

NQ
NATIONAL
QUALIFICATIONS

Objective Test **Higher Mathematics**

Objective Test

Higher Mathematics

Published date: May 2007
Publication code: BB3790

Published by the Scottish Qualifications Authority
The Optima Building, 58 Robertson Street, Glasgow G2 8DQ
Ironmills Road, Dalkeith, Midlothian EH22 1LE

www.sqa.org.uk

The information in this publication may be reproduced in support of SQA qualifications. If it is reproduced, SQA should be clearly acknowledged as the source. If it is to be used for any other purpose, then written permission must be obtained from the Publishing Team at SQA. It must not be reproduced for trade or commercial purposes.

For an up-to-date list of prices visit the **Publication Sales and Downloads** section of SQA's website.

This document can be produced, on request, in alternative formats, including large type, braille and numerous community languages.

For further details telephone SQA's Customer Contact Centre on 0845 279 1000.

SQA is committed to using plain English. We will try to make our publications as easy and straightforward to understand as we can, and will try to avoid all unnecessary jargon. If there is any language in this document that you feel is hard to understand, or could be improved, please write to Editor, Publishing Team, at the Glasgow address above or e-mail: **editor@sqa.org.uk**.

From 2008 onwards:

Paper 1 of the Higher Mathematics examination will

- Last for 1 hour 30 minutes
- Still be non-calculator
- Be split into two sections, sections A and B
- Section A will contain 20 objective questions worth 2 marks each, ie a total of 40 marks
- Section B will contain approximately three to five questions and be worth a total of 30 marks.

Paper 2 of the examination will

- Last for 1 hour 10 minutes
- Allow the use of a calculator
- Be worth a total of 60 marks.

This booklet has been written to let you see examples of the kind of questions that you will be asked, as it is possible that some of you will not be familiar with objective testing. It also contains advice on how to answer objective questions.

Objective Questions

An objective question offers you a choice of four answers. You have to choose the one you think is correct.

These questions are answered on a special sheet. Each answer sheet contains spaces for giving the answer A, B, C or D for each question.

It will look like this:

	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
etc				

A complete answer sheet is shown at the end of this booklet.

Recording your answer

Suppose for question 1 you decide the correct answer is given by A. You would record this by drawing a horizontal line with an **HB pencil** in box A for question 1. Your answer sheet would look like this:

	A	B	C	D
	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

It is very important that you check that you are using the correct row and the correct column.

Changing your answer

After you have recorded an answer you may decide that you have made an error and wish to make a change. If so:

- you should cancel the original answer by completely erasing your first choice, and
- if you now think that D is the correct answer then your answer sheet would now look like this:

	A	B	C	D
	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>



Working

In the majority of questions you will need to do some working before making your choice. Although this working will **not** be marked, it would be sensible to number the working and write neatly in case you need to go back and check it.

Method of Marking

Each correct answer is worth 2 marks. Each incorrect answer is worth 0 marks. It is to your advantage to answer every objective question although you may not be absolutely sure of the correct answer.

There is only one correct answer for each objective question. If you give two or more answers to any one question you will be marked wrong.

Types of Objective Questions

There are many different types of objective questions but only the following types will be used in the Higher Mathematics examination.

1 Direct question

example

A curve has equation $y = 4x^3 - 15x^2$. What is the gradient of the tangent at the point where $x = 2$?

- A 4
- B -12
- C -18
- D -24

Your “working” might look like this:

$$\frac{dy}{dx} = 12x^2 - 30x$$

$$m = 12 \times 2^2 - 30 \times 2 = 48 - 60 = -12$$

and you would therefore select answer B.

2 Direct question : options in a table

When there are two parts to an answer, the options may be listed in a table.

example

The graph with equation $y = pe^{qx}$ passes through the points (0, 1) and (1, 4).

What are the values of p and q ?

	p	q
A	1	$\log_e 4$
B	1	$4e$
C	$\frac{1}{e}$	$1 + \log_4$
D	$\frac{1}{e}$	$4e$

Your “working” might look like this:

$$\begin{aligned}(0, 1) &\Rightarrow 1 = p \times e^0 & (1, 4) &\Rightarrow 4 = 1 \times e^q \\ &1 = p \times 1 & &e^q = 4 \\ &\text{so } p = 1 & &q = \log_e 4\end{aligned}$$

and you would therefore select answer A.

3 Direct instruction

example

Find $\int \frac{2x^3 - 1}{x^2} dx$.

A $x^2 + x^{-1} + c$

B $3x - 3x^{-2} + c$

C $2x - 1 + c$

D $3x + c$

Your “working” might look like this:

$$\frac{2x^3 - 1}{x^2} = \frac{2x^3}{x^2} - \frac{1}{x^2} = 2x - x^{-2}$$

$$\int (2x - x^{-2}) dx = x^2 + x^{-1} + c$$

and you would therefore select answer A.

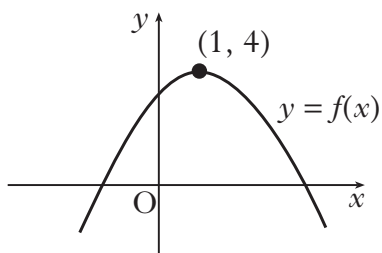
The majority of objective questions can be answered without any reference to the choices given by A, B, C or D.

Occasionally you will come across a question which makes direct reference to the choices. Often this will be a graphical question as shown in the next example.

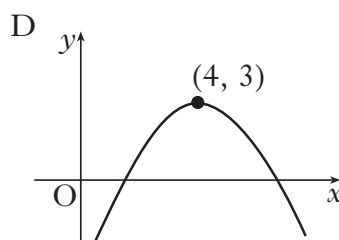
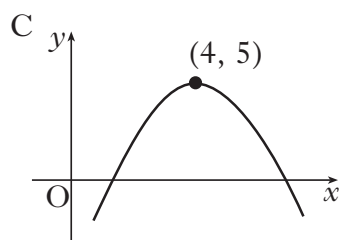
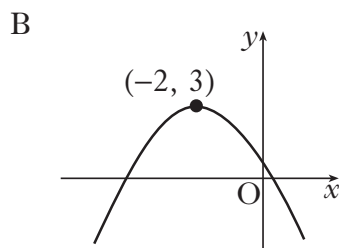
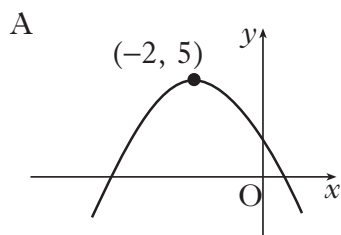
4 Choice reference

example

The diagram shows a sketch of the graph of $y = f(x)$.



Which of the diagrams below is most likely to show a sketch of the graph of $y = 1 + f(x - 3)$?



Your “working” might look like this:

The “ $f(x - 3)$ ” means translate graph 3 units to the right so $(1, 4)$ goes to $(4, 4)$.

The “ $+ 1$ ” means translate the graph 1 unit upwards.

So $(4, 4)$ goes to $(4, 5)$.

After examining the four choices you would therefore select C.

5 Simple multiple completion

example

Here are two statements about the equation $x^2 + 3kx + k^2 = 0$, where $k > 0$, k is real:

(1) the roots are real (2) the roots are equal.

Which one of the following is true?

- A Neither statement is correct
- B Only statement (1) is correct
- C Only statement (2) is correct
- D Both statements are correct

Your “working” might look like this:

$$b^2 - 4ac = (3k)^2 - 4 \times 1 \times k^2 = 9k^2 - 4k^2 = 5k^2$$

so roots are real but not equal.

You would therefore select answer B.

Preparation for the Objective Questions

To help you become familiar with the kind of questions you may be asked, this booklet includes a set of 20 specimen questions and you should time yourself, allowing about 40 to 45 minutes to complete them.

The correct answers, together with the possible working that you could do, are given at the end of the booklet.

Remember

- 1 Use an HB pencil for the objective question answer sheet.
- 2 Work steadily through the paper. Do not spend too long on any one question.
- 3 Be careful to mark the response in the correct row and column on the answer grid.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

Practice Questions

1. What is the integral of $(2x + 3)(2x - 5)$ with respect to x ?

A $\frac{4}{3}x^3 - 2x^2 - 15x + c$

B $x^4 - 2x^3 - 15x^2 + c$

C $8x - 4 + c$

D $8x^3 - 15x^2 + c$

2. What is the derivative of $\frac{6x^3 - 1}{3x}$ with respect to x ?

A $4x - 3x^{-2}$

B $4x + \frac{1}{3}x^{-2}$

C $2x^2 - 1$

D $6x^2$

3. \mathbf{u} and \mathbf{v} are vectors with components $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ respectively.

If t° is the angle between \mathbf{u} and \mathbf{v} , what is the value of $\cos t^\circ$?

A 0

B $-\sqrt{3}$

C $\frac{1}{2}$

D $-\frac{2}{\sqrt{6}}$

4. A curve has equation $y = 3x^2 - 7x - 2$.

What is the gradient of the tangent at the point where $x = 3$?

A 3

B 4

C 9

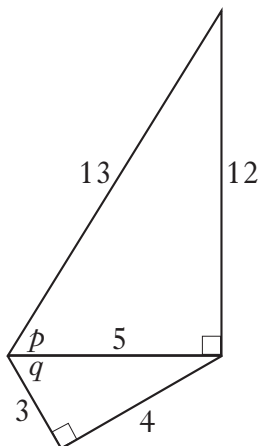
D 11

5. A circle with centre $(-1, 5)$ passes through the point $(2, 1)$.

What is the equation of the circle?

- A $(x + 1)^2 + (y - 5)^2 = 25$
B $(x - 1)^2 + (y - 5)^2 = 37$
C $(x + 1)^2 + (y - 5)^2 = 37$
D $(x - 1)^2 + (y + 5)^2 = 25$

6. The diagram shows two right-angled triangles with lengths as indicated.



What is the value of $\cos(p + q)$?

- A $-\frac{14}{65}$
B $\frac{63}{65}$
C $-\frac{33}{65}$
D $\frac{64}{65}$
7. $3x^2 + 6x - 10$ is expressed in the form $3(x + p)^2 + q$.
What is the value of q ?
- A -9
B -10
C -13
D -16

8. A sequence is defined by the recurrence relation $u_{n+1} = au_n + 3$ with $u_0 = 5$.

Find an expression, in terms of a , for u_2 .

- A $5a^2 + 3a + 3$
- B $10a + 30$
- C $5a^2 + 15$
- D $2a + 3$

9. Here are two statements about the functions $f(x) = \frac{2}{x}$ and $g(x) = -2x$:

(1) $f(x) = \frac{1}{g(x)}$

(2) $f'(1) = g'(1)$.

Which one of the following is true?

- A Neither statement is correct
- B Only statement (1) is correct
- C Only statement (2) is correct
- D Both statements are correct

10. k and a are given by

$$k \sin a = 3 \text{ and } k \cos a = -2 \text{ where } k > 0 \text{ and } 0 \leq a \leq \pi.$$

What is the value of k and the range for a ?

	k	Range of a
A	$\sqrt{13}$	$\frac{\pi}{2} < a < \pi$
B	$\sqrt{13}$	$0 < a < \frac{\pi}{2}$
C	$\sqrt{5}$	$\frac{\pi}{2} < a < \pi$
D	$\sqrt{5}$	$0 < a < \frac{\pi}{2}$

11. The point P(3, 1) lies on the circle with equation $(x - 2)^2 + (y - 3)^2 = 5$.

What is the gradient of the tangent at P?

A $\frac{1}{3}$

B $\frac{1}{2}$

C 1

D 2

12. A sequence is generated by the recurrence relation $u_{n+1} = 0.6u_n + 5$.

What is the limit of this sequence as $n \rightarrow \infty$?

A 5

B $\frac{3}{25}$

C $\frac{2}{5}$

D $\frac{25}{2}$

13. Find $\int 3\cos 4x dx$.

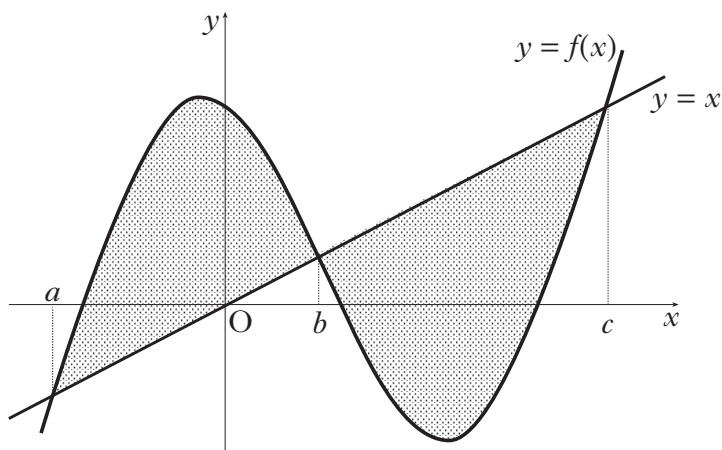
A $12\cos 4x + c$

B $\frac{3}{5}\sin 5x + c$

C $\frac{3}{4}\sin 4x + c$

D $\frac{3}{2}\cos^2 4x + c$

14. The diagram shows part of the graphs of a function with equation $y = f(x)$ and a line with equation $y = x$.



Which of the following gives the value of the shaded area?

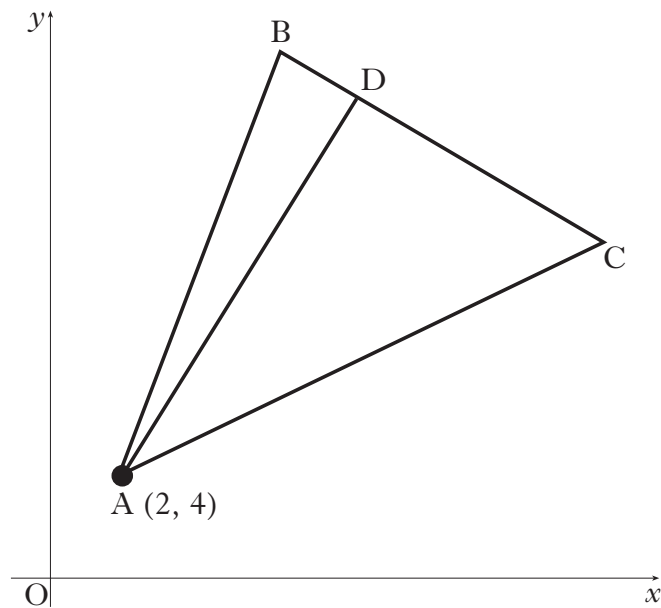
- A $\int_a^b (f(x) - x) dx + \int_b^c (x - f(x)) dx$
- B $\int_a^c (f(x) - x) dx$
- C $\int_a^b (f(x) - x) dx - \int_b^c (x - f(x)) dx$
- D $\int_a^c (x - f(x)) dx$
15. Functions f and g are given by $f(x) = 3x^2 - 1$ and $g(x) = x^2 + 2$.
Find an expression for $f(g(x))$.
- A $4x^2 + 1$
- B $9x^4 + 1$
- C $3x^4 + 5x^2 - 2$
- D $3x^4 + 12x^2 + 11$
16. Which of the following expressions is equal to $2 \cos\left(x + \frac{2\pi}{3}\right)$?
- A $2 \cos x - 1$
- B $2 \cos x + 1$
- C $\sqrt{3} \cos x + \sin x$
- D $-\cos x - \sqrt{3} \sin x$

17. Two circles have equations $x^2 + y^2 = 16$ and $(x - 2)^2 + y^2 = 4$.

Which of the following correctly describes the relative position of the two circles?

- A The circles do not touch or intersect
- B The two circles touch internally
- C The two circles intersect
- D The two circles touch externally

18. The diagram shows triangle ABC with altitude AD. A has coordinates (2, 4) and the gradient of BC is $-\frac{1}{3}$.



What is the equation of the altitude?

- A $3x - y - 2 = 0$
- B $3x - y - 14 = 0$
- C $x + 3y - 14 = 0$
- D $x - 3y + 2 = 0$

19. A, B and C have coordinates (1, -2, 4), (5, 4, -2) and (7, 7, -5) respectively.

Here are two statements about the points A, B and C:

(1) A, B and C are collinear

(2) $|\vec{AC}| : |\vec{BC}| = 2 : 1$.

Which of the following is true?

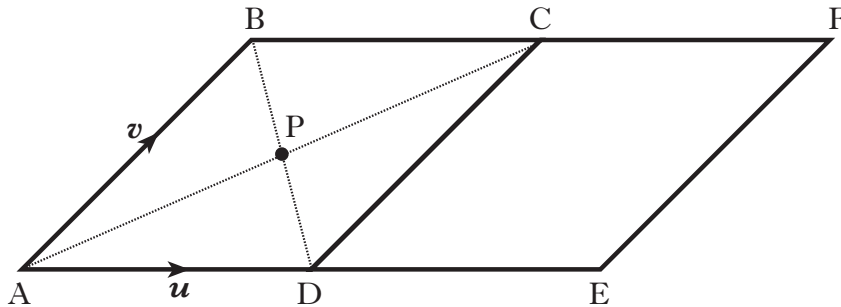
A Neither statement is correct

B Only statement (1) is correct

C Only statement (2) is correct

D Both statements are correct

20. The diagram shows two congruent parallelograms. Vectors \mathbf{u} and \mathbf{v} are represented by \vec{AD} and \vec{AB} .



Express \vec{EP} in terms of \mathbf{u} and \mathbf{v} .

A $-\frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}$

B $\frac{3}{2}\mathbf{u} - \frac{1}{2}\mathbf{v}$

C $-\frac{3}{2}\mathbf{u} - \frac{1}{2}\mathbf{v}$

D $\frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}$

Answers to practice questions

1 *Your working might look like:*

$$(2x + 3)(2x - 5) = 4x^2 - 4x - 15$$

$$\text{integrating gives } \frac{4}{3}x^3 - 2x^2 - 15x + c$$

so choose A

2 *Your working might look like:*

$$\frac{6x^3 - 1}{3x} = 2x^2 - \frac{1}{3}x^{-1}$$

$$\text{differentiating gives } 4x + \frac{1}{3}x^{-2}$$

so choose B

3 *Your working might look like:*

$$|\mathbf{u}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}, |\mathbf{v}| = 1, \mathbf{u} \cdot \mathbf{v} = 0 - 2 + 0$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = -\frac{2}{\sqrt{6}}$$

so choose D

4 *Your working might look like:*

$$\frac{dy}{dx} = 6x - 7$$

$$m_{x=3} = 6 \times 3 - 7 = 11$$

so choose D

5 *Your working might look like:*

$$\text{radius} = \sqrt{(-1-2)^2 + (5-1)^2} = \sqrt{25} = 5$$

$$(x+1)^2 + (y-5)^2 = 25$$

so choose A

6 *Your working might look like:*

$$\cos(p + q) = \cos p \cos q - \sin p \sin q$$

$$= \frac{5}{13} \times \frac{3}{5} - \frac{12}{13} \times \frac{4}{5} = \frac{15 - 48}{65} = -\frac{33}{65}$$

so choose C

7 *Your working might look like:*

$$3(x + p)^2 + q = 3x^2 + 6xp + 3p^2 + q$$

comparing coefficients

$$6p = 6, p = 1 \text{ and } 3p^2 + q = -10, q = -13$$

so choose C

8 *Your working might look like:*

$$u_1 = au_0 + 3 = 5a + 3$$

$$u_2 = au_1 + 3 = a(5a + 3) + 3 = 5a^2 + 3a + 3$$

so choose A

9 *Your working might look like:*

$$\frac{1}{g(x)} = (-2x)^{-1} = -\frac{1}{2x} \neq f(x) \text{ so (1) not correct}$$

$$f(x) = 2x^{-1}, f'(x) = -2x^{-2}, f'(1) = -2 \times 1 = -2$$

$$g(x) = -2x, g'(x) = -2, g'(1) = -2 \text{ so (2) correct}$$

so choose C

10 *Your working might look like:*

$$k^2 = 3^2 + (-2)^2 = \sqrt{13}$$

sin is positive, cos is negative, so 2nd quadrant

so choose A

11 *Your working might look like:*

$$m_{rad} = \frac{3-1}{2-3} = -2$$

$$m_{tgt} = \frac{1}{2}$$

so choose B

12 *Your working might look like:*

$$Limit = \frac{5}{1-0.6} = \frac{50}{4} = \frac{25}{2}$$

so choose D

13 *Your working might look like:*

$$Integral = (3 \sin 4x) \times \frac{1}{4} = \frac{3}{4} \sin 4x + c$$

so choose C

14 *Your working might look like:*

$$\text{left shaded} = \int_a^b (f(x) - x) dx$$

$$\text{right shaded} = \int_b^c (x - f(x)) dx$$

total area = sum of these

so choose A

15 *Your working might look like:*

$$\begin{aligned}f(g(x)) &= f(x^2 + 2) = 3(x^2 + 2)^2 - 1 \\ &= 3x^4 + 12x^2 + 11\end{aligned}$$

so choose D

16 *Your working might look like:*

$$\begin{aligned}2 \cos x \cos\left(\frac{2\pi}{3}\right) - 2 \sin x \sin\left(\frac{2\pi}{3}\right) \\ = 2 \cos x \times \left(-\frac{1}{2}\right) - 2 \sin x \times \left(\frac{\sqrt{3}}{2}\right) \\ = -\cos x - \sqrt{3} \sin x\end{aligned}$$

so choose D

17 *Your working might look like:*

$$\text{difference of radii} = 4 - 2 = 2$$

$$\text{distance between centres} = 2$$

so choose B

18 *Your working might look like:*

$$m_{AD} = 3$$

$$y - 4 = 3(x - 2)$$

$$y - 4 = 3x - 6 \Rightarrow 3x - y - 2 = 0$$

so choose A

19 *Your working might look like:*

$$\vec{AC} = \begin{pmatrix} 6 \\ 9 \\ -9 \end{pmatrix}, \vec{BC} = \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix} \text{ so } \vec{AC} = 3\vec{BC}$$

(1) is correct, (2) is incorrect

so choose B

20 *Your working might look like:*

$$\begin{aligned}\vec{EP} &= \vec{ED} + \vec{DP} \\ &= \vec{ED} + \frac{1}{2}\vec{DB} \\ &= -\mathbf{u} + \frac{1}{2}(-\mathbf{u} + \mathbf{v}) = -\frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}\end{aligned}$$

so choose A

ANSWER SHEET

Forename(s)

Surname

Date of birth


SCN

Full name of centre

Check the pre-printed details above and print your name and seat number below.

Candidate name

Number of seat

Indicate your choice of answer with a single mark as in the following example.  USE HB PENCIL ONLY. ERASE ALL ERRORS THOROUGHLY.

	A	B	C	D
1	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
14	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
15	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
16	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
17	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
18	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
19	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
20	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

SCOTTISH
QUALIFICATIONS
AUTHORITY



The Optima Building
58 Robertson Street
Glasgow
G2 8DQ

Ironmills Road
Dalkeith
Midlothian
EH22 1LE

Customer Contact Centre
tel: 0845 279 1000
fax: 0845 213 5000
e-mail: customer@sqa.org.uk

website: www.sqa.org.uk

BB3790

May 2007