



MATHEMATICS
Advanced Higher

Sixth edition – published January 2010



**NOTE OF CHANGES TO ARRANGEMENTS SIXTH EDITION
PUBLISHED JANUARY 2010**

COURSE TITLE: Mathematics (Advanced Higher)

**COURSE NUMBERS AND TITLES
FOR ENTRY TO COURSES :** C100 13 Mathematics: Maths 1, 2 and 3

National Course Specification

Course Details Some of the contents and notes sections have been amended to provide further exemplification of the depth of treatment appropriate to grades A and B.

National Unit Specification: N/A



National Course Specification

MATHEMATICS (ADVANCED HIGHER)

COURSE NUMBER C100 13 Mathematics: Maths 1, 2 and 3

COURSE STRUCTURE

C100 13 Mathematics: Maths 1, 2 and 3

This course consists of three mandatory units as follows:

<i>D321 13</i>	<i>Mathematics 1 (AH)</i>	<i>1 credit (40 hours)</i>
<i>D322 13</i>	<i>Mathematics 2 (AH)</i>	<i>1 credit (40 hours)</i>
<i>D323 13</i>	<i>Mathematics 3 (AH)</i>	<i>1 credit (40 hours)</i>

Administrative Information

Publication date: January 2010

Source: Scottish Qualifications Authority

Version: 06

© Scottish Qualifications Authority 2010

This publication may be reproduced in whole or in part for educational purposes provided that no profit is derived from reproduction and that, if reproduced in part, the source is acknowledged.

Additional copies of this course specification (including unit specifications) can be purchased from the Scottish Qualifications Authority for £7.50. **Note:** Unit specifications can be purchased individually for £2.50 (minimum order £5).

National Course Specification: general information (cont)

COURSE Mathematics (Advanced Higher)

In common with all courses, this course includes 40 hours for induction, extending the range of learning and teaching approaches, additional support, consolidation, integration of learning and preparation for external assessment. This time is an important element of the course and advice on the use of the overall 160 hours is included in the course details.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained a Higher Mathematics course award or its component units or equivalent. *Mathematics 1 (AH)* assumes knowledge of outcomes 2 and 3 of *Mathematics 3 (H)*.

CORE SKILLS

The course gives automatic certification of the following:

Complete core skills for the course Numeracy H

Additional core skills components for the course Critical Thinking H

For information about the automatic certification of core skills for any individual unit in this course, please refer to the general information section at the beginning of the unit.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications 2001/2002 BA0906 August 2001*.

National Course Specification: course details

COURSE Mathematics (Advanced Higher)

RATIONALE

As with all mathematics courses, Advanced Higher Mathematics aims to build upon and extend candidates' mathematical skills, knowledge and understanding in a way that recognises problem solving as an essential skill and enables them to integrate their knowledge of different aspects of the subject. The aim of developing mathematical skills and applying mathematical techniques in context will be furthered by exploiting the power of calculators and computer software where appropriate.

Because of the importance of these features, the grade descriptions for Advanced Higher Mathematics emphasise the need for candidates to undertake extended thinking and decision making to solve problems and integrate mathematical knowledge. The use of coursework tasks, therefore, to practise problem solving as set out in the grade descriptions, is strongly encouraged.

The course offers candidates, in an interesting and enjoyable manner, an enhanced awareness of the range and power of mathematics.

COURSE CONTENT

The syllabus is designed to build upon and extend candidates' mathematical learning in the areas of algebra, geometry, and calculus. The units, *Mathematics 1 (AH)* and *Mathematics 2 (AH)*, and *Mathematics 3 (AH)* are progressive and continue the development of algebra, geometry and calculus from Higher level.

COURSE Mathematics (Advanced Higher)

The outcomes and the performance criteria for each unit are statements of basic competence. Additionally, the course makes demands over and above the requirements of individual units. Within the 40 hours of flexibility time, candidates should be able to integrate their knowledge across the component units *Mathematics 1 (AH)*, *Mathematics 2 (AH)* and *Mathematics 3 (AH)*. The experience of extended thinking and decision making is important and will be enhanced when candidates are exposed to coursework tasks which require them to interpret problems, select appropriate strategies, come to conclusions and communicate these intelligibly.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

Detailed content

The content listed below should be covered in teaching the course. All of this content will be subject to sampling in the external assessment. Where comment is offered, this is intended to help in the effective teaching of the course.

References in this style indicate the depth of treatment appropriate to grades A and B.

CONTENT	COMMENT	TEACHING NOTES
<p>Mathematics 1 (AH)</p> <p>Algebra</p> <p>know and use the notation $n!$, ${}^n C_r$ and $\binom{n}{r}$</p> <p>know the results $\binom{n}{r} = \binom{n}{n-r}$</p> <p>and $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$</p> <p>know Pascal's triangle</p> <p>know and use the binomial theorem</p> $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r \text{ for } r, n \in \mathbf{N}$	<p>eg Calculate</p> <p>eg Solve, for $n \in \mathbf{N}$,</p> <p>Pascal's triangle should be extended up to $n = 7$.</p> <p>eg Expand $(x+3)^4$</p> <p>eg Expand $(2u-3v)^5$ [A/B]</p>	<p>Candidates should be aware of the size of $n!$ for small values of n, and that calculator results consequently are often inaccurate (especially if the formula ${}^n C_r = \frac{n!}{r!(n-r)!}$ is used). These results can be established numerically or from Pascal's triangle. This will be linked with elementary number theory in Mathematics 2 (AH), where they provide an opportunity to introduce the concept of direct proof.</p> <p>For assessment purposes $n \leq 5$.</p> <p>In the work on series and complex numbers (de Moivre's theorem) the binomial theorem will be extended to integer and rational indices.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>evaluate specific terms in a binomial expansion by using a formula such as</p> $\binom{n}{r} = a^{n-r}b^r$ <p>express a proper rational function as a sum of partial fractions (denominator of degree at most 3 and easily factorised)</p> <p>include cases where an improper rational function is reduced to a polynomial and a proper rational function by division or otherwise [A/B]</p>	<p>e.g Find the term in x^5 in $(x+3)^7$</p> <p>eg Find the term in x^7 in $\left(x + \frac{2}{x}\right)^9$ [A/B].</p> <p>eg Express $\frac{5-10x}{1-3x-4x^2}$ in partial fractions.</p> <p>eg Express $\frac{x^3 + 2x^2 - 2x + 2}{(x-1)(x+3)}$ in partial fractions [A/B].</p>	<p>The denominator may include a repeated linear factor or an irreducible quadratic factor. This is also required for integration of rational functions and useful in the context of differentiation and for graph sketching when asymptotes are present.</p> <p>When the degree of the numerator of the rational function exceeds that of the denominator by 1, non-vertical asymptotes occur.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>Differentiation know the meaning of the terms limit, derivative, differentiable at a point, differentiable on an interval, derived function, second derivative</p> <p>use the notation: $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$</p> <p>recall the derivatives of x^α (α rational), $\sin x$ and $\cos x$</p> <p>know and use the rules for differentiating linear sums, products, quotients and composition of functions:</p> <p>$(f(x) + g(x))' = f'(x) + g'(x)$ $(kf(x))' = kf'(x)$ where k is a constant</p> <p>the chain rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$ the product rule: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$</p> <p>the quotient rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$</p> <p>differentiate given functions which require more than one application of one or more of the chain rule, product rule and the quotient rule [A/B]</p>		<p>Candidates should be exposed to formal proofs of differentiation, although proofs will not be required for assessment purposes. Once the rules for differentiation have been learned, computer algebra systems (CAS) may be used for consolidation/extension. However, when CAS are being used for difficult/real examples the emphasis should be on the understanding of concepts rather than routine computation. When software is used for differentiation in difficult cases, candidates should be able to say which rules were used.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>know</p> <ul style="list-style-type: none"> the derivative of $\tan x$ the definitions and derivatives of $\sec x$, $\operatorname{cosec} x$ and $\cot x$ the derivatives of e^x ($\exp x$) and $\ln x$ <p>know the definition $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$</p> <p>know the definition of higher derivatives</p> $f^n(x), \frac{d^n y}{dx^n}$ <p>apply differentiation to:</p> <ul style="list-style-type: none"> a) rectilinear motion b) extrema of functions: the maximum and minimum values of a continuous function f defined on a closed interval $[a,b]$ can occur at stationary points, end points or points where f' is not defined [A/B] c) optimisation problems 	<p>eg Find the acceleration of a particle whose displacement s metres from a certain point at time t seconds is given by $s = 8 - 75t + t^3$.</p> <p>eg Find the maximum value of the function</p> $f(x) = \begin{cases} x^2, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases} \quad \mathbf{[A/B]}$	<p>Link with the graphs of these functions. The definitions of e^x and $\ln x$ should be revised and examples given of their occurrence.</p> <p>Candidates should be aware that not all functions are differentiable everywhere, eg $f(x) = x$ at $x = 0$. The use of software allows further exploration here.</p> <p>Candidates should also know that higher derivatives can have discontinuities and be aware of the graphical effects of this, ie lack of smoothness. An example of this is:</p> $f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ <p>for which f and f' are continuous but f'' is not.</p> <p>Optimisation problems should be linked with graph sketching.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>Integration know the meaning of the terms integrate, integrable, integral, indefinite integral, definite integral and constant of integration</p> <p>recall standard integrals of x^α ($\alpha \in \mathbf{Q}$, $\alpha \neq -1$), $\sin x$ and $\cos x$</p> $\int (af(x) + bg(x))dx = a \int f(x)dx + b \int g(x)dx \quad b \in \mathbf{R}$ $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \quad a < c < b$ $\int_b^a f(x)dx = -\int_a^b f(x)dx, \quad b \neq a$ $\int_a^b f(x)dx = F(b) - F(a), \text{ where } F'(x) = f(x)$ <p>know the integrals of e^x, x^{-1}, $\sec^2 x$</p>		<p>CAS may be used for consolidation/extension. However, when CAS are being used for difficult/real examples the emphasis should be on understanding of the concepts rather than routine computation. When software is being used for integration in difficult cases, candidates should be able to say which rules were used.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>integrate by substitution: expressions requiring a simple substitution</p> <p>expressions where the substitution will be given</p> <p>the following special cases of substitution</p> $\int f(ax + b)dx$ $\int \frac{f'(x)}{f(x)} dx$ <p>use an elementary treatment of the integral as a limit using rectangles</p> <p>apply integration to the evaluation of areas including integration with respect to y [A/B].</p>	<p>Candidates are expected to integrate simple functions on sight. eg $\int xe^{x^2} dx$</p> <p>eg $\int \cos^3 x \sin x dx, u = \cos x$</p> <p>eg $\int \frac{6\sin x}{\sqrt{1-4\cos^2 x}} dx, u = \cos x$ [A/B]</p> <p>eg $\int \sin(3x + 2)dx$</p> <p>eg $\int \frac{2x}{x^2 + 3} dx$</p> <p>Other applications may include</p> <ul style="list-style-type: none"> (i) volumes of simple solids of revolution (disc/washer method) (ii) speed/time graph [A/B]. 	<p>Where substitutions are given they will be of the form $x = g(u)$ or $u = g(x)$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>Properties of functions know the meaning of the terms function, domain, range, inverse function, critical point, stationary point, point of inflexion, concavity, local maxima and minima, global maxima and minima, continuous, discontinuous, asymptote</p> <p>determine the domain and the range of a function</p> <p>use the derivative tests for locating and identifying stationary points</p> <p>sketch the graphs of $\sin x$, $\cos x$, $\tan x$, e^x, $\ln x$ and their inverse functions, simple polynomial functions</p>	<p>ie Concave up $\Leftrightarrow f''(x) > 0$; concave down $\Leftrightarrow f''(x) < 0$; a necessary and sufficient condition for a point of inflexion is a change in concavity.</p>	<p>Candidates are expected to recognise graphs of simple functions, be able to sketch the graphs by hand and know their key features, eg behaviour of trigonometric and exponential functions. The assessment should be structured to ensure that candidates carry out and display the calculations required to identify the important features on the graph. Candidate learning can be enhanced through the use of calculators with a graphic facility and CAS.</p> <p>Care should be exercised when using the second derivative test in preference to the first derivative test. The second derivative may not exist, and even when it does and can easily be computed, it may not be helpful, eg the function $f(x) = x^4$ at $x = 0$. Here, $f''(0) = 0$ which is inconclusive. The first derivative test, however, easily shows there is a local minimum at $x = 0$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	COMMENT	TEACHING NOTES
<p>know and use the relationship between the graph of $y = f(x)$ and the graphs of</p> $y = kf(x),$ $y = f(x) + k,$ $y = f(x + k),$ $y = f(kx),$ <p>where k is a constant</p> <p>know and use the relationship between the graph of $y = f(x)$ and the graphs of</p> $y = f(x) $ $y = f^{-1}(x)$ <p>given the graph of a function f, sketch the graph of a related function</p> <p>determine whether a function is even or odd or neither and symmetrical and use these properties in graph sketching</p> <p>sketch graphs of real rational functions using available information, derived from calculus and/or algebraic arguments, on zeros, asymptotes (vertical and non-vertical), critical points, symmetry</p>	<p>ie Reflection in the line $y = x$ eg $f(x) = e^x, -\infty < x < \infty$</p> $f^{-1}(x) = \ln x, x > 0$ <p>For rational functions, the degree of the numerator will be less than or equal to three and the denominator will be less than or equal to two.</p>	<p>Care must be taken over the domain and range when finding inverses.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Systems of linear equations use the introduction of matrix ideas to organise a system of linear equations</p> <p>know the meaning of the terms matrix, element, row, column, order of a matrix, augmented matrix</p> <p>use elementary row operations (EROs)</p> <p>reduce to upper triangular form using EROs</p> <p>solve a 3×3 system of linear equations using Gaussian elimination on an augmented matrix</p> <p>find the solution of a system of linear equations $Ax = b$, where A is a square matrix, include cases of unique solution, no solution (inconsistency) and an infinite family of solutions [A/B].</p> <p>know the meaning of the term ill-conditioned [A/B].</p> <p>compare the solutions of related systems of two equations in two unknowns and recognise ill-conditioning [A/B]</p>	<p>Ill-conditioning can be introduced by comparing the solutions of the following systems:</p> <p>a. $x + 0.99y = 1.99$ $0.99x + 0.98y = 1.97$</p> <p>b. $x + 0.99y = 2.00$ $0.99x + 0.98y = 1.97$ [A/B]</p>	<p>Only 3×3 cases are required for assessment purposes, and at grade C they are restricted to those with a unique solution. Larger systems can be tackled using computer packages or advanced calculators.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Mathematics 2 (AH) Further differentiation know the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$</p> <p>differentiate any inverse function using the technique: $y = f^{-1}(x) \Rightarrow f(y) = x \Rightarrow (f^{-1}(x))'f'(y) = 1$, etc.,</p> <p>and know the corresponding result $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} \quad [\mathbf{A/B}].$</p> <p>understand how an equation $f(x, y) = 0$ defines y implicitly as one (or more) function(s) of x</p> <p>use implicit differentiation to find first and second derivatives $[\mathbf{A/B}]$ use logarithmic differentiation, recognising when it is appropriate in extended products and quotients and indices involving the variable $[\mathbf{A/B}]$.</p> <p>understand how a function can be defined parametrically</p>	<p>eg , $x^2 + y^2 = 1 \Leftrightarrow y = \pm\sqrt{1-x^2}$</p> <p>ie two functions defined on $[-1,1]$</p> <p>eg Find the derivatives of $y = \frac{x^2\sqrt{7x-3}}{1+x}, \quad y = 2^x \quad [\mathbf{A/B}].$</p>	<p>Link with the graphs of these functions in Mathematics 1 (AH).</p> <p>This technique should be initially used on the inverse trigonometric functions. Calculators with graphic facility and computer packages may be used to investigate these derivatives.</p> <p>Link the last result with the chain rule applied to $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ and with the logarithmic and exponential functions.</p> <p>Link with obtaining the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>understand simple applications of parametrically defined functions</p> <p>use parametric differentiation to find first and second derivatives [A/B], and apply to motion in a plane</p> <p>apply differentiation to related rates in problems where the functional relationship is given explicitly or implicitly</p> <p>solve practical related rates by first establishing a functional relationship between appropriate variables [A/B]</p>	<p>eg $x^2 + y^2 = r^2$, $x = r \cos \theta$, $y = r \sin \theta$</p> <p>eg If the position is given by $x = f(t)$, $y = g(t)$ then the velocity components are given by</p> $\frac{dx}{dt} \text{ and } \frac{dy}{dt}$ <p>and the speed by $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$</p> <p>The instantaneous direction of motion is given by</p> $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ <p>eg Explicitly: $V = \frac{1}{3}\pi r^2 h$; given $\frac{dh}{dt}$, find $\frac{dV}{dt}$.</p> <p>Implicitly: $x^2 + y^2 = r^2$ where x, y are functions of t; given $\frac{dx}{dt}$, find $\frac{dy}{dt}$ using $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.</p>	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Further integration</p> <p>know the integrals of $\frac{1}{\sqrt{1-x^2}}$, $\frac{1}{1+x^2}$</p> <p>use the substitution $x = at$ to integrate functions of the form $\frac{1}{\sqrt{a^2-x^2}}$, $\frac{1}{a^2+x^2}$</p> <p>integrate rational functions, both proper and improper, [A/B] by means of partial fractions; the degree of the denominator being ≤ 3 the denominator may include:</p> <ul style="list-style-type: none"> (i) two separate or repeated linear factors (ii) three linear factors with constant numerator and with non-constant numerator [A/B] (iii) a linear factor and an irreducible quadratic factor of the form x^2+a [A/B] <p>integrate by parts with one application</p> <p>integrate by parts involving repeated applications [A/B]</p>	<p>eg $\int x \sin x dx$</p> <p>eg $\int x^2 e^{3x} dx$ [A/B].</p>	<p>Link this with the derivatives of inverse trigonometric functions.</p> <p>Candidates' learning can be enhanced by completing the square but this will not be formally assessed.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>know the definition of differential equation and the meaning of the terms linear, order, general solution, arbitrary constants, particular solution, initial condition</p> <p>solve first order differential equations (variables separable)</p> <p>formulate a simple statement involving rate of change as a simple separable first order differential equation, including the finding of a curve in the plane, given the equation of the tangent at (x, y), which passes through a given point</p> <p>know the laws of growth and decay: applications in practical contexts</p>	<p>ie, equations that can be written in the form</p> $\frac{dy}{dx} = \frac{g(x)}{h(y)}$	<p>Link with differentiation. Begin by verifying that a particular function satisfies a given differential equation. Candidates should know that differential equations arise in modelling of physical situations, such as electrical circuits and vibrating systems, and that the differential equation describes how the system will change with time so that initial conditions are required to determine the complete solution.</p> <p>Link with motion in a straight line. Further similar contexts can be found in the Mechanics 1 (AH) unit.</p> <p>Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good motivating examples and can build on the knowledge and use of logarithms.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Complex numbers know the definition of i as a solution of $z^2 + 1 = 0$, so that $i = \sqrt{-1}$</p> <p>know the definition of the set of complex numbers as $C = \{a + ib : a, b \in R\}$</p> <p>know the definition of real and imaginary parts</p> <p>know the terms complex plane, Argand diagram</p> <p>plot complex numbers as points in the complex plane</p> <p>perform algebraic operations on complex numbers: equality (equating real and imaginary parts), addition, subtraction, multiplication and division</p> <p>evaluate the modulus, argument and conjugate of complex numbers</p>		<p>A suggested approach is through the solution of quadratics. The introduction of i then allows the provision of two solutions for all quadratics. Thereafter, an essentially geometric approach is recommended.</p> <p>Plotting complex numbers as points can be made easier because some software (and calculators) write complex numbers as ordered pairs.</p> <p>Multiplication by i is equivalent to rotation by 90° and demonstrates a link between operations and transformations in the plane. This could be linked with Matrices in Mathematics 3 (AH).</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>convert between Cartesian and polar form</p> <p>know the fundamental theorem of algebra and the conjugate roots property</p> <p>factorise polynomials with real coefficients</p> <p>solve simple equations involving a complex variable by equating real and imaginary parts</p> <p>interpret geometrically certain equations or inequalities in the complex plane</p>	<p>eg Find the roots of a quartic when one complex root is given.</p> <p>eg Solve $z + i = 2\bar{z} + 1$</p> <p>eg Solve $z^2 = 2\bar{z}$ [A/B]</p> <p>eg $z = 1$</p> <p>$z - a = b$</p> <p>$z - 1 = z - i$</p> <p>$z - a > b$ [A/B]</p>	<p>$x = r \cos \theta, y = r \sin \theta, r = \sqrt{x^2 + y^2}$ and $\tan \theta = \frac{y}{x}$.</p> <p>The commonly used practice of writing $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ should be used with reference to quadrants.</p> <p>The exponential form $z = re^{i\theta}$ provides a link with power series in Further sequences and series in Mathematics 3 (AH).</p> <p>The facility in CAS to produce complex factors of polynomials allows candidates to conjecture that a polynomial of degree n has n roots, and that these occur in conjugate pairs when the coefficients are real.</p> <p>The triangle inequality $z + w \leq z + w$ can be mentioned here.</p> <p>The proof of de Moivre's theorem for integer powers provides an opportunity to illustrate proof by induction. [Link with Further number theory in Mathematics 3(AH)]. For fractional powers the result can be stated without proof.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>know and use de Moivre's theorem with positive integer indices and fractional indices [A/B]</p> <p>apply de Moivre's theorem to multiple angle trigonometric formulae [A/B]</p> <p>apply de Moivre's theorem to find nth roots of unity [A/B]</p> <p>Sequences and series know the meaning of the terms infinite sequence, infinite series, nth term, sum to n terms (partial sum), limit, sum to infinity (limit to infinity of the sequence of partial sums), common difference, arithmetic sequence, common ratio, geometric sequence, recurrence relation</p> <p>know and use the formulae $u_n = a + (n - 1)d$ and $S_n = \frac{1}{2}n[2a + (n - 1)d]$ for the nth term and the sum to n terms of an arithmetic series, respectively</p>	<p>eg Expand $(\cos \theta + i \sin \theta)^3$ eg Expand $(\cos \theta + i \sin \theta)^{1/2}$ [A/B]</p> <p>eg Express $\sin 5\theta$ in terms of $\sin \theta$ only [A/B] Express $\cos^3 \theta$ in terms of $\cos \theta$ and $\cos 3\theta$ [A/B]</p>	<p>For assessment purposes only multiples and powers less than or equal to 5 are required.</p> <p>Link with the Argand diagram. An investigative approach to solving $z^n = 1$, $n \geq 2$, may be adopted, leading to discussion of the geometrical significance.</p> <p>Higher work on first order linear recurrence relations should be revised.</p> <p>Develop arithmetic and geometric sequences as special cases by considering $x_{n+1} = ax_n + b$ with $a = 1$, $b \neq 0$ for arithmetic sequences and then $a \neq 0$, $b = 0$ for geometric sequences.</p> <p>These results should be explored numerically, conjectures made and proved.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>know and use the formulae $u_n = ar^{n-1}$ and $S_n = \frac{a(1-r^n)}{1-r}$, $r \neq 1$, for the nth term and the sum to n terms of a geometric series, respectively</p> <p>know and use the condition on r for the sum to infinity to exist and the formula $S_\infty = \frac{a}{1-r}$ for the sum to infinity of a geometric series where $r < 1$ [A/B]</p> <p>expand $\frac{1}{1-r}$ as a geometric series and extend to $\frac{1}{a+b}$ [A/B]</p> <p>know the sequence $\left(1 + \frac{1}{n}\right)^n$ and its limit</p> <p>know and use the Σ notation</p> <p>know the formula $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$ and apply it to simple sums</p>	<p>eg $\sum_{r=1}^n (ar + b) = a \sum_{r=1}^n r + \sum_{r=1}^n b$</p>	<p>This result should be linked to the binomial expansion $(1-r)^{-1}$. In addition, link to Maclaurin expansions in Mathematics 3 (AH).</p> <p>The definition $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ should be explored numerically.</p> <p>Link with proof by mathematical induction in the next section.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Elementary number theory and methods of proof</p> <p>understand the nature of mathematical proof</p> <p>understand and make use of the notations \Rightarrow, \Leftarrow and \Leftrightarrow</p> <p>know the corresponding terminology implies, implied by, equivalence</p> <p>know the terms natural number, prime number, rational number, irrational number</p> <p>know and use the fundamental theorem of arithmetic</p> <p>disprove a conjecture by providing a counter-example</p> <p>use proof by contradiction in simple examples</p>	<p>Simple examples involving the real number line, inequalities and modulus. eg $x + y \leq x + y$</p> <p>eg The irrationality of $\sqrt{2}$. eg The infinity of primes.</p>	<p>Candidates should appreciate the need for proof in mathematics. A general strategy which could be used would be to explore a situation, make conjectures, verify (perhaps using software) and then finally prove. The triangle inequality can be linked with complex numbers.</p> <p>It should be emphasised that examples do not prove a conjecture except in proof by exhaustion.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>use proof by mathematical induction in simple examples</p> <p>prove the following results</p> $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$ <p>the binomial theorem for positive integers</p> <p>de Moivre's theorem for positive integers</p>	<p>Straightforward proofs could be asked for in assessments without guidance.</p> <p>eg $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$, for all $n \in \mathbf{N}$</p> <p>eg 8^n is a factor of $(4n)!$ for all $n \in \mathbf{N}$</p> <p>eg $n < 2^n$ for all $n \in \mathbf{N}$</p>	<p>The concept of mathematical induction may be introduced in a familiar context such as demonstrating that 2p coins and 5p coins are sufficient to pay any sum of money greater than 3p.</p> <p>Candidates should use the application of:</p> $\sum_{r=1}^n r = \frac{1}{2}n(n+1)$ <p>to prove results about other sums, such as the sum of the first n odd numbers is a perfect square.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Mathematics 3 (AH)</p> <p>Vectors know the meaning of the terms position vector, unit vector, scalar triple product, vector product, components, direction ratios/cosines</p> <p>calculate scalar and vector products in three dimensions</p> <p>know that $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$</p> <p>find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ in component form</p> <p>know the equation of a line in vector form, parametric and symmetric form</p> <p>know the equation of a plane in vector form, parametric and Cartesian form</p> <p>find the equations of lines and planes given suitable defining information</p> <p>find the angles between two lines, two planes and between a line and a plane [A/B]</p> <p>find the intersection of two lines, a line and a plane, and two or three planes [A/B]</p>	<p>eg Find the equation of a plane passing through a given point and perpendicular to a given direction (the normal).</p>	<p>Higher work on vectors should be revised.</p> <p>The triangle inequality can now be completed with the vector version: $\mathbf{a} + \mathbf{b} \leq \mathbf{a} + \mathbf{b}$</p> <p>This should be linked with determinants of matrices.</p> <p>Link with systems of equations.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Matrix algebra know the meaning of the terms matrix, element, row, column, order, identity matrix, inverse, determinant, singular, non-singular, transpose</p> <p>perform matrix operations: addition, subtraction, multiplication by a scalar, multiplication, establish equality of matrices</p> <p>know the properties of the operations: $A + B = B + A$ $AB \neq BA$ in general $(AB)C = A(BC)$ $A(B + C) = AB + AC$ $(A')' = A$ $(A + B)' = A' + B'$ $(AB)' = B'A'$ $(AB)^{-1} = B^{-1}A^{-1}$ $\det(AB) = \det A \det B$</p> <p>calculate the determinant of 2×2 and 3×3 matrices</p> <p>know the relationship of the determinant to invertability</p> <p>find the inverse of a 2×2 matrix</p>		<p>Link determinant with vector product.</p> <p>Link with Systems of linear equations in Mathematics 1 (AH).</p> <p>The result $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ should be known.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>find the inverse, where it exists, of a 3×3 matrix by elementary row operations</p> <p>know the role of the inverse matrix in solving linear systems</p> <p>use 2×2 matrices to represent geometrical transformations in the (x, y) plane</p> <p>Further sequences and series know the term power series</p> <p>understand and use the Maclaurin series:</p> $f(x) = \sum_{r=0}^{\infty} \frac{x^r}{r!} f^{(r)}(0)$ <p>find the Maclaurin series of simple functions: e^x, $\sin x$, $\cos x$, $\tan^{-1} x$, $(1+x)^\alpha$, $\ln(1+x)$, knowing their range of validity</p>		<p>Extension to larger matrices may be considered as a use of technology.</p> <p>Reference can also be made to the problem of finding the intersection of three lines or three planes in vector work and link its solubility to the existence of the matrix of coefficients.</p> <p>The transformations should include rotations, reflections, dilatations and the role of the transpose in orthogonal cases. Link with complex numbers in Mathematics 2 (AH).</p> <p>The approach taken should be through polynomials and their derivatives, linking coefficients to values at zero. Graphics software provides a powerful illustration of the concept of convergence. The difficulties of differentiating or integrating power series term by term should be stressed, pointing out that these processes are only valid within the interval of convergence</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>find the Maclaurin expansions for simple composites,</p> <p>use the Maclaurin series expansion to find power series for simple functions to a stated number of terms</p> <p>use iterative schemes of the form $x_{n+1} = g(x_n)$, $n = 0, 1, 2, \dots$ to solve equations where $x = g(x)$ is a rearrangement of the original equation</p> <p>use graphical techniques to locate approximate solution x_0</p> <p>know the condition for convergence of the sequence $\{x_n\}$ given by $x_{n+1} = g(x_n)$, $n = 0, 1, 2, \dots$</p>	<p>eg e^{2x}</p> <p>eg $e^{\sin x}$, $e^x \cos 3x$ [A/B]</p> <p>eg, $xe^x = 1 \Rightarrow x = e^{-x}$ to give the scheme</p> <p>$x_{n+1} = e^{-x_n}$ with $x_0 = 0.5$</p>	<p>Link $(1+x)^\alpha$ to $(1-r)^{-1}$ from series work and to the extension of the binomial theorem.</p> <p>Candidates should be made aware that the equation $f(x) = 0$ can be rearranged in a variety of ways and that some of these may yield suitable iterative schemes while others may not.</p> <p>Cobweb and staircase diagrams help to demonstrate the test for convergence of an iterative scheme for finding a fixed point α ($\alpha = g(\alpha)$) in a neighbourhood of α, namely $g'(x) < 1$.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Further ordinary differential equations solve first order linear differential equations using the integrating factor method</p> <p>find general solutions and solve initial value problems</p> <p>know the meaning of the terms: second order linear differential equation with constant coefficients, homogeneous, non-homogeneous, auxiliary equation, complementary function and particular integral</p>	<p>eg Write the linear equation</p> $a(x)\frac{dy}{dx} + b(x)y = g(x)$ <p>in the standard form</p> $\frac{dy}{dx} + P(x)y = f(x)$ <p>and hence as</p> $\frac{d}{dx}\left(e^{\int P(x)dx} y\right) = e^{\int P(x)dx} f(x)$ <p>eg Mixing problems, such as salt water entering a tank of clear water which is then draining at a given rate. eg Growth and decay problems, an alternative method of solution to separation of variables. eg Simple electrical circuits.</p> <p>eg The general solution of the homogeneous equation</p> $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0.$ <p>If $f(x)$ and $g(x)$ are solutions and $f(x) \neq Cg(x)$, ie linearly independent solutions, then the general solution is $C_1f(x) + C_2g(x)$.</p>	

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>solve second order homogeneous ordinary differential equations with constant coefficients</p> $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$ <p>find the general solution in the three cases where the roots of the auxiliary equation:</p> <ul style="list-style-type: none"> (i) are real and distinct (ii) coincide (are equal) [A/B] (iii) are complex conjugates [A/B] <p>solve initial value problems</p> <p>solve second order non-homogeneous ordinary differential equations with constant coefficients:</p> $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ <p>using the auxiliary equation and particular integral method [A/B]</p>	<p>eg $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 13y = 0$ with $y = -1$ and $\frac{dy}{dx} = 2$ when $x = 0$.</p>	<p>Search for a trial solution using $y = e^{mx}$ and hence derive the auxiliary equation $am^2 + bm + c = 0$.</p> <p>Link with Complex numbers in Mathematics 2 (AH).</p> <p>Context applications could include the motion of a spring, both with and without a damping term.</p> <p>The general solution is the sum of the general solution of the corresponding homogeneous equation (complementary function) and a particular solution.</p> <p>For assessment purposes, only cases where the particular solution can easily be found by inspection will be required, with the right-hand side being a low order polynomial or a constant multiple of $\sin x$, $\cos x$ or e^{kx}.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>Further number theory and further methods of proof know the terms necessary condition, sufficient condition, if and only if, converse, negation, contrapositive</p> <p>use further methods of mathematical proof: some simple examples involving the natural numbers</p> <p>direct methods of proof: sums of certain series and other straightforward results</p> <p>further proof by contradiction</p> <p>further proof by mathematical induction</p> <p>prove the following result</p> $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1); n \in \mathbf{N}$	<p>eg $x > 1 \Rightarrow x^2 > 1$, the triangle inequality, the sum to n terms of an arithmetic or geometric series.</p> <p>eg If $x, y \in \mathbf{R}$ such that $x + y$ is irrational then at least one of x, y is irrational.</p> <p>eg If m, n are integers and $mn = 100$, then either $m \leq 10$ or $n \leq 10$.</p> <p>eg The square matrices A and B are such that $AB=BA$. Prove by induction that $A^n B = B A^n$ for all integers $n \geq 1$ [A/B]</p> <p>eg Proof of the result $\sum_{r=1}^n r^3 = \left(\sum_{r=1}^n r\right)^2 = \frac{n^2(n+1)^2}{4}$ is a useful extension.</p>	<p>Number theory can be drawn upon for likely but unproved conjectures, and also for erroneous conjectures, such as:</p> <p>(i) Are there an infinite number of twin primes, n and $n + 2$? (unproved)</p> <p>(ii) Is $2^{2^n} + 1$ prime for all $n \in \mathbf{N}$? (erroneous, holds for $n = 1, 2, 3, 4$ but not for all higher values).</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

CONTENT	NOTES	TEACHING NOTES
<p>know the result $\sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$</p> <p>apply the above results and the one for $\sum_{r=1}^n r$ to prove by direct methods results concerning other sums</p> <p>know the division algorithm and proof</p> <p>use Euclid's algorithm to find the greatest common divisor (g.c.d.) of two positive integers</p> <p>know how to express the g.c.d. as a linear combination of the two integers [A/B]</p> <p>use the division algorithm to write integers in terms of bases other than 10 [A/B]</p>	<p>eg $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$.</p> <p>eg $\sum_{i=1}^4 (3i+1)$.</p> <p>eg $\sum_{r=1}^n r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$.</p> <p>eg $\sum_{r=1}^n r(r^2+2)$ [A/B]</p> <p>ie Use the division algorithm repeatedly.</p>	<p>The generalisation of these results provides an interesting extension exercise.</p> <p>The special cases of finding the g.c.d. of two Fibonacci numbers are possible extensions.</p> <p>A possible context for this is binary arithmetic.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

ASSESSMENT

To gain the award of the course, the candidate must pass all unit assessments as well as the external assessment. External assessment will provide the basis for grading attainment in the course award.

Where units are taken as component parts of a course, candidates will have the opportunity to achieve at levels beyond that required to attain each of the unit outcomes. This attainment may, where appropriate, be recorded and used to contribute towards course estimates and to provide evidence for appeals. Additional details are provided, where appropriate, with the exemplar assessment materials. Further information on the key principles of assessment are provided in the paper *Assessment*, published by HSDU in May 1996.

DETAILS OF THE INSTRUMENTS FOR EXTERNAL ASSESSMENT

The external assessment will take the form of an examination of up to three hours' duration. Candidates will sit a paper assessing *Mathematics 1 (AH)*, *2 (AH)* and *3 (AH)*. The examination will consist of a balance of short questions designed mainly to test knowledge and understanding, and extended response questions, which also test problem solving skills. These two styles of questions will include ones which are set in more complex contexts to provide evidence for performance at grades A and B.

GRADE DESCRIPTIONS FOR ADVANCED HIGHER MATHEMATICS

Advanced Higher Mathematics courses should enable candidates to solve problems which integrate mathematical knowledge across performance criteria, outcomes and units, and which require extended thinking and decision making. The award of grades A, B and C is determined by the candidate's demonstration of the ability to apply knowledge and understanding to problem solving. To achieve grades A and B in particular, this demonstration will involve more complex contexts including the depth of treatment indicated in the detailed content tables.

In solving these problems, candidates should be able to:

- a) interpret the problem and consider what might be relevant;
- b) decide how to proceed by selecting an appropriate strategy;
- c) implement the strategy through applying mathematical knowledge and understanding and come to a conclusion;
- d) decide on the most appropriate way of communicating the solution to the problem in an intelligible form.

Familiarity and complexity affect the level of difficulty of problems/assignments. It is generally easier to interpret and communicate information in contexts where the relevant variables are obvious and where their inter-relationships are known. It is usually more straightforward to apply a known strategy than to modify one or devise a new one. Some concepts are harder to grasp and some techniques more difficult to apply if they have to be used in combination.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

Exemplification at grade C and grade A

a) *Interpret the problem and consider what might be relevant*

At grade C candidates should be able to interpret and model qualitative and quantitative information as it arises within:

- the description of real-life situations
- the context of other subjects
- the context of familiar areas of mathematics

Grade A performance is demonstrated through coping with the interpretation of more complex contexts requiring a higher degree of reasoning ability in the areas described above.

b) *Decide how to proceed by selecting an appropriate strategy*

At grade C candidates should be able to tackle problems by selecting algorithms drawn from related areas of mathematics or apply a heuristic strategy.

Grade A performance is demonstrated through an ability to decide on and apply a more extended sequence of algorithms to more complex contexts.

c) *Implement the strategy through applying mathematical knowledge and understanding, and come to a conclusion*

At grade C candidates should be able to use their knowledge and understanding to carry through their chosen strategies and come to a conclusion. They should be able to process data in numerical and symbolic form with appropriate regard for accuracy, marshal facts, sustain logical reasoning and appreciate the requirements of proof.

Grade A performance is demonstrated through an ability to cope with processing data in more complex situations and sustaining logical reasoning, where the situation is less readily identifiable with a standard form.

d) *Decide on the most appropriate way of communicating the solution to the problem in an intelligible form*

At grade C candidates should be able to communicate qualitative and quantitative mathematical information intelligibly and to express the solution in language appropriate to the situation.

Grade A performance is demonstrated through an ability to communicate intelligibly in more complex situations and unfamiliar contexts.

National Course Specification: course details (cont)

COURSE Mathematics (Advanced Higher)

APPROACHES TO LEARNING AND TEACHING

The approaches to learning and teaching recommended for Higher level should be continued and reinforced whenever possible. Exposition to a group or class remains an essential technique. However, candidates should be more actively involved in their own learning in preparation for future study in higher education. Opportunities for discussion, problem solving, practical activities and investigation should abound in Advanced Higher Mathematics. There also exists much greater scope to harness the power of technology in the form of mathematical and graphical calculators and computer software packages.

Independent learning is further encouraged in the grade descriptions for the course. Coursework tasks and projects/assignments are recommended as vehicles for the introduction of new topics, the illustration or reinforcement of mathematics in context and for the development of extended problem solving, practical and investigative skills, as well as adding interest to the course.

SPECIAL NEEDS

This course specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessments Arrangements A0645/3 December 2001*.

SUBJECT GUIDES

A Subject Guide to accompany the Arrangements documents has been produced by the Higher Still Development Unit (HSDU) in partnership with the Scottish Consultative Council on the Curriculum (SCCC) and Scottish Further Education Unit (SFEU). The Guide provides further advice and information about:

- support materials for each course
- learning and teaching approaches in addition to the information provided in the Arrangements document
- assessment
- ensuring appropriate access for candidates with special educational needs

The Subject Guide is intended to support the information contained in the Arrangements document. The SQA Arrangements documents contain the standards against which candidates are assessed.



National Unit Specification: general information

UNIT	Mathematics 1 (Advanced Higher)
NUMBER	D321 13
COURSE	Mathematics (Advanced Higher)

SUMMARY

This unit is the first of three units, which comprise the Advanced Higher Mathematics course. This unit extends the calculus and graphicacy work from Higher level and introduces matrices for solving systems of linear equations. It provides a basis for progression to Mathematics 2 (AH).

OUTCOMES

- 1 Use algebraic skills.
- 2 Use the rules of differentiation on the elementary functions x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, e^x and $\ln x$ and their composites.
- 3 Integrate using standard results and the substitution method.
- 4 Use properties of functions.
- 5 Use matrix methods to solve systems of linear equations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained :

- Higher Mathematics award, including Mathematics 3 (H)

Administrative Information

Superclass:	RB
Publication date:	August 2003
Source:	Scottish Qualifications Authority
Version:	04

© Scottish Qualifications Authority 2001

This publication may be reproduced in whole or in part for educational purposes provided that no profit is derived from reproduction and that, if reproduced in part, the source is acknowledged.

Additional copies of this unit specification can be purchased from the Scottish Qualifications Authority. The cost for each unit specification is £2.50 (minimum order £5).

National Unit Specification: general information (cont)

UNIT Mathematics 1 (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher.

CORE SKILLS

Core skills for Advanced Higher remain subject to confirmation and details will be available at a later date.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications 2001/2002 BA0906 August 2001*.

National Unit Specification: statement of standards

UNIT Mathematics 1 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use algebraic skills.

Performance criteria

- (a) Expand an expression of the form $(x + y)^n$, $n \in \mathbf{N}$ and $n \leq 5$.
- (b) Express a proper rational function as a sum of partial fractions where the denominator is a quadratic in factorised form.

OUTCOME 2

Use the rules of differentiation on the elementary functions x^n , ($n \in \mathbf{Q}$), $\sin x$, $\cos x$, e^x and $\ln x$ and their composites.

Performance criteria

- (a) Differentiate a product.
- (b) Differentiate a quotient.
- (c) Differentiate a simple composite function using the chain rule.

OUTCOME 3

Integrate using standard results and the substitution method.

Performance criteria

- (a) Integrate an expression requiring a standard result.
- (b) Integrate using a substitution method where the substitution is given.
- (c) Integrate an expression requiring a simple substitution.

OUTCOME 4

Use properties of functions.

Performance criteria

- (a) Find the vertical asymptote of a rational function.
- (b) Find the non-vertical asymptote of a rational function.
- (c) Sketch the graph of a rational function including appropriate analysis of stationary points.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 1 (Advanced Higher)

OUTCOME 5

Use matrix methods to solve systems of linear equations.

Performance criteria

(a) Use Gaussian elimination to solve a 3×3 system of linear equations.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 1 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated and for checking of accuracy and sensibility of answers to be ever present.

In this unit the algebraic skills learnt at Higher level are extended in Outcome 1 to binomial expansions and partial fractions.

In Outcomes 2 and 3, the elementary calculus studied at Higher level is extended to differentiation of sums, products, quotients and composites of elementary functions and to integration using standard results and substitution methods respectively. In both of Outcomes 2 and 3, computer algebra systems can be used extensively for consolidation and extension.

In Outcome 4 the work at Higher level on using calculus methods to sketch graphs of functions is taken further, with enhancement through the use of graphic calculators recommended.

Outcome 5 is the only outcome which does not build upon Higher content. It provides an introduction to matrix methods leading to the use of Gaussian elimination to solve a 3×3 system of linear equations.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Mathematics 1 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – ‘How sensible is my answer?’
- estimate followed by a repeated calculation
- calculation in a different order

Further advice on learning and teaching approaches is contained within the subject guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessments Arrangements* A064513 December 2001.



National Unit Specification: general information

UNIT Mathematics 2 (Advanced Higher)

NUMBER D322 13

COURSE Mathematics (Advanced Higher)

SUMMARY

This unit is the second of three units, which comprise the Advanced Higher Mathematics course. It extends the calculus in Mathematics 1 (AH), extends the work on recurrence relations at Higher level and introduces complex numbers and mathematical proof.

OUTCOMES

- 1 Use further differentiation techniques.
- 2 Use further integration techniques.
- 3 Understand and use complex numbers.
- 4 Understand and use sequences and series.
- 5 Use standard methods to prove results in elementary number theory.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Mathematics 1 (AH)

CREDIT VALUE

1 credit at Advanced Higher.

Administrative Information

Superclass: RB

Publication date: August 2003

Source: Scottish Qualifications Authority

Version: 04

© Scottish Qualifications Authority 2001

This publication may be reproduced in whole or in part for educational purposes provided that no profit is derived from reproduction and that, if reproduced in part, the source is acknowledged.

Additional copies of this unit specification can be purchased from the Scottish Qualifications Authority. The cost for each unit specification is £2.50 (minimum order £5).

National Unit Specification: general information (cont)

UNIT Mathematics 2 (Advanced Higher)

CORE SKILLS

Core skills for Advanced Higher remain subject to confirmation and details will be available at a later date.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications 2001/2002 BA0906 August 2001*.

National Unit Specification: statement of standards

UNIT Mathematics 2 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use further differentiation techniques.

Performance criteria

- (a) Differentiate an inverse trigonometric function (involving the chain rule).
- (b) Find the derivative of a function defined implicitly.
- (c) Find the first derivative of a function defined parametrically.

OUTCOME 2

Use further integration techniques.

Performance criteria

- (a) Integrate a proper rational function where the denominator is a factorised quadratic.
- (b) Integrate by parts with one application.
- (c) Find a general solution of a first order differential equation (variables separable type).

OUTCOME 3

Understand and use complex numbers.

Performance criteria

- (a) Perform a simple arithmetic operation on two complex numbers of the form $a + bi$.
- (b) Evaluate the modulus and argument of a complex number.
- (c) Convert from cartesian to polar form.
- (d) Plot a complex number on an Argand diagram.

OUTCOME 4

Understand and use sequences and series.

Performance criteria

- (a) Find the n^{th} term and the sum of the first n terms of an arithmetic sequence.
- (b) Find the n^{th} term and the sum of the first n terms of a geometric sequence.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 2 (Advanced Higher)

OUTCOME 5

Use standard methods to prove results in elementary number theory.

Performance criteria

- (a) Disprove a conjecture by providing a counter-example.
- (b) Use proof by contradiction in a simple example.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessment, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 2 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated and for checking of accuracy and sensibility of answers to be ever present.

In this unit, the second of three progressive Mathematics units, outcome 1 extends the differentiation covered in Mathematics 1 (AH) to inverse functions and introduces implicit and parametric differentiation.

Integration is correspondingly extended in outcome 2 to integration by parts and partial fractions and first order differential equations are introduced.

In Outcome 3, candidates are introduced to the complex number system and are required to demonstrate competence in operations on complex numbers.

Higher level work on recurrence relations is extended to the formal study of arithmetic and geometric sequences in outcome 4 and the groundwork is laid for the study of Maclaurin expansions in Mathematics 3 (AH).

As candidates progress in mathematics they should acquire a growing awareness of the importance of mathematical proof and the need for mathematical rigour. It is for this reason that outcome 5 contains an introduction to elementary number theory and methods of proof.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.



National Unit Specification: general information

UNIT	Mathematics 3 (Advanced Higher)
NUMBER	D323 13
COURSE	Mathematics (Advanced Higher)

SUMMARY

This unit is the third of three units, which comprise the Advanced Higher Mathematics course. It introduces vector equations of lines and planes, matrices and their applications to geometrical transformations, and the Maclaurin series with simple applications, and extends number theory and proof. It also extends the work on differential equations from Mathematics 2 (AH).

OUTCOMES

- 1 Use vectors in three dimensions.
- 2 Use matrix algebra.
- 3 Understand and use further aspects of sequences and series.
- 4 Solve further ordinary differential equations.
- 5 Use further number theory and direct methods of proof.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained:

- Mathematics 1 (AH) and Mathematics 2 (AH)

CREDIT VALUE

1 credit at Advanced Higher.

Administrative Information

Superclass:	RB
Publication date:	August 2003
Source:	Scottish Qualifications Authority
Version:	04

© Scottish Qualifications Authority 2001

This publication may be reproduced in whole or in part for educational purposes provided that no profit is derived from reproduction and that, if reproduced in part, the source is acknowledged.

Additional copies of this unit specification can be purchased from the Scottish Qualifications Authority. The cost for each unit specification is £2.50 (minimum order £5).

National Unit Specification: general information (cont)

UNIT Mathematics 3 (Advanced Higher)

CORE SKILLS

Core skills for Advanced Higher remain subject to confirmation and details will be available at a later date.

Additional information about core skills is published in *Catalogue of Core Skills in National Qualifications 2001/2002 BA0906 August 2001*.

National Unit Specification: statement of standards

UNIT Mathematics 3 (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use vectors in three dimensions.

Performance criteria

- (a) Calculate a vector product.
- (b) Find the equation of a line in parametric form.
- (c) Find the equation of a plane in Cartesian form given a normal and a point in the plane.

OUTCOME 2

Use matrix algebra.

Performance criteria

- (a) Perform matrix operations of addition, subtraction and multiplication.
- (b) Calculate the determinant of a 3×3 matrix.
- (c) Find the inverse of a 2×2 matrix.

OUTCOME 3

Understand and use further aspects of sequences and series.

Performance criteria

- (a) Use the Maclaurin series expansion to find a stated number of terms of the power series for a simple function.
- (b) Solve a simple non-linear equation using a simple iteration of the form $x_{n+1} = g(x_n)$ where x_0 is given.

OUTCOME 4

Solve further ordinary differential equations.

Performance criteria

- (a) Solve a first order linear differential equation using the integrating factor.

National Unit Specification: statement of standards (cont)

UNIT Mathematics 3 (Advanced Higher)

OUTCOME 5

Use further number theory and direct methods of proof.

Performance criteria

- (a) Use proof by mathematical induction.
- (b) Use Euclid's algorithm to find the greatest common divisor of two positive integers.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessment candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 3 (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated, and for checking of accuracy and sensibility of answers to be ever present.

In this unit, the third of three progressive Mathematics units, the themes of earlier work at Higher and Advanced Higher are further developed. Outcome 1 builds on the vector content of Higher Mathematics and extends to the vector equations of lines and planes.

Matrices are studied in greater depth in Outcome 2 in applications to systems of equations.

The work on sequences and series in the previous unit is now, in Outcome 3, applied to Maclaurin expansions.

In Outcome 4, the study of first order differential equations in Mathematics 2 (AH) is continued.

The important topic of proof, also introduced in Mathematics 2 (AH), is further reinforced and developed in Outcome 5.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Mathematics 3 (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – ‘How sensible is my answer?’
- estimate followed by a repeated calculation
- calculation in a different order

Further advice on learning and teaching approaches is contained within the subject guide for Mathematics.

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements A0645/3 December 2001*.