Mathematics

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WHEN you are studying for your Maths exams, start by remembering that the biggest asset you have is your teacher – in the run up to the exams you must ask if there is something you aren't sure about. Help your teacher to help you, and use the advice and hints here as a guide. Remember though – this guide won't do the work for you. Only you can fully prepare yourself for the Maths exam, so get started with your revision. When you are studying, focus on certain topics at a time and try to master them. Make sure you also do as many past papers as possible and concentrate on the most recent ones.

Think about the *tools* you need for passing your maths exam, like all the different tools you would have in a tool box. If someone tells you to use a particular tool, then you have no difficulty – you can add, you can subtract, you can factorise, you can simplify, you can differentiate and so on. But when it comes to the exam, nobody is there to tell you what tool to use. You have to revise and learn how to use these tools in advance.

Remember that in Maths questions are based on knowledge. In some questions you have to make a decision about what you are being asked to do – we call these problems.

Maths exams are perhaps different from any other in that we all have that fear that we will not be able to do anything at all, that our mind will go blank and we will freeze. Everyone sitting a maths exam gets that feeling from time to time.

It is true that maths exams do not lend themselves to waffle: facts and figures are absolutes. So it is important to relax and read the questions carefully. Do not jump in and answer what you think or hope they are asking: answer what is being asked. Remember if you start to panic then stop, take a deep breath and look around you. Be positive – you are as capable as everyone else at answering the question; if they can do it so can you. If you are looking at a question and you don't know how to start it, look and see how many parts it is in – it might be possible to answer part (b) without part (a), which you can go back to later in the exam. Try to resist the temptation to flit from question to question too quickly. You have plenty of time, so use it. Once you have started writing, then it will become easier.

It is important to get off to a good start and getting the first question under your belt is the best way. In Standard Grade Paper 1, this is not too difficult as the first few questions will be short numerical ones. In the second paper and in both Higher papers it is important to get yourself settled. The effect of knowing (rightly or wrongly) that you have the first question correct boosts your confidence for the rest of the paper.

If you do not like the first question and feel it is not your favourite topic, then have a look at the next two or three; they may be better bets for you. However, the examiners will make the first few questions the easier, more straightforward ones and so it would not make sense to start at the end and work your way back.

The earlier questions in the exam are usually the standard ones, easy to recognise and hopefully easy to do. As you go through the paper the questions will steadily become more difficult. They are not impossible, just more challenging.

The middle group of questions then form the "battleground" of your exam. They are the ones you need to get correct to give you your pass. You should take care and try to answer each part. Give yourself time in this section and do not panic if they seem impossible – they are not, just think of your toolbox and decide what the crucial aspects of the question are and what tool to take from the box. You have probably done similar questions before: you just need to access the correct part of your brain to recall the method.

Look at how many marks are allocated to a question – this will give you a clue as to what you are expected to write. A question worth one mark is likely to be one in which you are expected simply to write down the answer (no working needed here). Questions worth two, three or four marks are likely to involve a few steps of working but should be fairly straightforward.

When you come to questions worth five, six, or seven marks, they may well be divided up into smaller parts or require quite sustained thinking and reasoning. These higher mark questions are not so common at Standard Grade.

Watch out for clues in the wording:

- Any word in **bold** is making a point! It is trying to draw your attention to something important.
- "*Hence*" means you must use the answer you have just found for the next part of the question.
- "*Evaluate*" simply means find the value of.
- "*Show*" means you must be very precise and put down all the steps of your reasoning.

Being allowed the use of a calculator does not mean that you can just write down the answer. Even correct answers could receive no credit if there is no working out shown.

Markers are looking for the steps involved and want to know how you got your answer. Marks are allocated to this part of your solution – it is not the case that all the marks are given for the final answer. If you write down only the answer and it is wrong, then you will receive no marks. The same wrong answer with working could gain you some marks.

STANDARD GRADE

GENERAL LEVEL

The following worked examples illustrate some of the commonly asked questions at this level:

2005 Paper I Question 1(a)

Carry out the following calculations:
(a) 209.3 – 175.48

You should set the answer out like this:

209.30 (Notice the insertion of 0)
– 175.48
= 33.82

The important bit here is writing 209.3 as 209.30 to fit in with the second number.

2005 Paper I Question 7

(a) Graham goes into a shop and buys a bottle of water and a cheese roll for £1.38. In the same shop, Alan pays £1.77 for two bottles of water and a cheese roll. How much does a bottle of water cost?

This is a reasoning question so you have to think your way through it and show your working. Remember your working shows how you are thinking and can gain you marks even if you don't finish the question.

(a) One water + cheese roll = £1.38
Two waters and cheese roll = £1.77
(An extra water has been purchased)
£1.77 – £1.38 = 39 pence
So one water alone costs 39 pence.

(b) Craig goes into the shop and buys four bottles of water and three cheese rolls. How much will this cost?

There are two ways we can do this:

From (a) we know that a bottle of water costs 39 pence, so we can work out that a cheese roll costs:
£1.38 – £0.39 = £0.99
So four bottles of water and three cheese rolls costs: 4 x 39 pence + 3 x 99 pence = £4.53

OR From the information in part (a) we see that we can work out the cost of four bottles of water and three cheese rolls by adding:
2 x £1.38 + £1.77 = £4.53
That is: 2 x (bottle of water + cheese roll) + (2 bottles of water + cheese roll).

2005 Paper I Question 8

John buys a football programme for £1.60 and

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sells it for £2.00. Calculate his percentage profit.

Profit = selling price minus buying price
= £2.00 – £1.60
= 40 pence

Percentage profit = profit/buying price x 100%
= 40/160 x 100%
= 25%

2005 Paper II Question 1

A night train from London to Edinburgh leaves at 23:21 and arrives at 06:51.

(a) How long does the train journey take?

Beware! You cannot use your calculator for time calculations. You must also be careful how you do time calculations without a calculator – time is not decimal. The following method is advised:

We have to work out how long it is between 23:21 and 06:51:

So, 23:21 → 24:00, 24:00 → 06:00, 06:00 → 06:51
= 39 minutes plus 6 hours plus 51 minutes
(= 6 hours 90 minutes)
= 7 hours 30 minutes

(b) The distance from London to Edinburgh is 644 kilometres. Find the average speed of the train in kilometres per hour. Give your answer correct to one decimal place.

Speed = distance / time

= 644/7.5

(Note change from 7 hours 30 minutes to 7.5 hours, because 30 minutes is 0.5 of an hour)

= 85.9 km/hour

2005 paper II Question 9

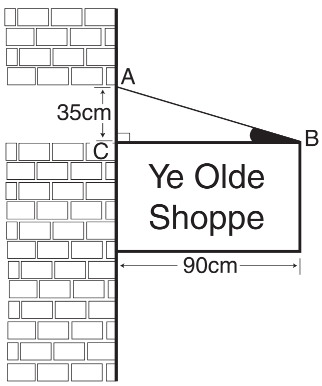
Serge drives from his home in Paris to Madrid, a journey of 1280 kilometres. His car has a 60-litre petrol tank and travels 13 kilometres per litre. Serge starts his journey with a full tank of petrol. What is the least number of times he has to stop to refuel? Give a reason for your answer.

60 litre tank
13 kilometres per litre
Distance car can travel
= 60 x 13 = 780 kilometres

The car starts with a full tank so it can travel 780 kilometres and then refuel. The car will then be able to travel another 780 kilometres. But the distance left to travel is 1280 – 780 = 500 kilometres, so Serge only needs to refuel once.

Note: Working should be shown in questions like this.

2005 Paper II Question 11



A rectangular shop sign is supported by a metal bar, AB. The length of the shop sign is 90 centimetres and the bar AB is attached to the wall 35 centimetres above the sign. Calculate the size of the shaded angle AB. Do not use a scale drawing.

When you see a right-angled triangle you should automatically think of two things: Pythagoras and trigonometry. You have to decide which has to be used. Here, because you are trying to connect angles and sides, it must be trigonometry.

Photograph: Rex

You have to decide what to use: sine, cosine or tangent. In this case, you have the opposite side and the adjacent side and you want to find the angle, so using SohCahToa gives you the tan ratio.

Tan ABC = 35/90
Angle ABC = 21.3˚

CREDIT LEVEL

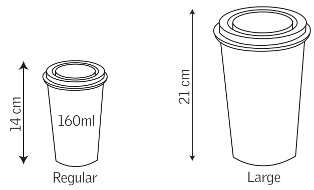
2006 PAPER I QUESTION 6

Solve the equation $x - 2(x + 1) = 8$

This should be a fairly straightforward question, but watch out for the negative sign.

$x - 2(x + 1) = 8$
$x - 2x - 2 = 8$
$-x - 2 = 8$
$-x = 10$
$x = -10$

2006 PAPER I QUESTION 7



Coffee is sold in regular cups and large cups. The two cups are mathematically similar in shape.

The regular cup is 14 centimetres high and holds 160 millilitres.

The large cup is 21 centimetres high. Calculate how many millilitres the large cup holds.

This is a similarity question, but it's related to similar volumes. In similarity questions we need to use a scale factor. Here we compare the two heights to get our factor.

The linear scale factor is $\frac{21}{14} = \frac{3}{2}$

However, since we are looking to find a volume we must use a volume scale factor of $(3/2)^3 = 27/8$.

So, large cup holds $\frac{27}{8} \times 160 = 540$ *millilitres*

2006 PAPER I QUESTION 11

(a) One session at the leisure centre costs £3. Write down an algebraic expression for the cost of x sessions.

One session costs £3 so x sessions will cost £3x. We write the algebraic expression as 3x.

(b) The leisure centre also offers a monthly card costing £20. The ***first 6*** *sessions are then free, with each additional session costing £2.*

(i) Find the ***total*** *cost of a monthly card and 15 sessions*

The total cost is £20 + (15 – 6) x £2
£20 + 9 x £2 = £38 which is the total cost.

(ii) Write down an alegbraic expression for the ***total*** *cost of a mothly card and x* ***sessions****, where x is greater than 6.*

An expression for the total cost is:
$20 + 2(x - 6) = 20 + 2x - 12$
$= 8 + 2x$

(c) Find the minimum number of sessions required for the monthly card to be the cheaper option. ***Show all working.***

This is the difficult part of this reasoning question and you are meant to try and solve it by applying an appropriate mathematical method.

You may be tempted to try a few numbers and see what works. This is called 'trial and improvement', and while this method may be appropriate for some problems, it's not what is being looked for at credit level. It's unlikely to gain you full marks in the exam, even if you do end up with the correct answer. You need to show extensive working and make sure you do enough trials to confirm your answer.

At credit level the method that should be applied is an algebra approach. In this case we are looking to solve an algebraic inequation.

For the monthly card to be a cheaper option, the cost you found in part *(b) (ii)* must be compared with cost you found in part *(a)*.

8 + 2x has to be less than 3x and we must find the lowest value of x that satisfies that condition:
$8 + 2x < 3x$
$8 < x$

So, the minimum number of sessions required is nine. This method is neater and better than a trial-and-improvement approach.

2006 PAPER II QUESTION 3

Harry bids successfully for a painting at an auction. An "auction tax" of 8% is added to his bid

price. He pays £324 in total. Calculate his bid price.

This type of question is very common and also badly done! Although it can appear to be a fairly straightforward percentage question, it's actually more a ratio question.

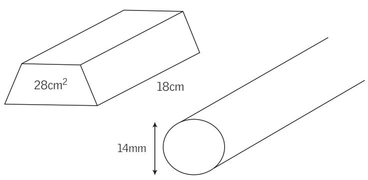
The £324 represents the bid price, plus 8% of the bid price.

So, £324 is actually 108% of the bid price. The bid price is 100%.

$$108\% = 324$$
$$100\% = \frac{324}{108} \times 100$$
$$= 300$$

So the bid price is £300.

2006 PAPER II QUESTION 7



(a) A block of copper 18 centimetres long is prism shaped as shown. The area of its cross section is 28 square centimetres. Find the volume of the block.

Volume of a prism = Area cross-section x length.

$$= 28 \times 18$$
$$= 504\ cm^3$$

(b) The block is melted down to make a cylindrical cable of diameter 14 ***millimetres****. Calculate the length of the cable.*

The main thing to watch here is the units. The diameter of the cable is in mm and the volume is in cm^3. We can change the 14mm diameter into 1.4 cm.

The Volume of the cable $= \pi r^2 \times length$

$$504 = \pi .\ 0.7^2 \times length$$

So: $$length = \frac{504}{3.14 \times 0.7^2} = 327.570\ldots$$

$$length = 327cm$$

2006 PAPER II QUESTION 9

The number of diagonals, d, in a polygon of n sides is given by the formula $d = \frac{1}{2}n(n-3)$

(a) How many diagonals does a polygon of 7 sides have?

$$d = \frac{1}{2}n(n-3)$$
$$= \frac{1}{2} \times 7 \times (7-3)$$
$$= \frac{1}{2} \times 7 \times 4$$
$$= 14$$

(b) A polygon has 65 diagonals. Show that for this polygon, $n^2 - 3n - 130 = 0$.

$$65 = \frac{1}{2}n(n-3)$$
$$130 = n(n-3)$$
$$130 = n^2 - 3n$$
$$n^2 - 3n - 130 = 0$$

(c) Hence find the number of sides in this polygon.

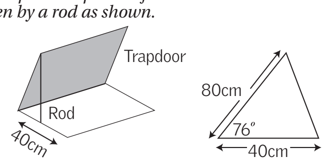
$$n^2 - 3n - 130 = 0$$
$$(n+10)(n-13) = 0$$
$$n = -10 \text{ or } n = 13$$

n cannot be negative, so n = 13

The number of sides in the polygon is 13.

2004 PAPER II QUESTION 7

A square trapdoor of side 80 cemtimetres is held open by a rod as shown.



The rod is attached to a corner of the trapdoor and placed 40 centimetres along the edge of the opening. The angle between the trapdoor and the opening is 76°. Calculate the length of the rod to 2 significant figures.

There is always a question on non right angled trigonometry in the Credit exam. This work covers the Sine rule, the Cosine rule and the area of a triangle. All the necessary formulae are given inside the front cover of the question paper.

Since you are being asked to find the length of a side in this triangle, you use either the Sine rule or Cosine rule. You normally try the Sine rule first, but in this case you would find that you do not have the necessary information. In fact, because the information you are given consists of two sides and the angle between them, you could move straight to the Cosine rule as the Sine rule will not work with that information (nor will it work if you are only given three sides or three angles).

The Cosine rule is given in the formula sheet as:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

In this question we will take the side we are wanting to calculate as 'a'.

So, $$a^2 = 80^2 + 40^2 - 2 \times 80 \times 40 \times \cos 76^o$$

Be careful to work out the whole of $2 \times 80 \times 40 \times \cos 76^o$ before subtracting it.

$$a^2 = 8000 - 1548.3$$
$$a^2 = 6451.7$$

Remember to take the square root!

$$a = 80.3$$

And … remember to round as the question asked for the answer to be given correct to two significant figures!

$$a = 80cm$$

2004 PAPER 1 QUESTION 4

Simplify $\frac{3}{m} + \frac{4}{(m+1)}$

This type of question is not normally well done, so practice is necessary. We need to have the same denominator before we add or subtract fractions. Here, the common denominator is found by multiplying the two denominators m and $(m+1)$. Remember to multiply the top of the fraction as well!

$$\frac{3}{m} + \frac{4}{(m+1)} = \frac{3(m+1)}{m(m+1)} + \frac{4m}{m(m+1)}$$
$$= \frac{3(m+1) + 4m}{m(m+1)}$$
$$= \frac{3m+3+4m}{m(m+1)}$$
$$= \frac{7m+3}{m(m+1)}$$

HIGHER GRADE

Remember that at Higher Grade, 65% of the marks are allocated at Grade C level, so there will be questions that you can do, and do well.

The questions in Paper I are likely to be shorter than those in Paper II – and more to the point. It should be your aim to build up a good mark in Paper I to make Paper II less daunting. The early questions in Paper I should be reasonably straightforward and explicit. As you work through the paper, the questions will get tend to harder.

In Paper II you will find the questions longer and more problem-based. This is particularly true of the later ones. If you have scored well in Paper I, then a couple of good questions early on in Paper II and you could be over the magic 50%.

WORKED EXAMPLES

The following 11 examples illustrate the different types of questions you can expect to meet in the exam. The first seven are graded C so you should all be able to attempt them with an expectation of success. The last two questions are more difficult (graded A) – it's important that you should try to get as much as possible out of every question and realise that you can gain partial marks. In other words, don't disregard questions because they look daunting. Parts of the question may be quite straightforward.

2005 PAPER I QUESTION 5

Differentiate $(1 + 2\sin x)^4$ with respect to x.

Let $f(x) = (1 + 2\sin x)^4$

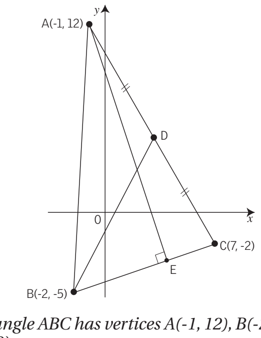
Using the chain rule to differentiate we get:

$$f'(x) = 4(1 + 2\sin x)^3 \times 2\cos x$$
$$= 8\cos x\,(1 + 2\sin x)^3$$

Remember that you must first differentiate the bracket then differentiate what is inside the bracket.

2006 PAPER I QUESTION 1

This question tests your knowledge of straight line equations. When you are asked to find the equation of a straight line you will usually use the standard equation $y - b = m(x - a)$ where (a,b) is a point on a line with gradient m. It's invariably finding the gradient that poses most problems. There are several ways of finding a gradient. We use two of them in this question.



Triangle ABC has vertices A(-1, 12), B(-2, -5) and C(7, -2).

(a) Find the equation of the median BD.

Since BD is a median, D is the mid-point of AC and we need to find it's co-ordinates:

$$D \text{ is } \left(\frac{-1+7}{2}, \frac{12-2}{2}\right) = (3,5)$$

We now find the gradient by means of the gradient formula:

$$m_{BD} = \frac{5-(-5)}{3-(-2)} = \frac{10}{5} = 2$$

We can now find the equation of BD by using $y - b = m(x-a)$

$$y + 5 = 2(x + 2)$$
$$y = 2x - 1$$

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(b) Find the equation of the altitude AE.

In this part we use the fact that AE is an altitude and is therefore at right angles to BC. So first we find the gradient of BC:

$m_{BC} = \frac{-2-(-5)}{7-(-2)} = \frac{3}{9} = \frac{1}{3}$

So, using the fact that the product of gradients of perpendicular lines is -1, we know that:

$m_{AE} = -3$

So, equation of AE is found by again using $y - b = m(x - a)$:

$y - 12 = -3(x + 1)$

$y = -3x + 9$

(c) Find the coordinates of the point of intersection of BD and AE.

Now we use simultaneous equations to find the point of intersection as this point lies on both the median and the altitude and so satisfies both equations:

$y = 2x - 1$

$y = -3x + 9$

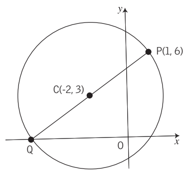
$2x - 1 = -3x + 9$

$5x = 10$

$x = 2$ *giving* $y = 3$ *and so the point of intersection is (2,3).*

2006 PAPER I QUESTION 2

In this question we must make use of the formula sheet printed on the inside cover of the paper, and we must decide which formula to use. For circle work there are two to choose from. For this question we choose the second one:



A circle has centre C(-2, 3) and passes through P(1, 6).

(a) Find the equation of the circle.

We use $(x - a)^2 + (y - b)^2 = r^2$

(a,b) is the centre and is (-2,3) so a = -2 and b = 3.

r is the radius and we need to use the distance formula to find it.

$r^2 = (1 - (-2))^2 + (6 - 3)^2$

$= 9 + 9$

$= 18$

So the equation is $(x + 2)^2 + (y - 3)^2 = 18$.

(b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.

A tangent is a straight line and so to find its equation we need a point and the gradient. We need to find both here.

Q is the only point we can find and it is diametrically opposite P.

So, from P to C the x co-ordinate goes down three and the y co-ordinate goes down three. From (-2,3) we go to (-5,0) and these are the co-ordinates of Q.

The gradient of the tangent is found from perpendicular lines as in question one. The tangent is perpendicular to the radius at the point of contact, so first we find the gradient of CQ.

$m_{CQ} = \frac{3-0}{-2-(-5)} = \frac{3}{3} = 1$

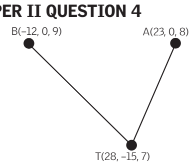
The gradient of the tangent must be -1.

The equation of the tangent is found again by using $y - b = m(x - a)$.

$y - 0 = -1(x + 5)$

$y = -x - 5$

2005 PAPER II QUESTION 4



The sketch shows the positions of Andrew (A), Bob (B) and Tracey (T) on three hill-tops,

MATHEMATICS EXAM TIMETABLE

Level/Paper	Time
Thursday May 3	
Foundation Paper 1 (Non-calculator)	9am-9.20am
Foundation Paper 2	9.40am-10.20am
General Paper 1 (Non-calculator)	10.40am-11.15am
General Paper 2	11.35am-12.30pm
Credit Paper 1 (Non-calculator)	1.30pm-2.25pm
Credit Paper 2	2.45pm-4.05pm

Level/Paper	Time
Tuesday May 15	
Higher Paper 1 (Non-calculator)	9am-10.10am
Higher Paper 2	10.30am-noon
Intermediate 1 Paper 1 (Non-calculator)	1pm-1.35pm
Intermediate 1 Paper 2	1.55pm-2.50pm
Intermediate 2 Paper 1 (Non-calculator)	1pm-1.45pm
Intermediate 2 Paper 2	2.05pm-3.35pm
Advanced Higher	1pm-4pm

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

In the dark, Andrew and Bob locate Tracey using heat-seeking beams.

(a) Express the vectors $\vec{TA}$ *and* $\vec{TB}$ *in component form.*

$$\vec{TA} = \begin{pmatrix} 23-28 \\ 0+15 \\ 8-7 \end{pmatrix} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$$

$$\vec{TB} = \begin{pmatrix} -12-28 \\ 0+15 \\ 9-7 \end{pmatrix} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$$

To find the angle between two vectors you consider the vectors represented by the lines and use the scalar product. The scalar product can be evaluated in two ways and by equating the two approaches it's possible to find the angle between the two vectors and hence the angle between the two lines.

(b) Calculate the angle between the two beams.

The lengths of the two vectors are found:

$|\vec{TA}| = \sqrt{(-5)^2 + 15^2 + 1^2} = \sqrt{251}$

$|\vec{TB}| = \sqrt{(-40)^2 + 15^2 + 2^2} = \sqrt{1829}$

Then we evaluate the scalar product by multiplying the corresponding components and adding:

$\vec{TA}.\vec{TB} = (-5) \times (-40) + 15 \times 15 + 1 \times 2$

$= 200 + 225 + 2$

$= 427$

Now we apply the other method of evaluating the product. This method involves multiplying the lengths of the two vectors and the cosine of the angle between them.

We apply the product and equate it to 427:

$TA \times TB \times \cos T° = 427$

So $\cos T° = \frac{427}{\sqrt{251 \times 1829}} = 0.6302 \ldots$

... and T = 50.9

2006 PAPER I QUESTION 7

Solve the equation $\sin x° - \sin 2x° = 0$ *in the interval* $0 \le x \le 360$.

Again we must make use of the formulae given to us. Remember you will never be guided towards the formulae sheet. You must be aware that it's there to help you and you should be familiar with the contents so you use it - and not make up your own formula!

From the sheet we see that $\sin 2A = 2\sin A\cos A$ and it's this double angle formula we use to help us solve the trig equation:

$\sin x° - \sin 2x° = 0$

$\sin x° - 2\sin x°\cos x° = 0$

We must now take out the common factor of $\sin x°$.

$\sin x° (1 - 2\cos x°) = 0.$

It's essential to ensure in these double angle trig equations that the right hand side is 0.

$\sin x° = 0$ or $1 - 2\cos x° = 0$

$\sin x° = 0$ or $\cos x° = \frac{1}{2}$

$x = 0, 180, 360$ or $x = 60, 300$

so $x = 0, 60, 180, 300, 360$.

2004 PAPER I QUESTION 2

$f(x) = x^3 - x^2 - 5x - 3$.

(a) (i) Show that (x + 1) is a factor of f(x).

To show that $(x + 1)$ is a factor of $f(x)$ we need to show that $f(-1) = 0$.

This can be done fairly easily by substitution, or by using synthetic division:

-1	1	-1	-5	-3
		-1	2	3
	1	-2	-3	0

The 0 at the end shows that f(-1) = 0 and so (x + 1) is a factor.

(ii) Hence or otherwise factorise f(x) fully.

To factorise fully we use the bottom line entries in the table.

The 1,-2 and -3 from the bottom line of the table are the coefficients of the quadratic factor left when (x +1) is removed.

$f(x) = (x + 1)(x^2 - 2x - 3)$

$= (x + 1)(x + 1)(x - 3)$

(b) One of the turning points of the graph of y = f(x) lies on the x-axis. Write down the coordinates of this turning point.

Two of the factors of f(x) are the same so this indicates that the turning point is found by equating the double factor to zero and so the answer is: (-1,0)

2006 PAPER II QUESTION 2

Find the value of k such that the equation kx2 +kx + 6 = 0, k ≠ 0, has equal roots.

We have to use the discriminant of a quadratic expression for this question.

The standard quadratic form is $ax^2 + bx + c$ and the discriminant is defined as $b^2 - 4ac$ and if the quadratic equation is to have equal roots then the discriminant must equal zero.

We compare the given equation $kx^2 + kx + 6 = 0$ with the standard form and see that $a = k, b = k$ and c = 6.

So $b^2 - 4ac = k^2 - 4.k.6$ and this must equal zero.

ie $k^2 - 24k = 0$

$k(k - 24) = 0$

$k = 0$ or $k = 24$

$k \ne 0$ so $k = 24$

2006 PAPER I QUESTION 8

(a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$.

This question is about completing the square, but can be tackled in two ways. We can look at the left hand side, $2x^2 + 4x - 3$ and complete the square as follows:

$$\begin{aligned} 2x^2 + 4x - 3 &= 2(x^2 + 2x) - 3 \\ &= 2[x^2 + 2x + 1 - 1] - 3 \\ &= 2[(x + 1)^2 - 1] - 3 \\ &= 2(x + 1)^2 - 2 - 3 \\ &= 2(x + 1)^2 - 5 \end{aligned}$$

(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.

The point is *(-1,-5)* which can be read directly from the completed square format.

2006 PAPER II QUESTION 5

The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x2$

The curve passes through the point (-1, 9). Express x in terms of y.

Although we see $\frac{dy}{dx}$ this is not a differentiating question.

We are given $\frac{dy}{dx}$ and have to find *y*, so it's an integrating question:

$$\frac{dy}{dx} = 4x - 6x^2$$

so, $$y = \int (4x - 6x^2)\, dx$$

$$y = \frac{4x^2}{2} - \frac{6x^3}{3} + c$$

(Remember to add c, the constant of integration)

$$y = 2x^2 - 2x^3 + c$$

To find the value of *c* we must look back at the question to see what other information we are given.

We are told the curve passes through *(-1,9)* and so $x = -1$ and $y = 9$ must satisfy our equation. Substituting these values will allow us to calculate *c:*

$$\begin{aligned} 9 &= 2(-1)^2 - 2(-1)^3 + c \\ 9 &= 2 + 2 + c \\ c &= 5 \end{aligned}$$

So equation is:

$$y = 2x^2 - 2x^3 + 5$$

2005 PAPER I QUESTION 4

Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on real set numbers.

(a) Find $h(x)$ where $h(x) = g(f(x))$.

$$f(x) = 3x - 1 \qquad g(x) = x^2 + 7$$

$$\begin{aligned} h(x) &= g(f(x)) \\ &= g(3x - 1) \\ &= (3x - 1)^2 + 7 \\ &= 9x^2 - 6x + 1 + 7 \\ &= 9x^2 - 6x + 8 \end{aligned}$$

(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.

We are asked to "write down" the co-ordinates. This means there should be no extra working to do. We should be able to look at our answer to (a) and see the answer to (b). The simplified answer to (a) is not helpful, however, as it does not allow us to "write down" what we need. If we look two lines before we should recognise the format $(x + a)^2 + b$. This is the completed square format. Completing the square is a fairly routine task and we should not have a problem if asked to do it, but recognising the completed square format is not so easy.

If we do recognise it then we can indeed just "write down" the answer. One of the uses of the completing the square is that it allows us to write down the turning point of a quadratic function. If we complete the square to get the form $(x + a)^2 + b$, then the turning point is $(-a, b)$.

So, from $(3x - 1)^2 + 7$ we can write down that the turning point is:

$$\left(\frac{1}{3}, 7\right)$$

(ii) Hence state the range of the function h.

Since this is a minimum turning point, the lowest value y can take is seven. So the range of the function (the possible values of y) must be:

$$y \geq 7$$

2005 PAPER II QUESTION 7

Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.

$$\log_4(5 - x) - \log_4(3 - x) = 2$$

$$\log_4 \frac{5 - x}{3 - x} = 2$$

This is using the fact that $\log A - \log B = \log \frac{A}{B}$

for same base logs. By subtracting the logs, you are dividing the numbers. Now we use the fact that $\log_a y = x$ is the same as $y = a^x$

$$\frac{5 - x}{3 - x} = 4^2$$

$$\frac{5 - x}{3 - x} = 16$$

$$5 - x = 16(3 - x)$$

$$5 - x = 48 - 16x$$

$$15x = 43$$

$$x = \frac{43}{15}$$