[C102/SQP249]

Higher Mathematics

Time: 1 hour 10 minutes

NATIONAL QUALIFICATIONS

Units 1, 2 and Statistics
Paper 1
(Non-calculator)
Specimen Question Paper **(Revised)**
for use in and after 2004

Read Carefully

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2 Full credit will be given only where the solution contains appropriate working.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Statistics:

Sample standard deviation $s = \sqrt{\frac{1}{n-1}\sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\sum x_i^{\,2} - \frac{1}{n}\left(\sum x_i\right)^2\right)}$ where n is the sample size.

Sums of squares and products:

$$S_{xx} = \sum\left(x_i - \bar{x}\right)^2 = \sum x_i^{\,2} - \frac{1}{n}\left(\sum x_i\right)^2$$

$$S_{yy} = \sum\left(y_i - \bar{y}\right)^2 = \sum y_i^{\,2} - \frac{1}{n}\left(\sum y_i\right)^2$$

$$S_{xy} = \sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = \sum x_i y_i - \frac{1}{n}\sum x_i \sum y_i$$

Linear regression: The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β, a and b, are given by:

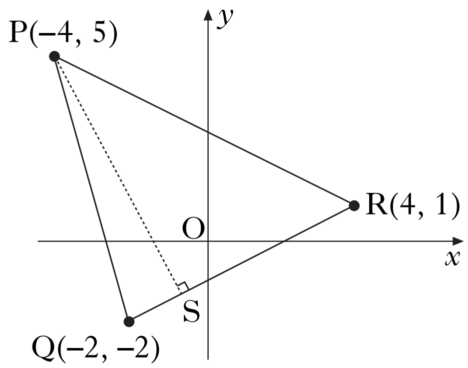
$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum\left(x_i - \bar{x}\right)^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient r: $r = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum\left(x_i - \bar{x}\right)^2 \sum\left(y_i - \bar{y}\right)^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

All questions should be attempted. *Marks*

1. P(−4, 5), Q(−2, −2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **(4)**



2. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}3u_n + 5$ with first term u_1.

(*a*) Explain why this sequence has a limit as n tends to infinity. **(1)**

(*b*) Find the **exact** value of this limit. **(2)**

3. (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. **(4)**

(*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes. **(2)**

(*c*) Find the stationary points of $y = f(x)$ and determine the nature of each. **(5)**

(*d*) Sketch the graph of $y = f(x)$. **(1)**

4. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x + 30)°$ is $\dfrac{4\sqrt{3} + 3}{10}$. **(4)**

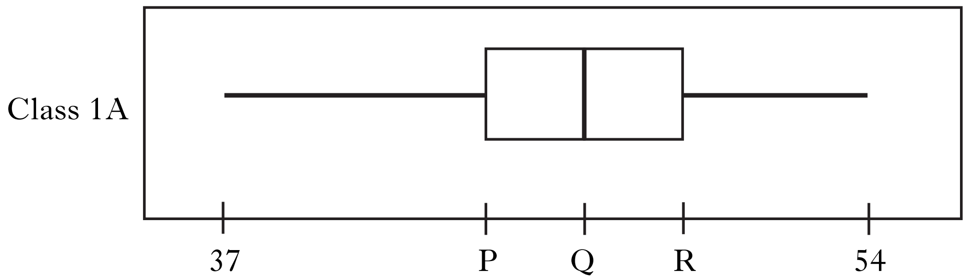
Marks

5. Class 1A sat a test out of 60. The marks are shown in the stem-and-leaf diagram below.

3	7											
4	0	1	2	2	4	4						
4	5	5	6	7	7	7	8	8	8	9	9	9
5	0	1	1	2	3	4						

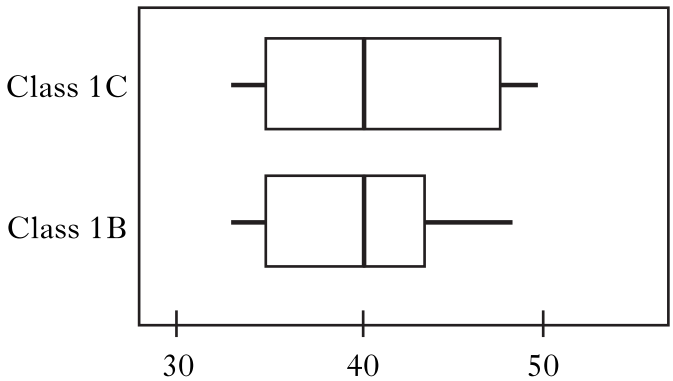
$n = 25$ 3 7 represents 37

The diagram below shows an incomplete boxplot for this data.



(*a*) Find the values associated with the points P, Q and R. **(3)**

(*b*) The boxplot below shows the data for classes 1B and 1C.

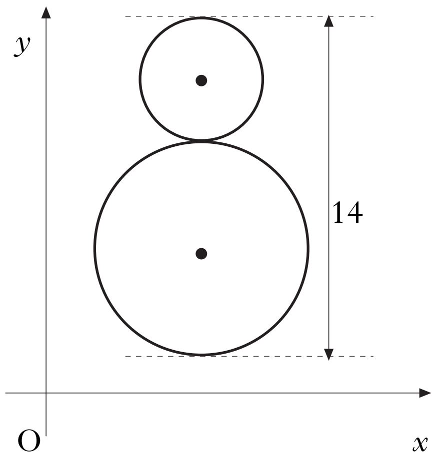


Compare the results of these two classes. **(2)**

Marks

6. A bakery firm makes ginger-bread men each 14 cm high with a circular "head" and "body". The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis.

Find the equation of the "head". **(6)**



7. Find the value of $\int_1^2 \frac{u^2+2}{2u^2}\,du.$ **(7)**

8. Sketch the graph of $y = 2\sin(x - 30)°$ for $0 \le x < 360$. **(4)**

Marks

9. A random device moves one unit to the right with probability 0·3, one unit to the left with probability 0·3 or remains in the same position after each trial.

(*a*) Tabulate the probability distribution of X, the position of the device, after one trial. **(2)**

(*b*) A calculator produces the following random numbers.

0·764	0·380	0·410	0·175	0·458
0·552	0·709	0·935	0·451	0·854

(i) Explain how you would use these numbers to simulate ten trials of this random experiment. **(3)**

(ii) List the results of your simulation. **(1)**

10. The total lifetime (in years) of 5 year old washing machines of a certain make is a random variable whose cumulative distribution function F is given by

$$F(x) = \begin{cases} 0 & \text{for } x \leq 5 \\ 1 - \dfrac{25}{x^2} & \text{for } x > 5 \end{cases}$$

(*a*) Find the probability that such a washing machine will be in service for:

(i) less than 8 years; **(1)**

(ii) more than 10 years. **(2)**

(*b*) Find the probability density function $f(x)$. **(3)**

(*c*) Calculate the exact value of the median lifetime of these washing machines. **(3)**

[*END OF SPECIMEN QUESTION PAPER*]

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$$r = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum\left(x_i - \bar{x}\right)^2 \sum\left(y_i - \bar{y}\right)^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

All questions should be attempted. *Marks*

1. ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC. **(4)**

2. Trees are sprayed weekly with the pesticide, "Killpest", whose manufacturers claim it will destroy 60% of all pests. Between the weekly sprayings, it is estimated that 300 new pests invade the trees.

A new pesticide, "Pestkill", comes onto the market. The manufacturers claim that it will destroy 80% of existing pests but it is estimated that 360 new pests per week will invade the trees.

Which pesticide will be more effective in the long term? **(6)**

3. (*a*) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants. **(3)**

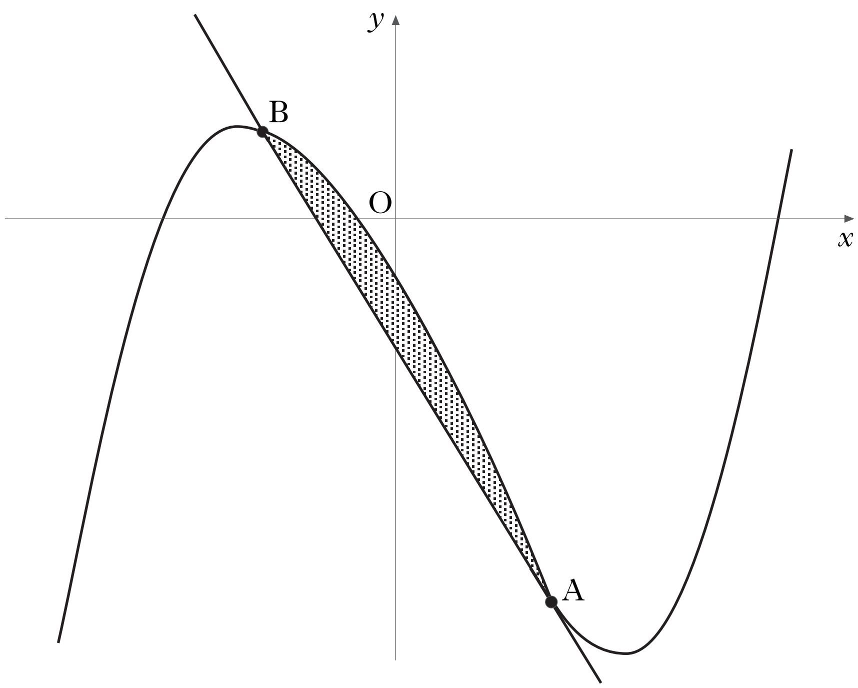
(*b*) Hence, or otherwise, find the coordinates of the turning point of the function f. **(1)**

4. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, −8).

(*a*) Find the equation of the tangent at A. **(4)**

(*b*) Hence find the coordinates of B. **(5)**

(*c*) Find the area of the shaded part which represents the land bounded by the river and the road. **(5)**



Marks

5. In an archery competition, the probability that a particular competitor hits the target with any shot is $\frac{3}{4}$. In the competition, she is allowed three shots.

(*a*) Find the probability that she hits the target:

(i) exactly twice; **(2)**

(ii) at least once. **(2)**

(*b*) State a statistical assumption that you have made. **(1)**

6. A market gardener wishes to investigate the relationship between the total weight of tomatoes produced by a tomato plant and the amount of fertiliser used. An experiment was carried out where known amounts of fertiliser were applied to 8 similar plants. The results are shown in the table.

Weight of fertiliser (g)	x	0	2	4	6	8	10	12	14
Tomato yield (kg)	y	4·44	5·13	5·45	5·27	5·81	6·04	5·90	6·23

A scatter diagram shows that a linear model is appropriate.

You may assume that $\sum y = 44 \cdot 27$, $\sum y^2 = 247 \cdot 3665$ and $\sum xy = 328 \cdot 58$

(*a*) Determine the equation of the least squares regression line of y on x. **(6)**

(*b*) Calculate the product moment correlation coefficient and comment on your answer. **(3)**

7. (*a*) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$. **(3)**

(*b*) **Hence**

(i) write the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in terms of $\sin x°$

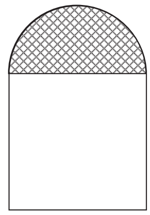
(ii) solve this equation in the interval $0 \le x < 90$. **(4)**

8. The roots of the equation $(x - 1)(x + k) = -4$ are equal.

Find the values of k. **(5)**

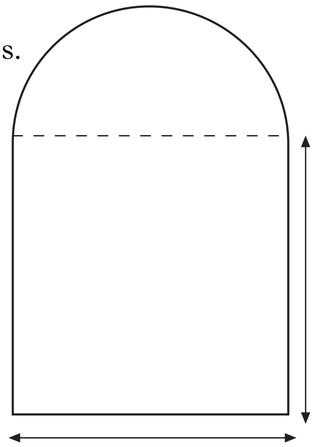
Marks

9. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.



The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.

The rectangle measures $2x$ metres by h metres.



(a) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(b) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(6)**

10. The random variable X has a probability density function

$$f(x) = \begin{cases} kx^2(1-x) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k. **(2)**

(b) Find the probability that X lies in the range $0 \le X \le \frac{2}{3}$. **(4)**

[*END OF SPECIMEN QUESTION PAPER*]

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