

# Scottish Qualifications Authority

## National Qualifications in Mathematics

Acceptable graphing calculator solutions to examination questions on

1. ROOTS & FACTORS OF POLYNOMIALS
2. RECURRENCE RELATIONS

This document extends the range of acceptable graphing calculator solutions to examination questions for National Qualifications in Mathematics to the topics of Roots & Factors of Polynomials and Recurrence Relations.

Earlier guidance is provided in the SQA document:-  
CLARIFICATION ON AMENDMENT TO RUBRIC-Updated and reissued in October 2002.

Issued December 2003

## FOREWORD

The Mathematics Update letter of November 2002 contained, as an Appendix, a reissue of an earlier document providing guidance on acceptable graphing calculator solutions to questions in examinations for National Qualifications in Mathematics. The update letter and the foreword to the appendix document both indicated that SQA would be issuing further illustrations of acceptable graphing calculator solutions to examination questions on a wider range of topics as they were approved by the Mathematics Advanced Calculator Working Group and the Mathematics and Statistics Assessment Panel.

This document is the first of this series of illustrative documents providing guidance on acceptable graphing calculator solutions to examination questions. This document covers the topics of roots and factors of polynomials and recurrence relations. SQA will be issuing further documents in due course.

Please note that this information does not constitute a change to the syllabus for Higher Mathematics. Graphing calculator solutions to examination questions are **not a requirement** of the Higher Mathematics course but are now considered, in certain instances, to be acceptable alternatives to the analytical solutions which appear in the marking Instructions for Paper 2. As at present, aspects of topics, where a calculator approach is not appropriate, will be examined in the non-calculator paper.

The guidance consists of examples of appropriate candidate responses and marking schemes which indicate the minimum working required for the award of each mark.

## GENERAL NOTES

As before, it is stressed that

- (a) full credit in examination questions will be given only where the solution contains appropriate working;
- (b) answers obtained by readings from scale drawings will not receive any credit.

Candidates will receive no credit for estimating values from the screen of a graphing calculator but will be given credit for a fully annotated sketch of a graph, clearly indicating relevant points obtained by interrogating the graph using calculator functions or "tracing".

### 1. ROOTS & FACTORS OF POLYNOMIALS

Finding roots using built-in calculation functions on the graphing calculator provides the expected "exact values". Using a "trace" function may not provide "exact values", however, where steps are taken to ensure that exact values are found and used, credit will be given.

Additional evidence is required to progress from roots of a family of polynomials to the factors of a particular polynomial.

### 2. RECURRENCE RELATIONS

In questions on recurrence relations where a limit is to be found, only an algebraic method is acceptable.

# 1. Roots & Factors of Polynomials

## Example 1

- (a) Show that  $x = 2$  is a root of  $f(x) = 2x^3 + x^2 - 13x + 6$   
 (b) Find the other roots.

4

### Traditional Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
2.1.1, 2.1.2	
ans: proof and $x = 2, x = -3, x = \frac{1}{2}$ 4 marks	
• <sup>1</sup> ss: synthetic division, long division or evaluation	• <sup>1</sup> $\begin{array}{r rrrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & & \\ \hline & 2 & 5 & -3 & 0 \end{array}$ or $f(2) = 2 \times 2^3 + \dots$ <b>explicit evidence of substitution required</b>
• <sup>2</sup> ic: complete proof	• <sup>2</sup> $\begin{array}{r} 2 \quad 2 \quad 1 \quad -13 \quad 6 \\ \quad \quad 4 \quad 10 \quad -6 \\ \quad \quad 2 \quad 5 \quad -3 \quad \underline{0} \end{array}$ or $f(2) = 16 + 4 - 26 + 6 = 0$
• <sup>3</sup> ic: state quadratic factor	• <sup>3</sup> $2x^2 + 5x - 3$
• <sup>4</sup> pd: state roots	• <sup>4</sup> $-3, \frac{1}{2}$

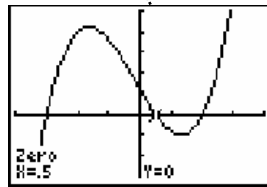
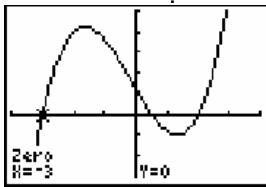
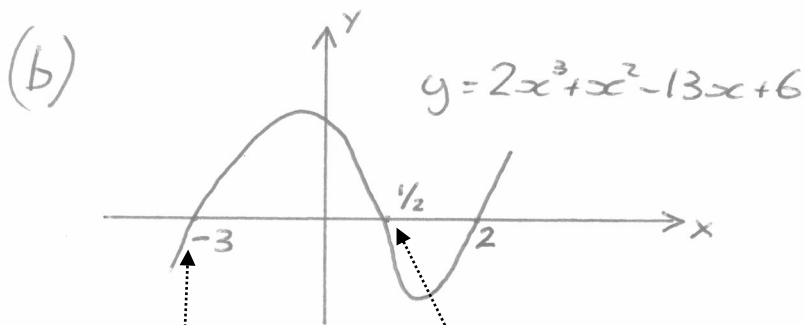
Graphing Calculator Solution

$$(a) \quad S(2) = 2 \times (2)^3 + 2^2 - 13 \times 2 + 6$$

$$= 0$$

$\therefore x = 2$  is a root

Plot1	Plot2	Plot3
Y1 = 2X^3 + X^2 - 13X + 6		
Y1(2)		0



$$S\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 13 \times \frac{1}{2} + 6$$

$$= 0$$

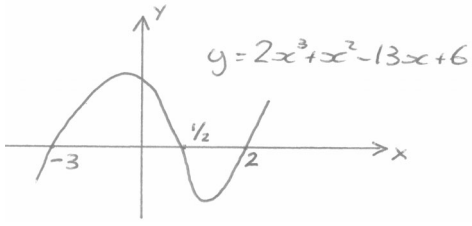
$$S(-3) = 2 \times (-3)^3 + (-3)^2 - 13 \times (-3) + 6$$

$$= 0$$

$\therefore$  OTHER ROOTS AT

$$\underline{\underline{x = \frac{1}{2}, \quad x = -3}}$$

Graphing Calculator Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
2.1.1, 2.1.2	
ans: proof and $x = 2, x = -3, x = \frac{1}{2}$ 4 marks	
• <sup>1</sup> ss: knowing to evaluate	• <sup>1</sup> $f(2) = 2 \times 2^3 \dots\dots\dots$ <b>explicit evidence of substitution required</b>
• <sup>2</sup> ic: complete evaluation	• <sup>2</sup> $f(2) = 2 \times 2^3 + \dots\dots\dots = 0$
• <sup>3</sup> ic: finding other roots and sketching annotated graph.	• <sup>3</sup>  <p>for a sketch of a cubic with the zeros indicated at 2, <math>\frac{1}{2}</math> and -3</p>
• <sup>4</sup> pd: state roots	• <sup>4</sup> -3, $\frac{1}{2}$

**Example 2**

- (a) Show that  $x + 2$  is a factor of  $2x^3 + 9x^2 + 7x - 6$ .  
 (b) Hence, or otherwise, factorise  $2x^3 + 9x^2 + 7x - 6$  completely.

**4**

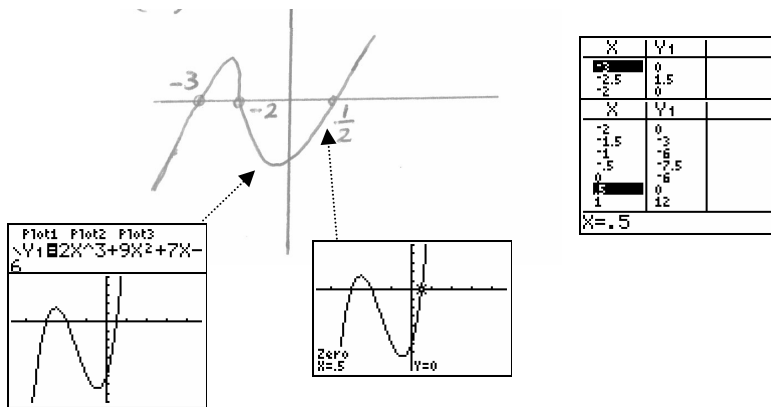
Traditional Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
<b>2.1.1, 2.1.3</b>	
<b>ans: proof and (x + 2)(x + 3)(2x - 1)      4 marks</b>	
<p>•<sup>1</sup> ss: synthetic division, long division or evaluation</p>	<p>•<sup>1</sup></p> $\begin{array}{r rrrrr} -2 & 2 & 9 & 7 & -6 \\ & & -4 & & \\ \hline & 2 & & & \end{array}$ <p>or</p> $f(-2) = 2 \times (-2)^3 \dots\dots\dots$
<p>•<sup>2</sup> ic: complete proof</p>	<p>•<sup>2</sup></p> $\begin{array}{r rrrrr} -2 & 2 & 9 & 7 & -6 \\ & & -4 & -10 & 6 \\ \hline & 2 & 5 & -3 & \underline{0} \end{array}$ <p>or</p> $f(-2) = -16 + 36 - 14 - 6 = 0$
<p>•<sup>3</sup> ic: state quadratic factor</p>	<p>•<sup>3</sup></p> $2x^2 + 5x - 3$
<p>•<sup>4</sup> pd: factorise fully</p>	<p>•<sup>4</sup></p> $(x + 2)(x + 3)(2x - 1) \text{ stated explicitly}$

Graphing Calculator Solution

$$\begin{aligned}
 (a) \quad f(-2) &= 2(-2)^3 + 9(-2)^2 + 7(-2) - 6 \\
 &= -16 + 36 - 14 - 6 \\
 &= \underline{\underline{0}}
 \end{aligned}$$

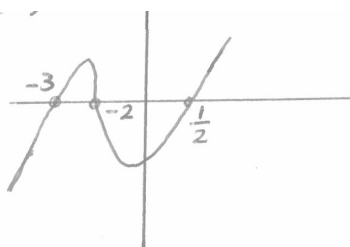
so  $(x+2)$  is a factor



$$\begin{aligned}
 2x^3 + 9x^2 + 7x - 6 &= k(x+2)(x+3)(x-\frac{1}{2}) \\
 \text{LET } x &= 0 \text{ (NOT A ROOT)} \\
 2(0)^3 + 9(0)^2 + 7(0) - 6 &= k(0+2)(0+3)(0-\frac{1}{2}) \\
 -6 &= -3k \\
 k &= 2
 \end{aligned}$$

FACTORS ARE  $(x+2)(x+3)(2x-1)$

Graphing Calculator Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
<b>2.1.1, 2.1.3</b>	
<b>ans: proof and (x + 2)(x + 3)(2x - 1)      4 marks</b>	
• <sup>1</sup> ss: knowing to evaluate	• <sup>1</sup> $f(-2) = 2 \times (-2)^3 \dots\dots\dots$ <b>explicit evidence of substitution required</b>
• <sup>2</sup> ic: complete evaluation	• <sup>2</sup> $f(-2) = 2 \times (-2)^3 + \dots\dots\dots = 0$
• <sup>3</sup> ic: finding other roots and sketching annotated graph.	• <sup>3</sup>  <p>for a sketch of a cubic with the zeros indicated at -3, -2, and <math>\frac{1}{2}</math>.</p>
• <sup>4</sup> pd: converting found roots into correct factors	• <sup>4</sup> $f(x) = k(x + 2)(x + 3)(x - \frac{1}{2})$ by comparing leading terms, for example, of $f(x) = k(x + 2)(x + 3)(x - \frac{1}{2})$ and $f(x) = 2x^3 + 9x^2 + 7x - 6$ gives $k = 2$  OR  evaluating at $x = 0$ , for example, $f(0) = 2(0)^3 + 9(0)^2 + 7(0) - 6 = -6$ and $f(0) = k(0 + 2)(0 + 3)(0 - \frac{1}{2}) = -6$ gives $k = 2$  so $f(x) = (x + 2)(x + 3)(2x - 1)$ <b>explicitly stated</b>

**Example 3**

- (a) Given that  $x + 2$  is a factor of  $2x^3 + x^2 + kx + 2$ , find the value of  $k$ . 3  
 (b) Hence solve the equation  $2x^3 + x^2 + kx + 2 = 0$  when  $k$  takes this value. 2

Traditional Marking Scheme

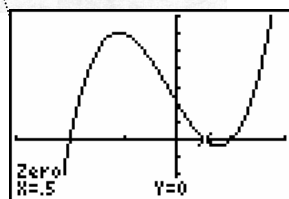
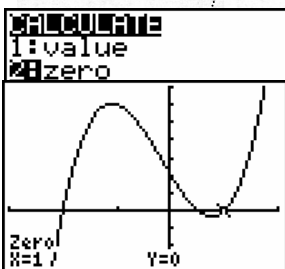
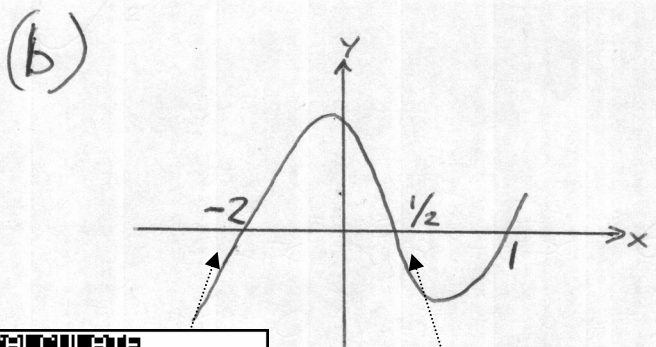
Give 1 mark for each •	Illustrations for awarding each •																																																																						
<b>2.1.2</b>																																																																							
<b>ans: <math>k = -5</math> and <math>x = -2, x = \frac{1}{2}, x = 1</math>      5 marks</b>																																																																							
(a)	<p><u>Method 1</u></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(-2) = 2 \times (-2)^3 + \dots\dots</math></li> <li>•<sup>2</sup> <math>2(-2)^3 + (-2)^2 - 2k + 2</math></li> <li>•<sup>3</sup> <math>k = -5</math></li> </ul> <p><u>Method 2</u></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <table style="margin-left: 20px;"> <tr><td>-2</td><td>2</td><td>1</td><td>k</td><td>2</td></tr> <tr><td></td><td></td><td></td><td></td><td>-4</td></tr> <tr><td></td><td>2</td><td>-3</td><td></td><td></td></tr> </table> </li> <li>•<sup>2</sup> <table style="margin-left: 20px;"> <tr><td>-2</td><td>2</td><td>1</td><td>k</td><td>2</td></tr> <tr><td></td><td></td><td></td><td></td><td>-4</td></tr> <tr><td></td><td></td><td></td><td>6</td><td>-2(k+6)</td></tr> <tr><td></td><td>2</td><td>-3</td><td>k+6</td><td>-2(k+6)+2</td></tr> </table> </li> <li>•<sup>3</sup> <math>k = -5</math></li> </ul> <p><u>Method 3</u></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <table style="margin-left: 20px;"> <tr><td>-2</td><td>2</td><td>1</td><td>k</td><td>2</td></tr> <tr><td></td><td></td><td></td><td></td><td>-4</td></tr> <tr><td></td><td></td><td></td><td>6</td><td></td></tr> <tr><td></td><td>2</td><td>-3</td><td></td><td></td></tr> </table> </li> <li>•<sup>2</sup> <table style="margin-left: 20px;"> <tr><td>-2</td><td>2</td><td>1</td><td>k</td><td>2</td></tr> <tr><td></td><td></td><td></td><td></td><td>-2</td></tr> <tr><td></td><td></td><td></td><td>1</td><td>0</td></tr> </table> </li> <li>•<sup>3</sup> <math>k = -5</math></li> </ul> <p>(b)</p> <ul style="list-style-type: none"> <li>•<sup>4</sup> find quadratic factor</li> <li>•<sup>5</sup> process</li> </ul> <p>•<sup>4</sup> <math>2x^2 - 3x + 1</math> or <math>2x^2 + 3x - 2</math> or <math>x^2 + x - 2</math>          •<sup>5</sup> <math>(2x - 1)(x - 1)</math> or <math>(2x - 1)(x + 2)</math> or <math>(x + 2)(x - 1)</math>          and <math>x = -2, \frac{1}{2}, 1</math></p>	-2	2	1	k	2					-4		2	-3			-2	2	1	k	2					-4				6	-2(k+6)		2	-3	k+6	-2(k+6)+2	-2	2	1	k	2					-4				6			2	-3			-2	2	1	k	2					-2				1	0
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Graphing Calculator Solution

$$(a) \quad \begin{aligned} f(-2) &= 2(-2)^3 + (-2)^2 + k(-2) + 2 \\ &= -10 - 2k \end{aligned}$$

$$\begin{aligned} \text{If } (x+2) \text{ is a factor } f(-2) &= 0 \\ \therefore -10 - 2k &= 0 \\ \underline{\underline{k &= -5}} \end{aligned}$$

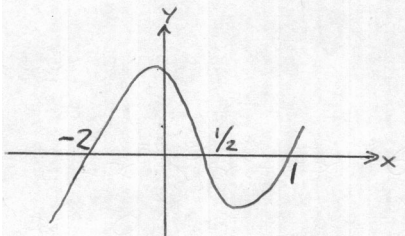
Plot1	Plot2	Plot3
Y1 = 2X^3 + X^2 + KX + 2		
0+K		0.00
Y1(-2)		-10.00



$$\begin{aligned} f(-2) &= 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0 \\ f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 = 0 \\ f(1) &= 2(1)^3 + (1)^2 - 5(1) + 2 = 0 \end{aligned}$$

$$\therefore \underline{\underline{x = -2, \frac{1}{2}, 1}}$$

Graphing Calculator Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
2.1.2	
ans: $k = -5$ and $x = -2, x = \frac{1}{2}, x = 1$ <b>5 marks</b>	
• <sup>1</sup> ss: knowing to evaluate	• <sup>1</sup> $f(-2) = 2 \times (-2)^3 \dots\dots\dots$ <b>explicit evidence of substitution required</b>
• <sup>2</sup> ic: complete evaluation	• <sup>2</sup> $-10 - 2k$
• <sup>3</sup> pd: process	• <sup>3</sup> $k = -5$
• <sup>4</sup> ic: sketching annotated graph.	• <sup>4</sup>  <p>for a sketch of a cubic with the zeros indicated at <math>-2, \frac{1}{2}</math> and <math>1</math></p>
• <sup>5</sup> pd: confirming roots by evaluation	• <sup>5</sup> $f(-2) = 2 \times (-2)^3 + (-2)^2 - 5(-2) + 2 = 0$ $f(\frac{1}{2}) = 2 \times (\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(\frac{1}{2}) + 2 = 0$ $f(1) = 2 \times (1)^3 + (1)^2 - 5(1) + 2 = 0$ <b>all explicitly stated</b>  so $x = -2, x = \frac{1}{2}, x = 1$

**Example 4**

Express  $x^4 - x$  in its fully factorised form

4

**Marking Scheme**

<b>Give 1 mark for each •</b>	<b>Illustrations for awarding each •</b>
<b>2.1.3</b>	
<b>ans: <math>x(x-1)(x^2+x+1)</math> 4 marks</b>	
• <sup>1</sup> process	• <sup>1</sup> $x(x^3 - 1)$
• <sup>2</sup> process	• <sup>2</sup> synthetic division or evaluation of $f(k)$
• <sup>3</sup> process	• <sup>3</sup> linear factor = $(x - 1)$
• <sup>4</sup> communication	• <sup>4</sup> full factorisation $x(x-1)(x^2+x+1)$

## Graphing Calculator Solution

7  $y = x^4 - x$

$f(0) = 0$   
 $f(1) = 0$

$\therefore x$  and  $(x-1)$  are factors

FUNCTION ONLY HAS TWO ROOTS SO ONLY TWO LINEAR FACTORS.

$$x(x^3 - 1)$$

$$\frac{x^3 - 1}{x - 1}$$

1	1	0	0	-1
		1	1	1
<hr/>				
	1	1	1	0

$$(x^2 + x + 1)$$

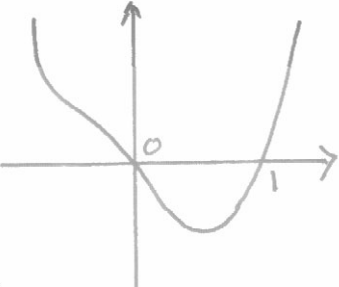
$\therefore \underline{\underline{x(x-1)(x^2+x+1)}}$

### Note of explanation

In this example although both roots can be obtained from the Graphing Calculator, candidates would be expected to comment that from the graph they can see that there are only two roots and hence only two linear factors.

In order to obtain the quadratic factor candidates would require to divide either algebraically or synthetically.

Graphing Calculator Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
2.1.3	
ans: $x(x-1)(x^2+x+1)$ 4 marks	
<p>•<sup>1</sup> for sketch fully annotated</p>	<p>•<sup>1</sup></p> 
<p>•<sup>2</sup> process <math>f(1) = 0 \Rightarrow (x-1)</math> is a factor</p>	<p>•<sup>2</sup> <math>(x-1)</math> is a factor <b>stated explicitly</b></p>
<p>•<sup>3</sup> process to find quadratic</p>	<p>•<sup>3</sup></p> $\begin{array}{cccccc} 1 & 1 & 0 & 0 & -1 & \\ & & 1 & 1 & 1 & \\ & & 1 & 1 & 1 & 0 \end{array}$ <p>and no further factorisation <b>stated explicitly</b></p>
<p>•<sup>4</sup> full factorisation and communication</p>	<p>•<sup>4</sup> full factorisation <math>x(x-1)(x^2+x+1)</math></p>

## 2. Recurrence Relations

### Example 1

On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

- (a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

Let  $u_n$  and  $u_{n+1}$  represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving  $u_n$  and  $u_{n+1}$

2

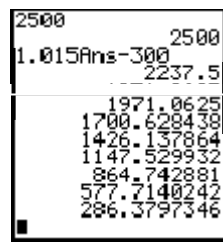
- (b) Find the date and the amount of the final payment.

4

### Traditional Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
<b>1.4.3</b>	
(a) • <sup>1</sup> interpret 1.5%	• <sup>1</sup> 1.015 stated or implied by start of (b)
• <sup>2</sup> state recurrence relation	• <sup>2</sup> $u_{n+1} = 1.015u_n - 300$ and initial value ( $u_0 = 2500$ ) stated or implied by start of (b)
(b) • <sup>3</sup> use recurrence relation	• <sup>3</sup> $u_1$ ie £2237.50
• <sup>4</sup> process	• <sup>4</sup> $u_2$ and $u_3$ ie £1971.06 and £1700.63
• <sup>5</sup> state final date	• <sup>5</sup> December or £286.38
• <sup>6</sup> process final payment	• <sup>6</sup> £290.68 for December payment

Graphing Calculator Solution



(a)  $u_{n+1} = 1.015u_n - 300$

(b)

$$u_1 = 2237.5$$

$$u_2 = 1971.06$$

$$u_3 = 1700.62$$

$$u_4 = 1426.13$$

$$u_5 = 1147.52$$

$$u_6 = 864.74$$

$$u_7 = 577.71$$

$$u_8 = 286.37.$$

$u_8$  represents Nov  
 Final payment Made  
 in December  
 will be  $286.37 + \text{interest}$   
 $286.37 \times 0.015 = 4.30$   
 Final Payment  
 $290.67.$

Graphing Calculator Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
<b>1.4.3</b>	
(a) • <sup>1</sup> interpret 1.5%	• <sup>1</sup> 1.015 stated or implied by start of (b)
• <sup>2</sup> state recurrence relation	• <sup>2</sup> $u_{n+1} = 1.015u_n - 300$ and initial value ( $u_0 = 2500$ ) stated or implied by start of (b)
(b) • <sup>3</sup> use recurrence relation	• <sup>3</sup> $u_1$ ie £2237.50
• <sup>4</sup> process	• <sup>4</sup> $u_1 = 2237.5$ $u_5 = 1147.52$ $u_2 = 1971.06$ $u_6 = 864.74$ $u_3 = 1700.62$ $u_7 = 577.71$ $u_4 = 1426.13$ $u_8 = 286.37.$
• <sup>5</sup> state final date	• <sup>5</sup> final payment will be in December
• <sup>6</sup> process final payment	• <sup>6</sup> £290.68 for December payment

**Example 2**

Two sequences are defined by these recurrence relations:

$$u_{n+1} = 3u_n - 0.4 \quad \text{with } u_0 = 1$$

$$v_{n+1} = 0.3v_n + 4 \quad \text{with } v_0 = 1$$

- (a) Explain why only one of these sequences approaches a limit as  $n \rightarrow \infty$  1  
 (b) Find algebraically the exact value of the limit. 2  
 (c) For the other sequence, find  
     (i) the smallest value of  $n$  for which the  $n^{\text{th}}$  term exceeds 1000, and  
     (ii) the value of that term. 2

**Traditional Marking Scheme**

Give 1 mark for each •	Illustrations for awarding each •
1.4.4 1.4.5 1.4.3	
(a) • <sup>1</sup> communication	• <sup>1</sup> only $v_n$ has limit because $-1 < 0.3 < 1$
(b) • <sup>2</sup> strategy for finding the limit	• <sup>2</sup> e.g. use $L = aL + b$
• <sup>3</sup> process	• <sup>3</sup> $L = \frac{40}{7}$
(c) • <sup>4</sup> strategy	• <sup>4</sup> evaluate enough terms to exceed 1000
• <sup>5</sup> process	• <sup>5</sup> $u_7 = 1749.8$

### Graphing Calculator Solution

(a) Only  $V_{n+1} = 0.3V_n + 4$  has a limit since  $-1 < 0.3 < 1$

(b) at limit  $V_{n+1} = V_n = L$

$$\therefore L = 0.3L + 4.$$

$$L(1 - 0.3) = 4.$$

$$L = \frac{4}{0.7}$$

$$= \frac{40}{7}.$$

#### Note of explanation

Candidates are expected to determine the limit of a Recurrence Relation algebraically.

Numerical determination of the limit through repeatedly pressing the [Enter] key, although useful in the teaching of the concept is not sufficiently rigorous to be worthy of examination credit.

(c) From Calculator

$$u_1 = 2.6$$

$$u_2 = 7.4$$

$$u_3 = 21.8$$

$$u_4 = 65$$

$$u_5 = 194.6$$

$$u_6 = 583.4$$

$$u_7 = 1749.8$$

When  $n=7$  sequence exceeds 1000.

3Ans=0.4	1
	2.6
	7.4
	21.8
	65
	194.6

	2.6
	7.4
	21.8
	65
	194.6
	583.4
	1749.8

Graphing Calculator Marking Scheme

Give 1 mark for each •	Illustrations for awarding each •
1.4.4 1.4.5 1.4.3	
(a) • <sup>1</sup> interpret 0.3	• <sup>1</sup> only $v_n$ has limit because $-1 < 0.3 < 1$
(b) • <sup>2</sup> state limit condition	• <sup>2</sup> at limit $u_{n+1} = u_n$
• <sup>3</sup> process	• <sup>3</sup> $L = \frac{40}{7}$ <b>do not accept</b> $L = 5.7142\dots\dots$
(c) • <sup>4</sup> process	• <sup>4</sup> $u_1 = 2.6$ $u_2 = 7.4$ $u_3 = 21.8$ $u_4 = 65$ $u_5 = 194.6$ $u_6 = 583.4$ $u_7 = 1749.8$
• <sup>5</sup> state $n = 7$	• <sup>5</sup> $u_7 = 1749.8$