23 March 2004

To: SQA Co-ordinator
Scottish Executive
Directors of Education
Mathematics Assessment Panel members
Higher Education Institutions

For the attention of all staff responsible for the delivery of National Qualifications in Mathematics

Action by Recipient	
	Response required
✓	**Note and pass on**
	None — update/information only

Contact Name: Noël Donoghue at Glasgow
Direct Line: 0141-242 2152
E-mail: noel.donoghue@sqa.org.uk

Dear Colleague

Advanced Higher Applied Mathematics

The Mathematics update letter of November 2003 indicated that for session 2004/05, there would be changes to the Advanced Higher Applied Mathematics framework and a new Unit: *Mathematics for Applied Mathematics (AH)* would be introduced.

I can now confirm that in session 2004/05, there will be three Courses in Applied Mathematics (AH).

1. C 202 13 Applied Mathematics: Statistics (AH)
   The Course will consist of three mandatory Units as follows:
   D326 13 *Statistics 1 (AH)*
   D330 13 *Statistics 2 (AH)*
   DE8Y 13 *Mathematics for Applied Mathematics (AH)*

2. C 203 13 Applied Mathematics: Numerical Analysis (AH)
   The Course will consist of three mandatory Units as follows:
   D328 13 *Numerical Analysis 1 (AH)*
   D329 13 *Numerical Analysis 2 (AH)*
   DE8Y 13 *Mathematics for Applied Mathematics (AH)*

3. C 204 13 Applied Mathematics: Mechanics (AH)
   The Course will consist of three mandatory Units as follows:
   D327 13 *Mechanics 1 (AH)*
   D331 13 *Mechanics 2 (AH)*
   DE8Y 13 *Mathematics for Applied Mathematics (AH)*

Each Course consists of two core Units (Statistics, Numerical Analysis or Mechanics), and a common component Unit *Mathematics for Applied Mathematics (AH)*. Candidates for Applied Mathematics (AH) will choose to enter for one of the Applied Mathematics (AH) Courses: either Applied Mathematics: Statistics (AH) or Applied Mathematics: Numerical Analysis (AH) or Applied Mathematics: Mechanics (AH).

The seven component Units of Advanced Higher Applied Mathematics Courses remain available to candidates as free standing Units.

A revised Applied Mathematics (AH) Arrangements document, which takes account of these structural changes, will be published in April 2004, and thereafter can be accessed through the NQ Mathematics subject-specific page on SQA's website (**www.sqa.org.uk**).

To support centres in becoming familiar with the content of the Unit *Mathematics for Applied Mathematics (AH)*, Appendix 1 of this letter contains:

- National Unit Specification: general information
- National Unit Specification: statement of standards
- National Unit Specification: support notes

Appendix 2 links the existing support material previously issued to centres through the Higher Still Development Programme, to Course content within the Unit *Mathematics for Applied Mathematics (AH)*. This support material is available from Learning and Teaching Scotland (LTS). Information about the material is on the Learning and Teaching Scotland website: www.ltscotland.org.uk.

The external assessment for each Applied Mathematics (AH) Course will take the form of an examination of up to three hours' duration. Candidates will sit an examination paper assessing *Statistics 1 (AH)*, *Statistics 2 (AH)* and *Mathematics for Applied Mathematics (AH)* **or** one assessing *Numerical Analysis 1 (AH)*, *Numerical Analysis 2 (AH)* and *Mathematics for Applied Mathematics (AH)* **or** one assessing *Mechanics 1 (AH)*, *Mechanics 2 (AH)* and *Mathematics for Applied Mathematics (AH)*. Each examination paper will contain a balance of short questions designed mainly to test knowledge and understanding and extended response questions which also test problem solving skills. The two styles of questions will include ones which are set in more complex contexts to provide evidence for performance at grades A and B.

To illustrate the style of the examination papers, a specimen question paper for the Course C 202 13 Applied Mathematics: Statistics (AH) is included in Appendix 3 of this letter. The paper uses questions from Section A of the 2002 Applied Mathematics Advanced Higher examination paper, and questions covering the content of the Unit *Mathematics for Applied Mathematics (AH)*. The examination papers for the Courses C 203 13 Applied Mathematics: Numerical Analysis (AH) and C 204 13 Applied Mathematics: Mechanics (AH) will be of a similar style. The rubric for the specimen paper (note 4) indicates that a booklet of Mathematical Formulae and Statistical Tables will be supplied for all candidates. The booklet will be issued for Courses C 202 13 Applied Mathematics: Statistics (AH) and C 203 13 Applied Mathematics: Numerical Analysis (AH) only. The rubric for C 204 13 Applied Mathematics: Mechanics (AH) will contain notes 1, 2 and 3 only.

NABs have been written for the new Unit: *Mathematics for Applied Mathematics*, and can be accessed online on a secure section of the SQA's website. If you encounter any problems with the secure area of the website, please have your SQA Co-ordinator get in touch with SQA's Customer Contact Centre:

Tel: 0845 279 1000
Fax: 0141-242 2244
E-mail: customer@sqa.org.uk

There was insufficient interest from centres to merit the holding of the AH Mathematics Conference referred to in the update letter of November 2003. I hope you will find the

information contained in the Appendices to this letter sufficient to complete the transition to the new AH Mathematics Arrangements.

Additional Question Banks at Higher, Intermediate 2 and Intermediate 1

The Mathematics update letter of November 2003 indicated that the Additional Question Banks which were originally issued through the Higher Still Development Programme would be updated.

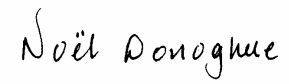
The new Bank of Additional Questions for Higher Mathematics consists of all questions from both the SCE Mathematics Higher Grade Examination Papers for 2000 and 2001, and the Mathematics Higher Examination Papers from 2000 to 2003. The new Bank of Additional Questions for Intermediate 2 and Intermediate 1 Mathematics consists of all questions from the Mathematics Intermediate 2 and Intermediate 1 Examination Papers from 2000 to 2002. The marking schemes from the examinations are included, and for each question the area of Course content covered, and the designated level of question, C, B/A is indicated.

The new item bank material for Higher can be accessed online on the secure section of SQA's website. The new item bank material for Intermediate 2 and Intermediate 1 will be available soon. If you encounter any problems with the secure area of the website, please have your SQA Co-ordinator get in touch with SQA's Customer Contact Centre:

Tel: 0845 279 1000
Fax: 0141-242 2244
E-mail: customer@sqa.org.uk

I hope you find the information in this letter helpful. If you require any further clarification please do not hesitate to contact me.

Yours faithfully



Noël Donoghue
Qualifications Manager
Mathematics and Science

Appendix 1

Appendix 1 to the update letter of February 2004 contains the information listed below for the Unit DE8Y 13 *Mathematics for Applied Mathematics:*

- National Unit Specification: general information
- National Unit Specification: statement of standards
- National Unit Specification: support notes

SCOTTISH QUALIFICATIONS AUTHORITY

National Unit Specification: general information

UNIT Mathematics for Applied Mathematics (Advanced Higher)

NUMBER DE8Y 13

COURSE Applied Mathematics: Statistics (Advanced Higher)
Applied Mathematics: Numerical Analysis (Advanced Higher)
Applied Mathematics: Mechanics (Advanced Higher)

SUMMARY

This unit is the third unit of each of the Advanced Higher Applied Mathematics courses. This unit extends the algebra and calculus work from Higher level and introduces matrix algebra which is applied to solving systems of linear equations.

OUTCOMES

1 Use algebraic skills.
2 Use matrix algebra.
3 Use the rules of differentiation on the elementary functions x^n ($n \in Q$), sin x, cos x, e^x and ln x and their composites.
4 Integrate using substitution and partial fractions and by parts.
5 Solve first order ordinary differential equations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained :

- Higher Mathematics award, including Mathematics 3 (H)

Administrative Information

Superclass: RB

Publication date: **March 2004**

Source: Scottish Qualifications Authority

Version: 01

National Unit Specification: general information (cont)

UNIT Mathematics for Applied Mathematics (Advanced Higher)

CREDIT VALUE

1 credit at Advanced Higher (8 SCQF credit points at SCQF level 7*)

**SCQF credit points are used to allocate credit to qualifications in the Scottish Credit and Qualifications Framework (SCQF). Each qualification in the Framework is allocated a number of SCQF credit points at an SCQF level. There are 12 SCQF levels, ranging from Access 1 to Doctorates.*

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	None	
Additional core skills components for the unit	Using Number	H

National Unit Specification: statement of standards

UNIT Mathematics for Applied Mathematics (Advanced Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

This unit is the third component unit of the three Advanced Higher Applied Mathematics Courses.

In this unit, you are required to demonstrate competence in five outcomes (topics) by achieving the performance criteria listed for each outcome.

OUTCOME 1

Use algebraic skills.

Performance criteria

(a) Expand an expression of the form $(x + y)^n$, $n \in \mathbf{N}$ and $n \leq 5$.

(b) Evaluate a simple sum of the form $\sum_{r=1}^{n}(ar + b)$; $r, n \in \mathbf{N}$

(c) Express a proper rational function as a sum of partial fractions where the denominator is a quadratic in factorised form.

OUTCOME 2

Use matrix algebra

Performance criteria

(a) Perform matrix operations of addition, subtraction and multiplication
(b) Calculate the determinant of a 3×3 matrix
(c) Find the inverse of a 2×2 matrix

OUTCOME 3

Use the rules of differentiation on the elementary functions x^n, $(n \in \mathbf{Q})$, $\sin x$, $\cos x$, e^x and $\ln x$ and their composites.

Performance criteria

(a) Differentiate a product.
(b) Differentiate a quotient.
(c) Find the first derivative of a function defined parametrically.

National Unit Specification: statement of standards (cont)

UNIT Mathematics for Applied Mathematics (Advanced Higher)

OUTCOME 4

Integrate using substitution and partial fractions and by parts.

Performance criteria

(a) Integrate using a substitution when the substitution is given.
(b) Integrate a proper rational function where the denominator is a factorised quadratic.

OUTCOME 5

Solve first order ordinary differential equations

Performance criteria

(a) Find a general solution of a first order differential equation (variables separable type).
(b) Solve a simple first order linear differential equation using an integrating factor.

Evidence requirements
Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics for Applied Mathematics (Advanced Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Advanced Higher level aims to build upon and extend candidates' mathematical knowledge and skills in a manner which reinforces the essential nature of problem solving. New mathematical concepts and skills are within theoretical or practical applications, and the importance of algebraic manipulative skills is emphasised throughout. At the same time, the benefits of advanced technology in securing and consolidating understanding are acknowledged and there are frequent references to the use of such technology throughout the course content. Equally important is the need, where appropriate, for the limitations of the technology to be demonstrated and for checking of accuracy and whether or not an answer is sensible to be ever present.

In this unit the algebraic skills learnt at Higher level are extended in Outcome 1 to binomial expansions, sigma notation and partial fractions.

Outcome 2, provides an introduction to matrix algebra.

In Outcomes 3 and 4, the elementary calculus studied at Higher level is extended to differentiation of sums, products, quotients and composites of elementary functions and to integration using substitution and partial fractions and by parts. In both of Outcomes 3 and 4, computer algebra systems can be used extensively for consolidation and extension.

Outcome 5 introduces first order ordinary differential equations of the variables separable type and where an integrating factor is used.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

National Unit Specification: support notes (cont)

UNIT Mathematics for Applied Mathematics (Advanced Higher)

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

The investigative approaches to teaching and learning consistently recommended at earlier levels are equally beneficial at Advanced Higher level mathematics.

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, to be aware of the limitations of the technology and always to apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the threshold of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to demonstrate attainment in the algebraic and calculus content of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

SPECIAL NEEDS

This unit specification is intended to ensure that there are no artificial barriers to learning or assessment. Special needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments or considering alternative outcomes for units. For information on these, please refer to the SQA document *Guidance on Special Assessment Arrangements* (A0645/4, September 2003).

Appendix 2

Mathematics for Applied Mathematics Unit

The content of the *Mathematics for Applied Mathematics (AH)* unit has been selected from the units *Mathematics 1 (AH)*, *Mathematics 2 (AH) and Mathematics 3 (AH)*.

Consequently, no new support materials have been written specifically for the *Mathematics for Applied Mathematics (AH)* unit. However, the following pages provide a link from the content items of the unit *Mathematics for Applied Mathematics (AH)* to the relevant pages of the following Higher Still support packs for Advanced Higher Mathematics which are available from Learning Teaching Scotland (LTS).

Pack M1	*Mathematics 1 (AH)* LTS Reference number 6804, available on Higher Still Support Materials CD
Pack M2	*Mathematics 2 (AH)* LTS Reference number 7533, available on Higher Still Support Materials CD
Pack M3	*Mathematics 3 (AH)* LTS Reference number 7534, available on Higher Still Support Materials CD

Each content item in the *Mathematics for Applied Mathematics (AH)* unit has been linked to the item code of the corresponding item in the Advanced Higher Mathematics units eg content item 2.3 in the *Mathematics for Applied Mathematics* (AH) corresponds to content item 3.2.4 in the Advanced Higher Mathematics course.

This linking facilitates access to additional relevant support materials in the Higher Still Supplementary Pack and the Additional Question Banks listed below.

Mathematics 1, 2 and 3 (Supplementary Pack)
LTS Reference number 7850, available on Higher Still Support Materials CD

Additional Question Bank - Mathematics 1, 2 and 3
LTS Reference number 7856, available on Higher Still Support Materials CD

Additional Question Bank - Mathematics 1, 2 and 3 - (Questions from pre-1992 CSYS Mathematics Papers)
LTS Reference number 8189, available on Higher Still Support Materials CD

The support material referred to above, which was originally issued to all centres through the Higher Still Development Programme, is now available from Learning Teaching Scotland. Further information is on the Learning Teaching Scotland website www.ltscotland.org.uk.

		Pack	*Page*	*Comment*	*AH Maths Content Ref*
1	**Algebra**				
1.1	know and use the notation $n!$, ${}^nC_r$ and $\binom{n}{r}$	**M1**	**3**		**1.1.1**
1.2	know and use the $\sum$ notation	**M1**	**7**	**- first mention. See 1.4, 1.5**	**2.4.7**
1.3	know Pascal's triangle. Pascal's triangle should be extended up to $n = 7$.	**M1**	**6**		**1.1.3**
1.4	know and use the binomial theorem $(a + b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r$, for $r, n \in \mathbf{N}$ **e.g. expand $(2u - 3v)^5$ [A/B]**	**M1**	**7**	**– specific term evaluation not required**	**1.1.4**
1.5	know the formulae $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$,	**M2**	**42**	**– 4th item for 1st formula (Proof-not reqd is M2 P49 (2.5.8))**	**2.4.8**
	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$	**M3**	**88**	**- 2nd item for 2nd formula (Proof-not reqd is M3 P86 (3.5.6)**	**3.5.8**
1.6	apply the above results to prove by direct methods results concerning other sums	**M3**	**88**	**– 1st and 2nd items for 3rd formula**	**3.5.7 3.5.8**
1.7	express a proper rational function as a sum of partial fractions (denominator of degree at most 3 and easily factorised).	**M1**	**9**		**1.1.6**
	include cases where an improper rational function is reduced to a polynomial and a proper rational function by division or otherwise. [A/B]	**M1**	**12**	**– for [A/B] material**	
2	**Matrices**				
2.1	know the meaning of the terms matrix, element, row, column, order, identity matrix, inverse, determinant, singular, non-singular	**M3**	**25**	**– omit transpose**	**3.2.1**
2.2	perform matrix operations: addition, subtraction, multiplication by a scalar, multiplication, establish equality of matrices	**M3**	**25**		**3.2.2**
2.3	calculate the determinant of 2×2 and 3×3 matrices	**M3**	**27**	**– 2nd item**	**3.2.4**
2.4	know the relationship of the determinant to invertability	**M3**	**27**	**– 3rd item**	**3.2.5**
2.5	find inverses, where they exist, of 2×2 matrices and 3×3 matrices	**M3** **M3**	**27** **30**	**– 4th item (for 2 × 2)** **– 1st item (for 3 × 3)**	**3.2.6** **3.2.7**
2.6	use inverse matrices in solving linear systems	**M3**	**30**	**– 2nd item**	**3.2.8**
3	**Differentiation**				
3.1	know the meaning of the terms limit, derivative, differentiable at a point, differentiable on an interval, derived function, second derivative	**M1**	**14**		**1.2.1**
3.2	use the notation: $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$	**M1**	**14**		**1.2.2**
3.3	recall the derivatives of x^α (α rational), $\sin x$ and $\cos x$	**M1**	**14**		**1.2.3**
3.4	know and use the rules for differentiating linear sums, products, quotients and composition of functions: $(f(x) + g(x))' = f'(x) + g'(x)$; $(kf(x))' = kf'(x)$, where k is a constant; the chain rule: $(f(g(x)))' = f'(g(x))g'(x)$; the product rule: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$; the quotient rule: $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$ **differentiate given functions which require more than one application of one or more of the chain rule, product rule and the quotient rule. [A/B]**	**M1**	**15**		**1.2.4**
3.5	know • the derivative of $\tan x$ • the definitions and derivatives of $\sec x$, $\operatorname{cosec} x$ and $\cot x$ • the derivatives of e^x ($\exp x$) and $\ln x$	**M1**	**17**		**1.2.5**

		Pack	*Page*	*Comment*	*AH Maths Content Ref*
3.6	apply differentiation to simple rates of change, e.g. rectilinear motion, optimisation problems	**M1**	**20**	**- Omit (b) – extrema of functions**	**1.2.8**
3.7	understand how a function can be defined parametrically	**M2**	**9**		**2.1.6**
3.8	understand simple applications of parametrically defined functions e.g. $x^2 + y^2 = r^2, x = r\cos\theta, y = r\sin\theta$	**M2**	**9**		**2.1.7**
3.9	use parametric differentiation to find first **and second derivatives [A/B]**, and apply to motion in a plane	**M2**	**9**		**2.1.8**
3.10	apply differentiation to related rates in problems where the functional relationship is given	**M2**	**11**		**2.1.9**
3.11	**solve practical related rates by first establishing a functional relationship between appropriate variables [A/B]**	**M2**	**11**		**2.1.10**
4	**Integration**				
4.1	know the meaning of the terms integrate, integrable, integral, indefinite integral, definite integral and constant of integration	**M1**	**23**		**1.3.1**
4.2	recall standard integrals of x^α ($\alpha \in \mathbf{Q}, \alpha \neq -1$), $\sin x$ and $\cos x$ and know the following $\int(af(x)+bg(x))dx = a\int f(x)dx + b\int g(x)dx, a, b \in \mathbf{R}$. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, a < c < b$ $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx, b \neq a$ $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$	**M1**	**23**		**1.3.2**
4.3	know the integrals of e^x, x^{-1}, $\sec^2 x$.	**M1**	**23**		**1.3.3**
4.4	integrate by substitution: expressions requiring a simple substitution candidates are expected to integrate simple functions on sight. expressions where the substitution will be given the following special cases of substitution $\int f(ax+b)dx$ $\int \frac{f'(x)}{f(x)}dx$	**M1**	**25**		**1.3.4**
4.5	apply integration, e.g. to the evaluation of areas; **volumes of simple solids of revolution (disc/washer method) [A/B]**	**M1**	**27**	**– Omit 'integration with respect to *y*' and '(ii) speed/time graph'**	**1.3.6**
4.6	integrate rational functions, both proper and improper, by means of partial fractions; the degree of the denominator being $\leqslant 3$; the denominator may include: (i) two separate or repeated linear factors (ii) **three linear factors [A/B]**	**M2**	**15**	**– (i) and (ii) only (Note: 4.6 does NOT include all of 2.2.1)**	**2.2.1**
4.7	integrate by parts with one application	**M2**	**17**		**2.2.2**
4.8	**integrate by parts involving repeated applications [A/B]**	**M2**	**17**		**2.2.3**
5	**Ordinary differential equations**				
5.1	know the definition of a differential equation and the meaning of the terms linear, order, general solution, arbitrary constants, integrating factor, particular solution, initial condition	**M2**	**19**		**2.2.4**
5.2	solve first order differential equations (variables separable)	**M2**	**19**		**2.2.5**
5.3	solve first order linear differential equations using the integrating factor method	**M3**	**61**		**3.4.1**
5.4	formulate a simple statement involving rate of change as a simple separable first order differential equation	**M2**	**22**	**(Note: 5.4 does NOT include all of 2.2.6)**	**2.2.6**
5.5	know the laws of growth and decay: applications in practical contexts	**M2**	**22**		**2.2.7**
5.6	find general solutions and solve initial value problems	**M3**	**61**		**3.4.2**

Appendix 3

Appendix 3 to the update letter of February 2004 is a Specimen Question Paper, for use in and after 2005, for the Course Applied Mathematics: Statistics (AH).

C202/SQP229

Advanced Higher Applied Mathematics: Statistics

Time: 3 hours

NATIONAL QUALIFICATIONS

Specimen Question Paper
for use in and after 2005

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

   Section A assesses the Units Statistics 1 and 2
   Section B assesses the Unit Mathematics for Applied Mathematics
3. **Full credit will be given only where the solution contains appropriate working.**
4. A booklet of Mathematical Formulae and Statistical Tables is supplied for all candidates. It contains Numerical Analysis formulae and Statistical formulae and tables.

SCOTTISH QUALIFICATIONS AUTHORITY

Answer all the questions.

SECTION A

Marks

1. Coloured cards are placed in three boxes as follows.

	Red	*White*	*Blue*
Box 1	3	4	3
Box 2	4	1	3
Box 3	2	3	5

A box is selected at random and a card drawn at random from it is found to be red.

Find the probability that Box 1 was selected. **5**

2. Scientists observed that the mean number of alpha particles emitted from a radioactive source during time intervals of 7·5 seconds was 4. Assuming that the situation can be modelled by a Poisson distribution, obtain the probability that during a randomly selected interval of 7·5 seconds the number of alpha particles emitted was

(*a*) zero, **1**

(*b*) four, **2**

(*c*) greater than four. **2**

What is the standard deviation of the number of alpha particles emitted during intervals of duration 7·5 seconds? **1**

3. In order to carry out a study of farms in a region, the farms were classified as large, medium and small. Local authority records revealed the following numbers of farms in each category.

Category of farm	*Number of farms in category*
Large	30
Medium	320
Small	150

Explain how you would set about taking a stratified random sample of 50 farms in the region, specifying the number of farms in each category which you would sample. **3**

State **two** advantages of stratified random sampling over simple random sampling. **2**

Marks

4. (*a*) Write down an expression for the 95% confidence interval for a population proportion p. **2**

(*b*) Show that the maximum value of $p(1 - p)$ is 0·25. **2**

(*c*) Use these results to show that the maximum width of such a 95% confidence interval is approximately 0·06 when the sample size is 1000. **2**

5. A market researcher believes that in a population of users of laptop computers 30% prefer manufacturer A, 60% prefer manufacturer B with the remainder preferring other manufacturers. A simple random sample of 600 users selected from the population showed the following preferences.

Manufacturer	A	B	Other
Number preferring this manufacturer	192	342	66

Do these data provide evidence to cast doubt on the researcher's belief? **5**

6. In a population of healthy adult males, systolic blood pressure, X (mmHg), is normally distributed with mean 140 and standard deviation 15. For the 99·9% normal range of systolic blood pressure in the population, physicians use the interval (a, b), with mid-point 140, such that $P(a < X < b) = 0{\cdot}999$. Calculate the values of a and b. **5**

7. A machine used for the automatic filling of breakfast cereal packs is known to deliver amounts of cereal with weights which are distributed with a standard deviation of 3 grams. When set for a production run, with packs labelled as 500 grams weight, the cereal manufacturer uses 505 grams as a target for the mean weight of the amounts. This is to ensure compliance with statutory regulations concerning under-filling of packs. During set-up, a trial run was carried out and a random sample of packs was found to have the following content weights.

509·9 507·7 506·7 507·5 506·7 506·1 507·0 506·5 505·2 510·5

Stating any assumption required, test the null hypothesis that the machine is operating "on target" using a two-tail test. **5**

Explain why it would not be in the manufacturer's interest to simply carry out a one-tail test with alternative hypothesis that the mean amount is less than 505. **1**

Marks

8. There is constant probability p that a computer "chip" produced at a factory is defective. State the distribution of X, the number of defective chips, in random samples of size n and show that the sample proportion of defective chips has mean p and variance pq/n. **4**

The production manager estimates that, for the current manufacturing process, p is 0·20 and she wishes to monitor the defective rate using daily random samples of 80 chips.

Find the probability that, with the specified proportion, a sample contains between 9 and 23 defective chips (inclusive), justifying, and showing clearly, your method. **6**

9. In the nineteenth century, the Scottish physicist J. D. Forbes investigated the feasibility of estimating altitude in the mountains by observing the temperature at which water boiled. The table below gives data he collected on atmospheric pressure, x (inches Hg), and the corresponding boiling point of water, y (°F), from 17 locations in Scotland and the Alps. The table also gives the residuals from fitting the least squares regression of $y = 155{\cdot}3 + 1{\cdot}902x$.

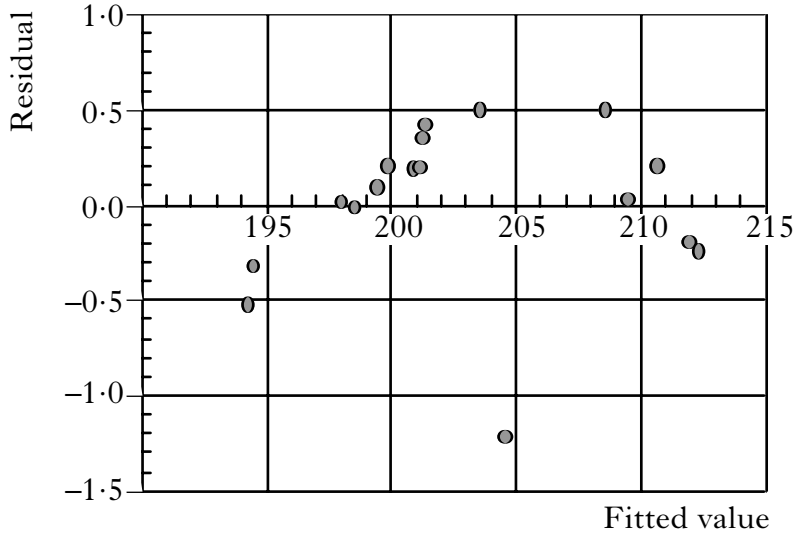
x	y	*Residual*
20·79	194·5	–0·335
20·79	194·3	–0·535
22·40	197·9	0·004
22·67	198·4	–0·010
23·15	199·4	0·077
23·35	199·9	0·197
23·89	200·9	0·170
23·99	201·1	0·180
24·02	201·4	0·423
24·01	201·3	0·342
25·14	203·6	0·493
26·57	204·6	–1·227
28·49	209·5	0·022
27·76	208·6	0·510
29·04	210·7	0·176
29·88	211·9	–0·222
30·06	212·2	–0·264

(*a*) State the underlying model. Given that the sum of the squares of the residuals is 2·960, obtain an estimate of σ^2 in the model. **2**

(*b*) Given that $S_{xx} = 145{\cdot}94$, test the null hypothesis that the slope parameter in the model is zero, stating any assumption required. **5**

9. (continued)

(*c*) A plot of residuals against fitted values is shown below. Comment. **2**



10. In an experiment, designed to investigate the effect of caffeine on manual dexterity, twenty male students were trained to carry out a finger-tapping exercise. Following the training, ten of the students were given a drink containing a 200 mg dose of caffeine, while the remaining students were given a similar drink containing no caffeine. Two hours later each student was observed carrying out the finger-tapping exercise and the following ordered data recorded.

Caffeine dose (mg)	*Number of finger-taps per minute*									
0	242	242	242	244	244	245	246	247	248	248
200	245	246	246	248	248	248	250	250	250	252

(*a*) State **two** procedures which would constitute good practice during the running of the experiment. **2**

(*b*) Analyse the data using the Mann-Whitney test. **7**

(*c*) Describe briefly an alternative design for the experiment and state how you would analyse the resulting data. **2**

Marks

SECTION B

11. Differentiate with respect to x

(a) $f(x) = (1 + 2x) \ln (1 + 2x)$, $x > -\frac{1}{2}$, **3**

(b) $g(x) = e^{\cot 2x}$, $0 < x < \frac{\pi}{2}$. **2**

12. Expand

$$\left(x^2 - \frac{1}{x}\right)^3, \quad x \neq 0$$

and simplify as far as possible. **3**

13. (a) Obtain partial fractions for

$$\frac{x}{x^2 - 1}, \qquad x > 1. \quad \textbf{2}$$

(b) Use the result of (a) to find

$$\int \frac{x^3}{x^2 - 1}\, dx, \quad x > 1. \quad \textbf{4}$$

14. Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 1 \\ 4 & -2 & -2 \\ -3 & 2 & 1 \end{pmatrix}.$$

Show that $AB = kI$ for some constant k, where I is the 3×3 identity matrix. Hence obtain (i) the inverse matrix A^{-1}, and (ii) the matrix A^2B. **4**

15. Obtain

$$\int 2x \sin 4x\, dx. \quad \textbf{4}$$

Marks

16. A chemical plant food loses effectiveness at a rate proportional to the amount present in the soil. The amount M grams of plant food effective after t days satisfies the differential equation

$$\frac{dM}{dt} = kM, \text{ where } k \text{ is a constant.}$$

(*a*) Find the general solution for M in terms of t where the initial amount of plant food is M_0 grams. **3**

(*b*) Find the value of k if, after 30 days, only half the initial amount of plant food is effective. **3**

(*c*) What percentage of the original amount of plant food is effective after 40 days? **2**

(*d*) The plant food has to be renewed when its effectiveness falls below 25%. Is the manufacturer of the plant food justified in calling its product "sixty day super food"? **2**

[*END OF SPECIMEN QUESTION PAPER*]