1}$$

C5. Applying the Inverse Square Law of Gravitation

$$ma_A = \frac{GmM}{(3R + R)^2 \times 1000^2} \qquad ma_B = \frac{GmM}{(5R + R)^2 \times 1000^2} \qquad \mathbf{1}$$

so

$$\frac{GM}{1000^2} = 16R^2 a_A \qquad \text{and} \qquad \frac{GM}{1000^2} = 36R^2 a_B \qquad \mathbf{1}$$

But

$$a_A = 4R\omega_A^2 \times 1000 \qquad \text{and} \qquad a_B = 6R\omega_B^2 \times 1000 \qquad \mathbf{1}$$

so

$$\omega_A^2(4R)^3 = \omega_B^2(6R)^3 \qquad \mathbf{1}$$

$$\Rightarrow 64\omega_A^2 = 216\omega_B^2 \qquad \mathbf{1}$$

$$\text{i.e.}\ 8\omega_A^2 = 27\omega_B^2.$$

C6. (a) Note that the tension in the elastic string is given by

$$T = \frac{\lambda x}{l} \qquad \mathbf{1}$$

so by Newton II

$$m\frac{d^2x}{dt^2} = mg - T \qquad \mathbf{1}$$

$$\Rightarrow \frac{d^2x}{dt^2} + \left(\frac{\lambda}{ml}\right)x = g$$

$$\text{so} \qquad \omega^2 = \frac{\lambda}{ml}. \qquad \mathbf{1}$$

(b) The mass is in equilibrium when $\dfrac{d^2x}{dt^2} = 0.$ **1**

$$\text{so} \qquad \omega^2 x_e = g$$

$$\Rightarrow \qquad x_e = \frac{g}{\omega^2} \qquad \mathbf{1}$$

C7. Initially we have

$$\mathbf{r}_a = \mathbf{0}; \qquad \mathbf{r}_h = -50\sqrt{2}\,\mathbf{i} + \mathbf{k} \qquad \mathbf{1}$$

Also, for the aircraft,

$$\mathbf{v}_a = 100\sqrt{2}\,\mathbf{j} \qquad \mathbf{1}$$

$$\Rightarrow \qquad \mathbf{r}_a = 100\sqrt{2}t\,\mathbf{j} \qquad (\text{as } \mathbf{r}_a(0) = \mathbf{0}) \qquad \mathbf{1}$$

and, for the helicopter,

$$\mathbf{v}_h = 100\left(\cos 45^\circ\,\mathbf{i} + \sin 45^\circ\,\mathbf{j}\right) = 50\sqrt{2}\,(\mathbf{i} + \mathbf{j}) \qquad \mathbf{1}$$

$$\Rightarrow \qquad \mathbf{r}_h = 50\sqrt{2}t\,(\mathbf{i} + \mathbf{j}) + \mathbf{c}$$

Therefore $\mathbf{c} = \mathbf{r}_h(0) = -50\sqrt{2}\,\mathbf{i} + \mathbf{k}$ so

$$\mathbf{r}_h(t) = 50\sqrt{2}\,(t - 1)\,\mathbf{i} + 50\sqrt{2}t\,\mathbf{j} + \mathbf{k} \qquad \mathbf{1}$$

The position of the helicopter relative to the aircraft is

$$\mathbf{r}_h - \mathbf{r}_a = 50\sqrt{2}\,(t - 1)\,\mathbf{i} - 50\sqrt{2}t\,\mathbf{j} + \mathbf{k} \qquad \mathbf{1}$$

C8. (a) From the equations of motion

$$x = V\cos\alpha\, t \qquad \mathbf{1}$$

and

$$y = H + V\sin\alpha t - \tfrac{1}{2}gt^2. \qquad \mathbf{1}$$

Eliminating t gives

$$y = H + x\tan\alpha - \frac{1}{2}\frac{gx^2}{V^2\cos^2\alpha} \qquad \mathbf{1}$$

but since $V^2 = 2gH$ and $\dfrac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$

$$y = H + x\tan\alpha - \frac{\left(1 + \tan^2\alpha\right)x^2}{4H}. \qquad \mathbf{1}$$

(b) Given that $x = 2H$ when $y = 0$ then the equation of the trajectory gives

$$0 = H + 2H\tan\alpha - \left(1 + \tan^2\alpha\right)H \qquad \mathbf{1}$$

$$\Rightarrow \qquad \tan\alpha\,(2 - \tan\alpha) = 0 \qquad \mathbf{1}$$

$$\Rightarrow \qquad \tan\alpha = 0 \text{ or } \tan\alpha = 2$$

but since $\alpha > 0$, $\tan\alpha = 2$. **1**

(c) Putting $\tan\alpha = 2$ in the equation of the trajectory gives

$$y = H + 2x - \frac{5}{4H}x^2 \qquad \mathbf{1}$$

The maximum height is attained when $\frac{dy}{dx} = 0$. **1**

$$\text{i.e. when } x = \frac{4H}{5}. \qquad \mathbf{1}$$

So the maximum height is

$$y = H + \frac{8H}{5} - \frac{5}{4H}\cdot\frac{16H^2}{25} \qquad \mathbf{1}$$

$$= \tfrac{9}{5}H.$$

C9. (a) By Newton II

$$ma = F - mkv \qquad \mathbf{1}$$

and with $F = \dfrac{P}{v}$ we have **1**

$$m\frac{dv}{dt} = \frac{P}{v} - mkv$$

$$\text{or} \qquad m\frac{dv}{dt} = \frac{P - mkv^2}{v} \qquad \mathbf{1}$$

(b) Separating the variables and integrating gives

$$\int \frac{mv\,dv}{P - mkv^2} = \int dt \qquad \mathbf{1}$$

$$\Rightarrow \qquad -\frac{1}{2k}\int \frac{-2mkv}{P - mkv^2}\,dv = \int dt \qquad \mathbf{1}$$

so

$$\ln\left|P - mkv^2\right| = -2k(t + c) \qquad \mathbf{1}$$

from which we obtain

$$P - mkv^2 = e^{-2kt - 2kc}. \qquad \mathbf{1}$$

Using $v = 0$ when $t = 0$ gives

$$e^{-2kc} = P \qquad \mathbf{1}$$

and hence

$$P - mkv^2 = Pe^{-2kt}$$

$$\Rightarrow \qquad v^2 = \frac{P}{mk}\left(1 - e^{-2kt}\right). \qquad \mathbf{1}$$

(c) As $t \to \infty$, $v^2 \to \dfrac{P}{mk}$ so $v \to \sqrt{\dfrac{P}{mk}}$. **1**

C10. (a) Let v be the speed of the bead released from P when it reaches A. By conservation of energy

$$\frac{1}{2}mv^2 = mg(R + R\cos 30°) \qquad \mathbf{1}$$

$$\Rightarrow \quad v^2 = 2gR(1 + \tfrac{\sqrt{3}}{2})$$

$$\Rightarrow \quad v = \sqrt{gR(2 + \sqrt{3})} \qquad \mathbf{1}$$

Momentum before the collision = mv.
Momentum after the collision = $3mv_a$.
So by the conservation of momentum

$$3mv_a = mv \qquad \mathbf{1}$$

$$\Rightarrow \quad v_a = \tfrac{1}{3}\sqrt{(2 + \sqrt{3})\,gR} \qquad \mathbf{1}$$

The kinetic energy before the collision is

$$K_b = \tfrac{1}{2}mgR(2 + \sqrt{3})$$

and the kinetic energy after the the collision

$$K_a = \tfrac{1}{2}\,3m\,\tfrac{1}{9}\,(2 + \sqrt{3})\,gR$$

$$= \tfrac{1}{6}\,mgR(2 + \sqrt{3})\,. \qquad \mathbf{1}$$

The loss in kinetic energy is

$$K_b - K_a = \tfrac{1}{2}mgR(2 + \sqrt{3})(1 - \tfrac{1}{3})$$

$$= \tfrac{1}{3}(2 + \sqrt{3})\,mgR. \qquad \mathbf{1}$$

So the fraction of kinetic energy lost is

$$\frac{K_b - K_a}{K_b} = \frac{2}{3}. \qquad \mathbf{1}$$

(b) If H is the vertical height attained, by conservation of energy

$$3mgH = \tfrac{1}{2}(3m) \times \tfrac{1}{9}(2 + \sqrt{3})\,gR$$

$$H = \frac{(2 + \sqrt{3})\,R}{18}. \qquad \mathbf{1}$$

Then

$$H = R(1 - \cos\phi) \qquad \mathbf{1}$$

$$\Rightarrow \; R(1 - \cos\phi) = \frac{R}{18}(2 + \sqrt{3}) \qquad \mathbf{1}$$

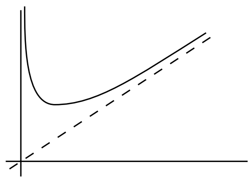
$$\Rightarrow \quad \cos\phi = \frac{16 - \sqrt{3}}{18}. \qquad \mathbf{1}$$

SECTION D (Mathematics 1)

	Solution	Marks
D1.	(a) $y = \dfrac{\ln(1 + x^2)}{(1 + x^2)}$	
	$\dfrac{dy}{dx} = \dfrac{\frac{2x}{1+x^2}(1+x^2) - 2x\ln(1+x^2)}{(1+x^2)^2}$	1 method 2E1 for the rest
	$= \dfrac{2x[1 - \ln(1+x^2)]}{(1+x^2)^2}$	
	(b) $f(x) = e^{\sec x}$	
	$f'(x) = \sec x \tan x \, e^{\sec x}$	1 method 1 for the rest Total 5
D2.	$\binom{n}{2} - \binom{n}{1} = 2$	1 for answer only. 1 for further evidence of checking.
	$\dfrac{n!}{(n-2)!2!} - \dfrac{n!}{(n-1)!1!} = 2$	1 for this line
	$n(n-1) - 2n = 4$	1 for a quadratic
	$n^2 - 3n - 4 = 0$	
	$(n-4)(n+1) = 0$	1 for solving
	So $n = 4$.	1 for final statement (lost if $n = -1$ is included) Total 4
D3.	$x = 1 + t^2$	
	$dx = 2t\,dt$	1 for the derivative
	$\displaystyle\int \frac{x}{\sqrt{x-1}}\,dx = \int \frac{1+t^2}{t}\,2t\,dt$	1 for substitution
	$= 2\displaystyle\int (1 + t^2)\,dt$	
	$= 2\left(t + \frac{1}{3}t^3\right) + c$	1 for integration
	$= \frac{2}{3}t(3 + t^2) + c$	
	$= \frac{2}{3}(x + 2)\sqrt{x-1} + c$	1 for solution Total 4
	or $2\sqrt{x-1} + \frac{2}{3}(x-1)^{3/2} + c$	

D4.	$$a + 0.99b = 9.99 \quad (1)$$ $$0.99a + 0.98b = 9.89 \quad (2)$$ $$(1) \Rightarrow 0.99a + 0.9801b = 9.8901 \quad (3)$$ $$(3) - (2) \Rightarrow 0.0001b = 0.0001$$ $$\Rightarrow b = 1 \text{ and } a = 9$$	1 for solving
	$$a + 0.99b = 10 \quad (1)$$ $$0.99a + 0.98b = 9.89 \quad (2)$$ $$0.99a + 0.9801b = 9.9 \quad (3)$$ $$(3) - (2) \Rightarrow 0.0001b = 0.01$$ $$\Rightarrow b = 100 \text{ and } a = -89$$	1 for solving
	The equations are ill-conditioned. A small change in a coefficient results in a big change in the solutions.	1 for ill-conditioned 1 for reason Total 4
D5.	$$\frac{2x^2 - 3x + 2}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$ $$2x^2 - 3x + 2 = Ax(x-1) + B(x-1) + Cx^2$$	1 for method
	$$x = 0 \Rightarrow B = -2$$ $$x = 1 \Rightarrow C = 1$$ $$x = 2 \Rightarrow 8 - 6 + 2 = 2A + (-2) + 4$$ $$\Rightarrow A = 1$$	3E1 for values
	$$\frac{2x^2 - 3x + 2}{x^2(x-1)} = \frac{1}{x} - \frac{2}{x^2} + \frac{1}{x-1}$$	1 for statement
	$$\frac{2x^2 - 3x + 2}{x^2(x-1)} = \frac{x-2}{x^2} + \frac{1}{x-1}$$ does not gain final mark.	Total 5

D6.

	Solution	Marks	
(a)	$f(x) = 3x + \frac{3}{x}$ Asymptotes are $x = 0$ and $y = 3x$	2E1 for asymptotes	
(b)	$f'(x) = 3 - \frac{3}{x^2} = 0$ at SV $3(x^2 - 1) = 0$ So $a = 1$ and $f(1) = 6$ $f''(x) = \frac{6}{x^3} > 0$ so a minimum	1 for derivative 1 for a 1 for nature	
(c)		1 for sketch	Inclusion of negative branch not penalised.
(d)	Volume $= \int \pi y^2\,dx$ $= 9\pi \int_1^3 (x^2 + 2 + x^{-2})\,dx$ $= 9\pi \left[\frac{1}{3}x^3 + 2x - \frac{1}{x}\right]_1^3$ $= 9\pi\left[\left(9 + 6 - \frac{1}{3}\right) - \left(\frac{1}{3} + 2 - 1\right)\right]$ $= 3\pi[45 - 1 - (1 + 3)]$ $= 120\pi$	1 for method 1 for integral 1 for integration 1 for evaluation	Where possible, marked out of 2 if an incorrect formula was used.
		Total 10	

SECTION E (Statistics 1)

E1. $P(R) = P(R \mid B_1)\,P(B_1) + P(R \mid B_2)\,P(B_2) + P(R \mid B_3)\,P(B_3)$ **1**

$= \frac{3}{10} \times \frac{1}{3} + \frac{4}{8} \times \frac{1}{3} + \frac{2}{10} \times \frac{1}{3}$ **1**

$= \frac{1}{3}$ **1**

$P(R \mid B_1) = \frac{P(B_1 \mid R)\,P(B_1)}{P(R)}$ **1**

$= \frac{\frac{3}{10} \times \frac{1}{3}}{\frac{1}{3}} = \frac{3}{10}$ **1**

(formulae need not be quoted, other methods eg Venn or Tree acceptable)

E2.

(a) $P(X = 0) = F(0) = 0.0183$ **1**

(b) $P(X = 4) = F(4) - F(3)$ **1**

$= 0.6288 - 0.4335 = 0.1953$ **1**

(c) $P(X > 4) = 1 - F(4)$ **1**

$= 1 - 0.6288 = 0.3712 \approx 0{\cdot}37$ **1**

$V(X) = \mu = 4$ so $\sigma = \sqrt{4} = 2.$ **1**

E3. A simple *random* sample of farms would be selected from *each* category. **1**

(Both words needed.)

Total number of farms is 500.

Numbers from each category would be 50/500 of number in the category. **1**

Numbers would be 3 large, 32 medium and 15 small. **1**

If the strata are homogeneous then stratified sampling will provide more precise estimates of population parameters than simple random sampling with the same total sample size. **1**

A random sample might exclude one category altogether. **1**

(Not and exhaustive list, many other possible comments acceptable.)

E4.	The assumption that content weights are normally distributed (or same standard deviation) is required. (Independence not enough.)	**1**
	The sample has mean 507.38.	
	$\left.\begin{array}{l} H_0 : \mu = 505 \\ H_1 : \mu \neq 505 \end{array}\right\}$	**1**
	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{507.38 - 505}{3 / \sqrt{10}} = 2.51$	**1**
	The critical values for a two-tailed test at the 5% level of significance are ±1·96.	**1**
	Since z exceeds 1·96 the null hypothesis would be rejected at the 5% level of significance so the data provide *evidence* that the machine is not operating 'on target'. (Keyword *evidence*.)	**1**
	(The alternative of setting up a confidence interval and rejecting if outwith, is acceptable.) (A t-test is not appropriate.)	
	The manufacturer might overlook situations where packs were being overfilled with consequent losses.	**1**

E5.	$X \sim \mathrm{B}(n, p)$ (parameters desirable.)	**1**
	$\mathrm{E}(X) = np$ so that $\mathrm{E}(X/n) = np/n\,(= p)$	**1**
	$\mathrm{V}(X) = npq$	**1**
	so that $\mathrm{V}(X/n) = npq/n^2\,(= pq/n)$	**1**
	(Other methods are, of course, acceptable.)	
	Use normal approximation to binomial **since *np* and *nq* are both > 5**.	**1**
	For the binomial distribution with parameters $n = 80$ and $p = 0.20$ the mean and standard deviation are 16 and 3.58.	**1,1**
	$\mathrm{P}(9 \leqslant X \leqslant 23)$ where $X \sim \mathrm{N}(16, 3.58^2)$	
	$\approx \mathrm{P}\left(\dfrac{8.5 - 16}{3.58} \leqslant Z \leqslant \dfrac{23.5 - 16}{3.58}\right)$	*c.c.* **1**
	$= \mathrm{P}(-2.09 \leqslant Z \leqslant 2.09)$	**1**
	$= \Phi(2.09) - \Phi(-2.09)$	
	$= 0.9817 - (1 - 0.9817)$	
	≈ 0.96	**1**

SECTION F (Numerical Analysis 1)

F1.

$$f(x) = \frac{1}{(5x - 4)} \qquad f'(x) = \frac{-5}{(5x-4)^2} \qquad f''(x) = \frac{50}{(5x-4)^3}$$

Taylor polynomial is $p(x) = p(1 + h) = 1 - 5h + 25h^2$. **3**

For $f(0.99)$, $h = -0.01$; $p(0.99) = 1 + 0.05 + 25 \times (-0.01)^2 = 1.0525$. **2**

Coefficient of h in Taylor polynomial is substantially greater than 1.
Hence $f(x)$ is likely to be sensitive to small changes in x near $x = 1$. **1**

F2.

$$L(3) = \frac{(3-2)(3-5)}{(0-2)(0-5)} 1.3271 + \frac{(3-0)(3-5)}{(2-0)(2-5)} 1.5238 + \frac{(3-0)(3-2)}{(5-0)(5-2)} 1.8516$$

$$= \frac{-1.3271}{5} + 1.5238 + \frac{1.8516}{5} = 1.629$$ **3**

F3.

$\Delta^2 f_0 = \Delta f_1 - \Delta f_0 = (f_2 - f_1) - (f_1 - f_0) = f_2 - 2f_1 + f_0$
$\Delta^3 f_0 = (f_3 - 2f_2 + f_1) - (f_2 - 2f_1 + f_0) = f_3 - 3f_2 + 3f_1 - f_0$ **2**

Maximum error is $\varepsilon + 3\varepsilon + 3\varepsilon + \varepsilon = 8\varepsilon$. **1**

$\Delta^3 f_0 = 1.658 - 3 \times 1.532 + 3 \times 1.416 - 1.311 = -0.001$
Maximum rounding error in $\Delta^3 f_0 = 8 \times 0.0005 = 0.004$. **2**

$\Delta^3 f_0$ is probably not significantly different from zero. **1**

F4.

(a) Difference table is:

i	x	$f(x)$	diff1	diff2	diff3	diff4
0	1.0	1.263	193	47	8	2
1	1.1	1.456	240	55	10	-4
2	1.2	1.696	295	65	6	
3	1.3	1.991	360	71		
4	1.4	2.351	431			
5	1.5	2.782				

3

(b) $p = 0.8$;

$$f(1.18) = 1.456 + 0.8(0.240) + \frac{(0.8)(-0.2)}{2}(0.055) + \frac{(0.8)(-0.2)(-1.2)}{6}(0.010)$$

$$= 1.456 + 0.192 - 0.004 + 0.0003$$

$$= 1.644$$ **3**

F5. (a)

$$\int_{x_0}^{x_1} f(x)\,dx = \int_0^1 f(x+ph)h\,dp = h\int_0^1 \left[f(x_0) + f'(x_0)ph + \tfrac{1}{2}f''(x_0)p^2h^2\right]dp$$

$$= h\left[f(x_0)p + f'(x_0)hp^2/2 + f''(x_0)h^2p^3/6\right]_0^1$$

$$= h\left[f(x_0) + \tfrac{1}{2}f'(x_0)h + \tfrac{1}{6}f''(x_0)h^2\right]$$

$$= h\left[f(x_0) + \tfrac{1}{2}f(x_1) - \tfrac{1}{2}f(x_0) - \tfrac{1}{4}f''(x_0)h^2 + \tfrac{1}{6}f''(x_0)h^2\right]$$

$$= \frac{h(f_0+f_1)}{2} - \frac{h^3f''(x_0)}{12}$$

(using $f(x_1) = f(x_0) + hf'(x_0) + \tfrac{1}{2}h^2f''(x_0) + \ldots$). **5**

(b) Trapezium rule calculation is:

x	$f(x)$	m	$mf(x)$
1	0	1	0
1.1	0.1153	2	0.2307
1.2	0.2625	2	0.5251
1.3	0.4434	2	0.8868
1.4	0.6595	1	0.6595
			2.3020

Hence $I = 2.3020 \times 0.1/2 = 0.1151$. **2**

(c) $f(x) = x^2 \ln x \qquad f'(x) = 2x \ln x + x$

$f''(x) = 2 \ln x + 3$ which has maximum on $[1, 1.4]$ at $x = 1.4$.

$f''(1.4) = 3.673$

$|\text{truncation error}| = 0.1^2 \times 0.4 \times 3.673/12 \approx 0.0012$ **3**

Hence estimate for I is $I = 0.115$ **1**

SECTION G (Mechanics 1)

G1. If $\mathbf{a} = 2t\mathbf{i}$ then, as $\mathbf{v}(0) = \mathbf{0}$,

$$\mathbf{v}(t) = t^2\mathbf{i} \qquad \mathbf{1}$$

and, as $\mathbf{r}(0) = \mathbf{0}$,

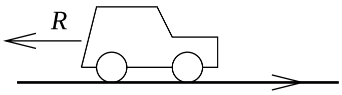
$$\mathbf{r}(t) = \frac{1}{3}t^3\mathbf{i}. \qquad \mathbf{1}$$

When $|\mathbf{v}| = 1$, $t = 1$ which gives $\mathbf{r}(1) = \frac{1}{3}\mathbf{i}$.

When $|\mathbf{v}| = 9$, $t = 3$ and hence $\mathbf{r}(3) = 9\mathbf{i}$. **1**

Therefore the distance travelled is $\frac{26}{3}$ m. **1**

G2.



By Newton II, the acceleration is $a = -\dfrac{R}{m}$. **1**

This is constant so use

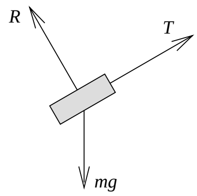
$$v^2 = u^2 + 2ax \qquad \mathbf{1}$$

with $v = 0$ and $u = U$

$$x = -\frac{U^2}{2a} = \frac{mU^2}{2R}. \qquad \mathbf{1}$$

Since U and R are fixed, the stopping distance increases directly as the mass of the car increases. **1**

G3. (a)



The reaction force is

$$R = mg\cos 30° \qquad \mathbf{1}$$

and then resolving forces parallel to the sloping surface when $\theta = 30°$ gives

$$T + \mu R = mg\sin 30° \qquad \mathbf{1}$$

or

$$T = \tfrac{1}{2}mg\left(1 - \sqrt{3}\,\mu\right). \qquad (*) \qquad \mathbf{1}$$

(b) Again, resolving forces parallel to the sloping surface

$$2T + \mu mg\cos 45° = mg\sin 45° \qquad \mathbf{1}$$

$$\Rightarrow T = \frac{mg}{2\sqrt{2}}(1 - \mu) \qquad (**) \qquad \mathbf{1}$$

(c) Equating (*) and (**) gives

$$1 - \sqrt{3}\mu = \frac{1}{\sqrt{2}}(1 - \mu) \qquad \mathbf{1}$$

and rearranging

$$\mu = \frac{\sqrt{2} - 1}{\sqrt{6} - 1} \approx 0{\cdot}29. \qquad \mathbf{1}$$

G4. Initially we have

$$\mathbf{r}_a = \mathbf{0}; \qquad \mathbf{r}_h = -50\sqrt{2}\,\mathbf{i} + \mathbf{k} \qquad \mathbf{1}$$

Also, for the aircraft,

$$\mathbf{v}_a = 100\sqrt{2}\,\mathbf{j} \qquad \mathbf{1}$$

$$\Rightarrow \qquad \mathbf{r}_a = 100\sqrt{2}t\,\mathbf{j} \qquad (\text{as } \mathbf{r}_a(0) = \mathbf{0}) \qquad \mathbf{1}$$

and, for the helicopter,

$$\mathbf{v}_h = 100\left(\cos 45^\circ\,\mathbf{i} + \sin 45^\circ\,\mathbf{j}\right) = 50\sqrt{2}\,(\mathbf{i} + \mathbf{j}) \qquad \mathbf{1}$$

$$\Rightarrow \qquad \mathbf{r}_h = 50\sqrt{2}t\,(\mathbf{i} + \mathbf{j}) + \mathbf{c}$$

Therefore $\mathbf{c} = \mathbf{r}_h(0) = -50\sqrt{2}\,\mathbf{i} + \mathbf{k}$ so

$$\mathbf{r}_h(t) = 50\sqrt{2}\,(t - 1)\,\mathbf{i} + 50\sqrt{2}t\,\mathbf{j} + \mathbf{k} \qquad \mathbf{1}$$

The position of the helicopter relative to the aircraft is

$$\mathbf{r}_h - \mathbf{r}_a = 50\sqrt{2}\,(t - 1)\,\mathbf{i} - 50\sqrt{2}t\,\mathbf{j} + \mathbf{k} \qquad \mathbf{1}$$

G5. (a) From the equations of motion

$$x = V\cos\alpha\, t \qquad \mathbf{1}$$

and

$$y = H + V\sin\alpha t - \tfrac{1}{2}gt^2. \qquad \mathbf{1}$$

Eliminating t gives

$$y = H + x\tan\alpha - \frac{1}{2}\frac{gx^2}{V^2\cos^2\alpha} \qquad \mathbf{1}$$

but since $V^2 = 2gH$ and $\dfrac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$

$$y = H + x\tan\alpha - \frac{\left(1 + \tan^2\alpha\right)x^2}{4H}. \qquad \mathbf{1}$$

(b) Given that $x = 2H$ when $y = 0$ then the equation of the trajectory gives

$$0 = H + 2H\tan\alpha - \left(1 + \tan^2\alpha\right)H \qquad \mathbf{1}$$

$$\Rightarrow \qquad \tan\alpha\,(2 - \tan\alpha) = 0 \qquad \mathbf{1}$$

$$\Rightarrow \qquad \tan\alpha = 0 \text{ or } \tan\alpha = 2$$

but since $\alpha > 0$, $\tan\alpha = 2$. **1**

(c) Putting $\tan\alpha = 2$ in the equation of the trajectory gives

$$y = H + 2x - \frac{5}{4H}x^2 \qquad \mathbf{1}$$

The maximum height is attained when $\frac{dy}{dx} = 0$. **1**

$$\text{i.e. when } x = \frac{4H}{5}. \qquad \mathbf{1}$$

So the maximum height is

$$y = H + \frac{8H}{5} - \frac{5}{4H}\cdot\frac{16H^2}{25}$$

$$= \tfrac{9}{5}H. \qquad \mathbf{1}$$

[END OF MARKING INSTRUCTIONS]