16 November 2005

To: SQA Co-ordinator (Secondary Schools,
Special Schools and Further Education Colleges)
Scottish Executive
HMIE
Directors of Education
Mathematics Assessment Panel members
Higher Education Institutions — Department of Education

For the attention of all staff responsible for the delivery of National Qualifications in Mathematics

Action by Recipient
Response required
✓ **Note and pass on**
None – update/information only

Contact Name: Noël Donoghue at Glasgow
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E-mail: noel.donoghue@sqa.org.uk

Dear Colleague

N**ational Qualifications Update — Mathematics**

This letter is intended to provide an update on National Qualifications, and should be passed to the member of staff responsible for Mathematics.

1. Update on NQ Review recommendations.

Since the last update letter, recommendations arising form the NQ Review for Mathematics have progressed as follows:

(i) Course frameworks for Advanced Higher Mathematics and Advanced Higher Applied Mathematics

Advanced Higher Mathematics

The Advanced Higher Mathematics examination, based on the Units Mathematics 1 (AH), Mathematics 2 (AH) and Mathematics 3 (AH), was successfully introduced for Diet 2004. The Advanced Higher Mathematics examination in Diet 2005, designed to mirror the style and format of the Advanced Higher Mathematics examination in 2004, performed as expected.

Advanced Higher Applied Mathematics

The Advanced Higher Applied Mathematics examination, based on one of the Applied Mathematics options, Applied Mathematics: Statistics (AH) or Applied Mathematics: Numerical Analysis (AH), or Applied Mathematics: Mechanics (AH) along with the common component Unit Mathematics for Applied Mathematics (AH), was successfully introduced for diet 2005. The separate examination paper for each of the three Applied Mathematics options was well received, and the separate examination papers allow candidates to concentrate exclusively on their option questions and the common Section B.

This recommendation is now complete.

Design of questions in examination papers in Mathematics

Further work arising from the Mathematics Subject Review led to the appointment of a consultant to investigate alternative forms of assessment. Information was gathered on forms of assessment employed in other countries by interviewing a number of key personnel with knowledge of practices in other countries. Some interesting variations were revealed, but the investigation revealed no possibility of importing an improved assessment system from abroad. However discussions and further information led the investigation in the direction of objective testing. Key personnel in mathematics education consulted on the use of objective testing in examinations raised no objections, and acknowledged that it could have much to offer in monitoring and maintaining standards from year to year.

Given the time constraints for NQ examinations, objective testing holds the most promise in terms of making the best use of the examination time available in Mathematics (maximum of three hours including break as laid down for all NQ subjects by the Scottish Executive). Objective testing is generally more efficient in the use of time, and allows for a wider sampling of the Course in a relatively short time, freeing up the rest of the examination time for assessment of other aspects of the Course, eg extended questions, sustained thinking, problem solving.

The Subject Advisory Group (SAG) was set up, as discussed and agreed at the Mathematics and Statistics Assessment Panel (MSAP) meeting of May 2004, to progress issues arising from the National Qualifications Review of Mathematics. The group comprises members of the MSAP, the Principal Assessor for Mathematics at Higher level, a Consultant, and SQA's Qualifications Manager and Qualifications Officer for Mathematics. The Principal Assessors for Intermediate 1 and Intermediate 2 were consulted and briefed on the work of the SAG. A member of HMIE was invited to attend in an advisory role, and a representative from Learning Teaching Scotland was also invited. The SAG met in September and December 2004, and April 2005, and reported to the MSAP in May 2005.

Appendix 1 of this letter gives an overview of the work carried out to date in further investigating the use of objective testing (multiple choice questions) within examination papers in Mathematics. Appendix 1A is an exemplar of a Higher examination paper where objective testing forms part of the paper.

I would urge you to discuss Appendix 1 and Appendix 1A with your colleagues in the Mathematics department in your centre, and if you have particular views or comments about the introduction of objective testing into external assessment or the exemplar Higher examination paper, please contact me directly at the above address. As you communications will be considered and discussed at the January 2006 meeting of the SAG, I would urge you to send your views or comments to myself by **6 January 2006**.

Please note that there is **no change** to the format of Higher, Intermediate 2 or Intermediate 1 Mathematics examinations at this time. Centres will be kept informed of any developments.

(iii) Advances in calculator technologies and capabilities

Graphing calculator questionnaire

Thank you to all centres who responded to the consultation on the use of graphing calculators. Your responses were read with interest, analysed, and a report prepared for consideration by the Mathematics and Statistics Assessment Panel in May 2005. The Mathematics and Statistics Assessment Panel approved the report and the recommendations contained therein.

The consultation feedback indicated strong support for, and stressed the benefit of the use of graphing calculators in learning and teaching, as a tool to aid understanding and to enrich the learning and

teaching process. However there was insufficient evidence to support a radical change to SQA examination arrangements where access to a graphing calculator is assumed for all candidates.

The main proposals were that:

1. A group is set up to investigate the writing of Unit(s) at Higher level and/or Advanced Higher level to test the full power of current technology.
2. At this time, there should be no change regarding the use of technology to the current structure of National Courses in Mathematics or the existing arrangements for SQA examinations in Mathematics.
3. Acceptable graphing calculator solutions should, where appropriate, continue to be incorporated into marking instructions for SQA examinations.
4. Advice and guidance documents on acceptable graphing calculator solutions to examination questions should continue to be issued to centres where appropriate.
5. The situation regarding the use of graphing calculators in centres should be kept under review, and centres surveyed to determine an accurate profile of current activity in this area within approximately five years (dependant on developments in this area).

The full report is now available on the Mathematics homepage of SQA's website: **www.sqa.org.uk**

If you have particular views or comments about the report, please contact me directly at the above address. Centres will be kept informed of any developments.

2. Principal Assessor and Senior Moderator reports

The Principal Assessor (PA) and Senior Moderator (SM) reports for the 2005 diet of examinations are available on the Mathematics homepage of SQA's website.

The Principal Assessor reports include details of: statistical information on entries; grade boundaries at C, B and A; and comments on candidate performance, including areas where candidates performed well in the external assessment, areas where candidates had difficulty with the external assessment and areas of common misunderstanding.

The SM reports (one for Visiting Moderation, and one for Central Moderation) identifies specific issues arising from moderation and includes general comments on the moderation activity and feedback to centres.

3. Standard Grade Mathematics: Credit

In Standard Grade examination papers, marks are allocated to the individual elements Knowledge and Understanding (KU), and Reasoning and Enquiry (RE) across the two question papers. The total mark allocation for the RE element in Standard Grade Credit was increased to 45 in and after 2004. The additional marks available were designed to reward candidates for demonstrated attainment where previously the marking was tight, and to improve accessibility to extended reasoning questions. To access these additional marks candidates must be actively encouraged to commit working to paper. The Principal Assessor report for Standard Grade Mathematics in 2005 indicated in the 'Feedback to centres' section:

'Candidates are also more practised in writing down steps of working. This allows examiners to award partial marks….
However, candidates at Credit level (now having 90 marks) should be more actively encouraged to commit working to paper. Credit candidates have more marks to be awarded but also more to lose should no attempt be made at a question.'

4. Marking instructions

The marking instructions for the Mathematics (Standard Grade Foundation, General and Credit, Intermediate 1, Intermediate 2, Higher and Advanced Higher) and Applied Mathematics (Advanced Higher) 2005 examinations are available on SQA's website (**www.sqa.org.uk**).

5. Use of mathematical instruments in examinations

Markers report that there is evidence in candidate scripts that some candidates do not have access to a protractor during the examination. Indeed some candidates write a message to that effect on their script.

I would like to take this opportunity to remind you that protractors should be issued to all candidates for Standard Grade Mathematics at General and Foundation level, and National Qualifications at Intermediate 1 level. For Standard Grade at Credit level and National Qualifications at Intermediate 2 level protractors should be available and issued to candidates on request.

These instructions were clearly printed in the *Handbook for Invigilators 2005*, and will be in the *Handbook for Invigilators 2006*. I would ask for your support and vigilance in this matter.

6. Intermediate 1 and Intermediate 2 Courses

You may recall that in diet 2004 there were a few centres where a number of candidates for the Intermediate 1 and Intermediate 2 examinations were presented with the incorrect question paper. I am pleased to report that there were very few instances of this in diet 2005.

I would like to take this opportunity to remind you that by **24 February 2006** you should ensure that candidates are entered for the correct National Qualification, ie **C100** or **C101**, and advise SQA of any changes of Course. Course amendments are possible without late entry charges up to **31 March 2005**. (Details in the *Operational Guide for Schools 2005/2006*, available on SQA's website: **www.sqa.org.uk**)

In centres where there are candidates entered for both options, it would be useful to draw this to the attention of the Chief Invigilator prior to the examination.

I appreciate your support and vigilance in this matter.

7. Progression route to Higher Mathematics

Some centres are using Intermediate 2 as a progression route to Higher. Candidates progressing through this route were:

- in 2002 — 1624 candidates
- in 2003 — 1818 candidates
- in 2004 — 1842 candidates
- in 2005 — 2192 candidates

The number of centres who opt to replace Standard Grade Mathematics with Intermediate 2 is rising and the increase is expected to continue next year and in future years.

Success at Higher depends heavily on a full and in depth understanding and competence in the algebraic content of Standard Grade Credit level or Intermediate 2. Teachers of Standard Grade Credit level or Intermediate 2 students intending to progress to Higher are advised to give more attention to algebraic content than the minimum necessary for success in the Standard Grade Credit level or Intermediate 2 examinations.

In previous update letters, progression data for candidates from Standard Grade and Intermediate 2 to Higher was provided. The progression data for 2005 Higher Mathematics candidates by Qualification level attempted in 2004, ie Standard Grade and Intermediate 2, is included in **Appendix 2** of this letter.

8. E-assessment of NABs

The Pass-IT project, funded by the Scottish Executive (August 2002 – January 2005) was a partnership between SQA, Learning and Teaching Scotland, BBC Scotland, Scottish Further Education Unit and Scottish Centre for Research into Online Learning and Assessment (based at Heriot-Watt University). The project investigated how best to use ICT-based assessment in schools and colleges to enhance flexibility, improve attainment and support teaching and learning. It did this through a process of developing online assessments for a range of qualifications. Further information about Pass-it can be found on the project website: www.pass-it.org.uk

One part of the project progressed the development of ICT-based assessments, extending into NAB assessments from Access to Advanced Higher in Mathematics. Project staff worked with subject specialists (Principal Assessors and the Senior Moderator for Mathematics) to explore ways in which existing paper-based NABs could be re-designed for an ICT-based approach. A number of schools were approached and invited to participate in piloting e-assessment with their pupils.

After a successful pilot of the moderation of online candidate material, a business case to develop and deliver NQ Units as summative e-assessments for all NABs in Mathematics at Higher level (Units 1, 2 and 3) in 2005/2006 was approved by SQA. Approximately 20 centres who have previously expressed interest in e-assessment were invited to send staff to a training/information event in Stirling. Delegates, both mathematics specialists and management colleagues, were briefed on the practical and technical delivery issues, and given time to allow them to become familiar with the e-assessments and associated paperwork.

If you would like further information about e-assessment of NABs, please contact Campbell White, Centre Liaison Officer.

E-mail: campbell.white@sqa.org.uk
Tel: 0131-271 6763

Further information can be found in the SQA document *SQA Guidelines on e-assessment for Schools* (June 2005), which is being sent to all schools in the Autumn.

9. Understanding Standards

To learn more about SQA standards and how they are applied, you can visit the Understanding Standards website (**www.understandingstandards.org.uk**). The website is available to all those delivering SQA Qualifications. It is designed specifically for teachers/lecturers rather than pupils or parents It aims to explain the principles underlying the marking of assessment evidence and to develop, interactively, the user's skills in applying these principles to a selection of candidate responses.

The site has been developed for Mathematics at Higher and Intermediate 2 levels, and will be extended to Intermediate 1 levels next year.

Your SQA Coordinator will give you a user name and password. If you have any problems using the site, please contact the Customer Contact Centre by phone on tel: 0845 279 1000, or e-mail the project team at: usw@sqa.org.uk

10. Professional development workshops

SQA is again offering professional development workshops in Mathematics, aimed at helping teachers and lecturers understand the standards that National Qualifications are based on. The aim of the workshops is to deliver quality staff training and development which can be used by delegates to improve delivery of their subject and increase their knowledge of SQA processes and procedures. In the third year of the programme, workshops are again planned for Mathematics at Higher level and additionally in 2005 for Mathematics at Advanced Higher and Intermediate 2 levels.

The workshops are led by the Principal Assessor for Advanced Higher, Higher or Intermediate 2, and supported by members of the relevant examining team. They provide a mixture of practical inputs and discussion opportunities.

Delegates at each of the workshops are asked for feedback at the end of the workshop, and feedback (as in previous years) is read and acted upon in the next year's programme. If you have any comments regarding the workshops (eg dates, venue, levels offered) please contact me at the above address and I will forward these on to the relevant team.

11. Appeals 2005

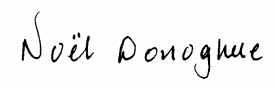
I would like to take this opportunity to remind you of two publications which centres would find helpful to refer to when preparing and collating evidence for Appeals purposes.

I would direct you to the NQ Mathematics subject-specific page of SQA's website (**www.sqa.org.uk**) and the publication *Estimates, Absentees and external Assessment Appeals: guidance on evidence requirements* (April 2004). The publication provides guidance (both generic and subject-specific) for centres which are delivering National Courses.

On the secure area of SQA's website, you can access the publication pack *Exemplification of Appeals: Mathematics* (June 2004). The pack is designed to support teachers and lecturers of mathematics in submitting evidence to support appeals.

I hope you find the information in this letter helpful. If you require any further clarification please do not hesitate to contact me.

Yours faithfully



Noël Donoghue
Qualifications Manager
Mathematics and Science

Appendix 1

Objective testing in Mathematics

Objective tests: multiple choice

There are a variety of formats for multiple choice items, all of which require a candidate to select from a set of given responses rather than writing an answer. The multiple choice items considered for use in external assessment for Mathematics are similar to those used in Science, ie the question stem and options type. The characteristic components of the question stem and options type are:

Stem	defines the task set and may be in the form of a question or sentence to be completed.
Options	are the four possible answers from which the candidate chooses.
Key	is the correct answer.
Distractors	are the incorrect answers.

eg

If $x = 3$ and $y = 1$, the value of $x^2 + y^2$ is:

A 16 B 10 C 8 D 5

(Key: B)

Subject Advisory Group (SAG): progress to date

The SAG was set up, as discussed and agreed at the Mathematics and Statistics Assessment Panel (MSAP) meeting of May 2004, to progress issues arising from the National Qualifications Review of Mathematics. The group comprises members of the MSAP, the Principal Assessor for Mathematics at Higher level, a Consultant, and SQA's Qualifications Manager and Qualifications Officer for Mathematics. The Principal Assessors for Intermediate 1 and Intermediate 2 were consulted, and briefed on the work of the SAG. A member of HMIE was invited to attend in an advisory role, and a representative from Learning Teaching Scotland was also invited. The SAG met in September and December 2004, and April 2005, and reported to the MSAP in May 2005.

The SAG considered the benefits and advantages of objective testing in examinations in Mathematics, and the current role of objective testing in National Qualifications in Physics, Chemistry, Biology, Human Biology and Biotechnology. The Principal Assessors for Mathematics at Higher, Intermediate 2 and Intermediate 1 were consulted and their views considered. The subject update letter for Mathematics (Autumn 2004) asked centres for their views and comments on the subject, which were passed onto and considered by the SAG.

The SAG looked at developments in the use of technology to support electronic item banks of objective test questions, the pretesting of the objective test questions and the electronic capture of data about the questions. Since objective test questions once written are pre-tested, question analyses can be done to determine how the individual

test items will perform, and the facility value and discriminating index of each item calculated. (The facility value is a measure of the difficulty of a question expressed as a percentage of the pre-test candidates who got the question correct, and the discriminating index measures the power of the item to discriminate between more able and less able candidates.)

The SAG spoke to the Project Manager within SQA about the electronic marking of the objective test component in examinations in the sciences proposed for diet 2005 and beyond. The capture of data from the objective test component in external assessment could be used to track performance of candidates from year to year, and in reporting on standards.

Candidate results in the objective test component of an external assessment could be used for quality assurance purposes, by matching candidate results in different components, and identifying differences in performance.

The main advantages of multiple choice items in examinations were identified as:

1. They allow wide sampling across the contents of the Arrangements. This is useful to ensure coverage of Course content in combination with the rest of the question paper.
2. The items can be pre-tested so the Principal Assessor and setting and vetting teams will know how the items will perform in the examination. The degree of difficulty of each item and how well it discriminates between candidates can be determined in advance.
3. Over time the same items can be used in different years to compare the populations over time.
4. The electronic marking of multiple choice items allows for the speedy and accurate capture of data about individual items.

The SAG consulted with the Principal Assessors for at Higher, Intermediate 2 and Intermediate 1, and the Research and Information Services within SQA as to the style and format of multiple choice items. Both four and five option multiple choice items were considered. It was recognised that more options increases the reading and comprehension time for candidates at the expense of the mathematics. When reviewing five option multiple choice items, often one distractor attracted little or no candidates (ie was not a good distractor). It was recognised that it is often difficult to write four good distractors.

It was further recognised that the potential for e-assessment is a rapidly developing area, as revealed in the Pass-IT project and the e-assessment of NQ Mathematics NABs at Higher level by 20 centres in 2005/2006. The introduction of objective testing may indeed be a precursor to a future review of external assessment practices to take advantage of evolving technological advances.

The Principal Assessor for Mathematics at Higher level was charged with deconstructing the 2004 Higher level question paper, and rewriting the examination to incorporate an element of objective testing. The proposed revised format question paper was considered by the SAG and recommended to the MSAP as a suitable model to be issued to centres for comment.

The proposed revised format question paper format is:

Paper 1	Non-calculator		
	Section A	20 objective test items	40 marks
	Section B	Calculator protected items	30 marks
Time: 90 mins		**Total marks**	**70 marks**
Paper 2	Calculator		60 marks
Time: 70 mins		**Total marks**	**60 marks**

Please see Appendix 1A for the proposed revised format question paper at Higher level.

The SAG will be meeting in the spring of 2006 and would welcome your views and comments on the proposed revised format question paper. Please send me your views and comments by post or e-mail to the address at the top of this update letter.

A review of existing objective test items held by SQA (from Ordinary and Higher Grade) resulted in a number of items being identified (with minor amendments) to be vetted for inclusion in a bank of multiple choice items. After vetting, although a number of items were discarded, a number were placed in the bank, and a number identified for pre-testing.

A number of schools were approached and asked to take part in the pre-testing of multiple choice items at Higher level (as happens each year in the sciences) in March 2005. Twenty centres and approximately 1,000 candidates successfully pre-tested 60 Higher objective test items, and no difficulties were reported.

Subject Advisory Group (SAG): future developments

The Principal Assessors for Intermediate 2 and Intermediate 1 are currently deconstructing the 2005 Intermediate 2 and Intermediate 1 level question papers, and proposed revised format question papers will be issued to centres for comment in due time.

A number of markers of Higher, Intermediate 2 and Intermediate 1 Mathematics have been approached and invited to attend a training weekend on the writing of objective test items. A training weekend for Higher level will be held in November 2005, and for Intermediate levels in February 2006.

It is anticipated that pre-testing of multiple choice items will take place in March 2006, and if you are approached I hope you can support the process. Informal feedback from centres involved in 2005 is that the tests were useful as revision for the forthcoming examination, and candidates enjoyed the experience.

Subject Advisory Group (SAG): support for centres

The introduction of an element of objective testing using multiple choice items in Mathematics examinations (Higher, Intermediate 2 and Intermediate 1) is being considered by the SAG and MSAP. However it is fully recognised that prior to any change to the format of question papers centres will require not only Specimen Question Papers but also an item bank of objective test items on which to draw in the creation of prelim examinations. It is anticipated that such an item bank would be developed and available to centres on the secure section of SQA's website.

Please note that it is envisaged that the adoption of an element of multiple choice in mathematics examinations would result in **no change** to Course contents.

Please note that there is **no change** to the format of Higher, Intermediate 2 or Intermediate 1 Mathematics examinations at this time. I would stress that centres will be kept fully informed of any developments, and changes to the format of examinations in due course.

Appendix 1A

Proposed revised format question paper
— based on the 2004 Higher examination

Paper 1 Section A (Each question is worth 2 marks)

1 The straight lines with equations $x+3y+1=0$ and $2x+5y=0$ intersect at Q.
The coordinates of Q are

$A \quad (2,-1) \qquad B \quad (5,-2) \qquad C \quad (-4,1) \qquad D \quad (-7,2)$

2 The line through P $(7,a)$ and Q $(4,-5)$ has a gradient of 3. The value of a is

$A \quad -4 \qquad B \quad 14 \qquad C \quad 4 \qquad D \quad 38$

3 A line L is perpendicular to the line with equation $4y=-2x+1$. The gradient of line L equals

$A \quad 2 \qquad B \quad -2 \qquad C \quad -\frac{1}{2} \qquad D \quad \frac{1}{2}$

4 When $\left(x^3-x^2-5x-3\right)$ is divided by $(x+3)$, the remainder is

$A \quad -24 \qquad B \quad 0 \qquad C \quad -36 \qquad D \quad 30$

5 On dividing $f(x)$ by $(x-1)$, the remainder is zero and the quotient is x^2-4x-5.
In its fully factorised form $f(x)$ equals

$A \quad (x-1)(x-1)(x+5) \qquad B \quad (x+1)(x-5)$
$C \quad (x-1)(x-1) \qquad D \quad (x-1)(x+1)(x-5)$

6 Given that $\tan x=-\sqrt{3},\ 0<x<2\pi,$ then the solutions for x are

$A \quad \frac{7\pi}{3},\frac{11\pi}{3} \qquad B \quad \frac{4\pi}{3},\frac{5\pi}{3} \qquad C \quad \frac{2\pi}{3},\frac{5\pi}{3} \qquad D \quad \frac{5\pi}{3},\frac{7\pi}{3}$

7 $A=(-3,4,7),\ B=(-1,8,3)$ and $C=(0,10,1)$. The ratio in which B divides AC is

$A \quad 2:1 \qquad B \quad 3:-1 \qquad C \quad 1:2 \qquad D \quad 3:1$

8 A $=(-3,4,7)$ and B $=(-1,8,3)$. If $\overrightarrow{AD}=4\overrightarrow{AB}$ then the coordinates of D are

$A \quad (-19,52,47) \qquad B \quad (-8,16,-16) \qquad C \quad (8,-16,16) \qquad D \quad (5,20,-9)$

9 Given that $y=3\cos 5x,$ then $\frac{dy}{dx}$ equals

$A \quad 15\cos 5x \qquad B \quad -15\sin 5x \qquad C \quad -3\sin 5x \qquad D \quad 3\cos 5$

Paper 1 Section A cont

10 If c is a constant of integration then $\int \left(4x+1\right)^{\frac{1}{2}}\,dx$ equals

$A \quad \frac{1}{6}\left(4x+1\right)^{\frac{3}{2}}+c \qquad B \quad \frac{1}{4}\left(4x+1\right)+c \qquad C \quad \frac{1}{4}\left(4x+1\right)^{\frac{3}{2}}+c \qquad D \quad 2\left(4x+1\right)^{-\frac{3}{2}}+c$

11 Given that $\int \left(3x+1\right)^{-\frac{1}{2}}\,dx = \frac{2}{3}\left(3x+1\right)^{\frac{1}{2}}+c$, then $\int_0^1 \left(3x+1\right)^{-\frac{1}{2}}\,dx$ equals

$A \quad \frac{2}{3} \qquad B \quad \frac{4}{3} \qquad C \quad 2 \qquad D \quad \sqrt{2}$

12 When written in the form $\left(x+b\right)^2+c$, $x^2-16x+27$ equals

$A \quad \left(x+8\right)^2+7 \qquad B \quad \left(x-8\right)^2-37 \qquad C \quad \left(x-4\right)^2+41 \qquad D \quad \left(x-4\right)^2-41$

13 Which one of the following statements is true for the function g where $g'(x) = x^2+2x+1$:

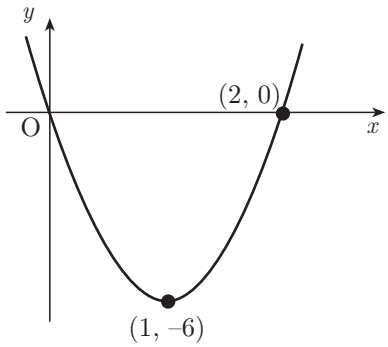
A never increasing
B decreasing then increasing
C increasing then decreasing
D never decreasing

14 $\log_2(x+1)-2\log_2 3$ equals

$A \quad \log_2\left(\frac{x+1}{9}\right) \qquad B \quad \log_2\left(x-8\right) \qquad C \quad \log_2\left(x-2\right) \qquad D \quad \log_2 9(x+1)$

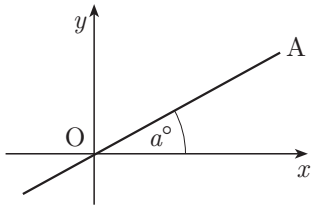
15 The equation of the parabola shown is of the form $y = ax(x-2)$.
The value of a is

$A \quad -1, \qquad B \quad 1 \qquad C \quad -6 \qquad D \quad 6$



16 The diagram shows the line OA with equation $2y = x$.
The angle between OA and the positive direction of the x-axis is $a°$.
The gradient of OA is given by

$A \quad \tan^{-1}\frac{1}{2} \qquad B \quad \tan^{-1}1 \qquad C \quad \tan^{-1}2 \qquad D \quad -\tan^{-1}\frac{1}{2}$



Paper 1 Section A cont

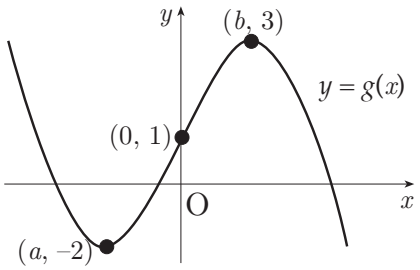
17 Points P and Q have coordinates $(1,3,-1)$ and $(2,5,1)$.
If T is the midpoint of PQ then the position vector of T is

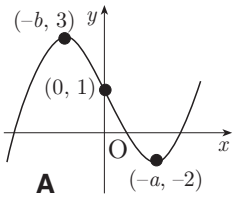
$A \begin{pmatrix} -\frac{3}{2} \\ -4 \\ 0 \end{pmatrix}$ $B \begin{pmatrix} 3 \\ 8 \\ 0 \end{pmatrix}$ $C \begin{pmatrix} -\frac{1}{2} \\ -1 \\ -1 \end{pmatrix}$ $D \begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$

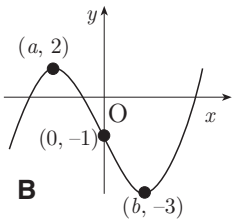
18 The recurrence relation $u_{n+1} = 0{\cdot}4u_n + 3$ has a limit equal to

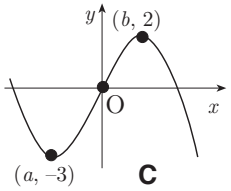
$A \quad -\frac{1}{5}$ $B \quad \frac{15}{7}$ $C \quad 5$ $D \quad \frac{1}{10}$

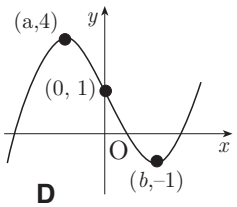
19 The diagram shows the graph of $y = g(x)$.
Which diagram below shows the graph of $y = -g(x)$?



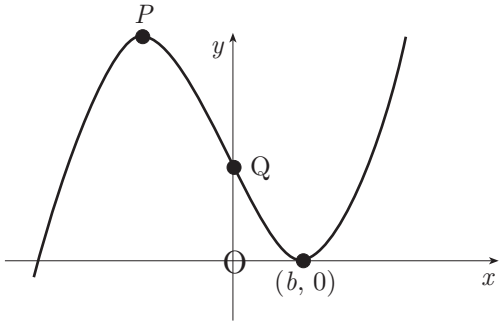








20 The diagram shows the graph of $y = 3 - g(x)$ where the graph of $y = g(x)$ is shown in question 19.
P is the image of $(a,-2)$ and Q is the image of (0,1).
The coordinates of P and Q are



$A \quad P(a,4)\ Q(0,1)$ $B \quad P(a,5)\ Q(0,2)$ $C \quad P(a,-5)\ Q(0,-2)\ D \quad P(a,1)\ Q(0,4)$

[END OF PAPER 1 SECTION A]

Paper 1 Section B

21 Solve the equation $\log_x 8 + \log_x 4 = 5$ **4**

22 Solve the equation $\sin 2x - \cos x = 0$ for $0 \leq x \leq \frac{\pi}{2}$ **6**

23 (*a*) Find the stationary points on the curve with equation $y = x^3 + 3x^2 - 9x + 5$ and justify their nature. **6**

b) The curve passes through the point (–5, 0). Sketch the curve. **2**

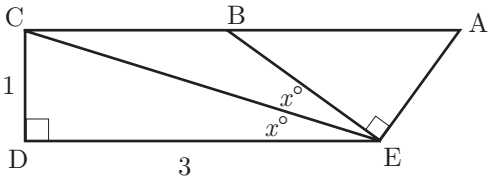
24. In the diagram,

angle DEC = angle CEB = $x°$ and

angle CDE = angle BEA = 90°.

CD = 1 unit; DE = 3 units.

By writing angle DEA in terms of $x°$, find the exact value of $\cos\left(\hat{\text{DEA}}\right)$. **7**

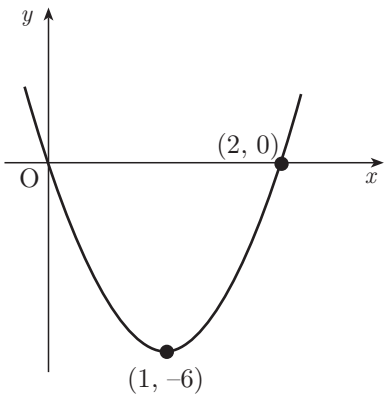


25. The diagram shows a parabola with equation

$y = 6x(x - 2)$

This parabola is the graph of $y = f'(x)$.

Given that $f(1) = 4$, find the formula for $f(x)$. **5**



[END OF PAPER 1 SECTION B] **70**

Paper 2

All questions should be attempted

1. Given that $\overrightarrow{QP} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$ and $\overrightarrow{QR} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$ find the size of angle PQR. **5**

2. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

3. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

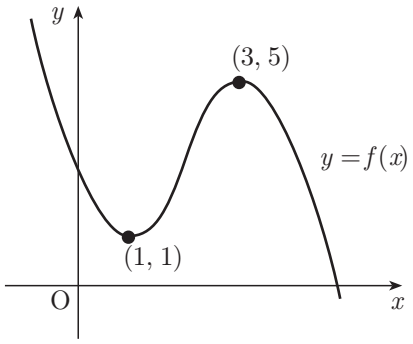
(a) Find the value of x for which the gradient of the tangent at P is 12. **5**

(b) Hence find the equation of the tangent at P. **2**

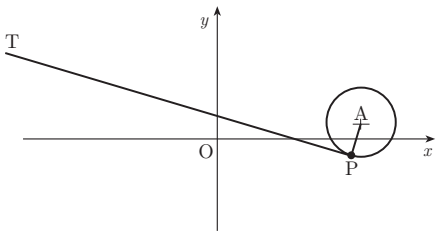
4. (a) Express $3\cos x^\circ + 5\sin x^\circ$ in the form $k\cos\left(x^\circ - a^\circ\right)$ where $k > 0$ and $0 \le a \le 90$. **4**

(b) Hence solve the equation $3\cos x^\circ + 5\sin x^\circ = 4$ for $0 \le x \le 90$. **3**

5. The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5). Sketch the graph of $y = f'(x)$. **3**

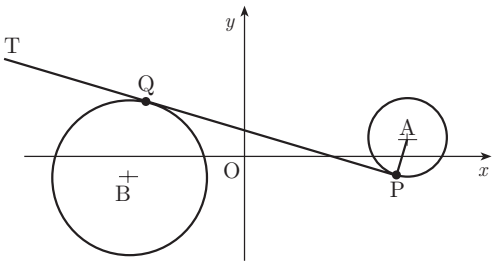


6. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$ .The line PT is a tangent to this circle at the point P(5, –1).



(a) Show that the equation of this tangent is $x + 2y = 3$. **4**

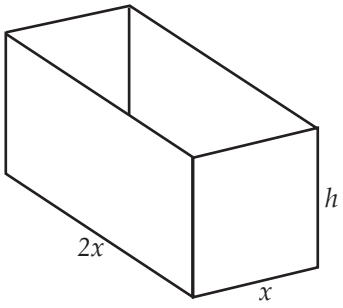
The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.



(b) Show that PT is also a tangent to this circle. **5**

(c) Q is the point of contact. Find the length of PQ. **2**

7. An open cuboid measures internally x units by $2x$ uits by h units and has an inner surface area of 12 units^2.



(a) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x\left(6 - x^2\right)$. **3**

(b) Find the exact value of x for which this volume is a maximum. **5**

8. The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0 e^{-0.002t}$, where A_0 is the amount present initially.

(a) If 600 micrograms are left after 1000 years, how many micrograms were present initially? **3**

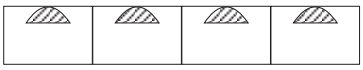
(b) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? **4**

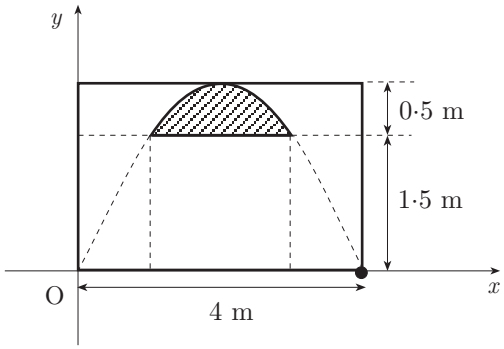
9. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.

The second diagram shows one such window. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.

Find the area in square metres of the glass in one window.





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[END OF PAPER 2 QUESTION PAPER]

Appendix 2

Progression data for 2005 Higher Mathematics candidates by Qualification level attempted in 2004

1. **Standard Grade**
2. **Intermediate 2**