Scottish Qualifications Authority

National Qualifications in Mathematics

This document replaces the document shown below which was issued in 1992

SCOTTISH EXAMINATION BOARD

SCOTTISH CERTIFICATE OF EDUCATION
MATHEMATICS (REVISED) ON THE HIGHER GRADE

NOTES OF CLARIFICATION ON AMENDMENT TO RUBRIC OF PAPERS I AND II FOR EXAMINATIONS IN AND AFTER 1993

OCTOBER 1992

This document does not convey any changes to the existing examination arrangements for SQA Mathematics courses – it simply updates and provides further exemplification of the guidance in the original document on acceptable calculator solutions to examination questions.

November 2002

FOREWORD

Recent years have seen a growth in the use of Graphing calculators both in the teaching process and by students themselves. There have been a number of initiatives to support the use of such calculators following the publication of the Advanced Calculators and Mathematics Education (ACME) document. National training has recently been carried out by Learning and Teaching Scotland on behalf of the Scottish Executive Education Department. Local Authorities supported this national training and are currently involved in extending the training to a wider audience.

This document has been updated and reissued to facilitate discussion and consideration of the next steps in relation to the use of technology in mathematics examinations and to act as a reminder to staff that Graphing calculator solutions are currently acceptable in some instances.

It should also be noted that, where appropriate, the guidance contained in this document also applies to the new National Qualifications at Intermediate 1 and Intermediate 2 levels.

This document does not convey any changes to the current conditions and arrangements for examinations, nor to what is considered to be an appropriate calculator solution to an examination question. Some refinements have been made to the original document to accommodate the major changes in the ease of use and functionality of technology since October 1992. Additional exemplars of acceptable solutions to examination questions, some with SQA marking schemes, are provided.

It is accepted that the content of the original document, now some ten years old, reflects the historic use of Graphing calculators and to some extent is superseded by the currently available technology and its current usage. Although this document amounts to no more than a restatement of the current position, SQA is currently giving consideration to extending the range of examination questions for which Graphing calculator solutions will be acceptable.

Clearly the proliferation in use and availability of Graphing calculators continues. The technology itself is changing rapidly and the increasing functionality suggests that there is significant work to do in managing the appropriate extension of the use of this technology in the examination system. The increasing availability of hand held Computer Algebra Systems (CAS) is also under consideration. .

In response to these advances, SQA has established a group, representative of key stakeholders, which will consult widely in order to formulate longer term strategies.

Introductory Comments

(a) The rubric of Mathematics Higher Paper 2 reads as shown below:

1.	Full credit will be given only where the solution contains appropriate working.
2.	Calculators may be used.
3.	Answers obtained by reading from scale drawings will not receive any credit.

This set of notes provides clarification on each of these rubric items instructions by giving illustrative situations in which they might be applicable.

(b) Square-ruled paper is not issued to candidates sitting Mathematics Higher Level examinations. Any diagram which is either

- a scale drawing or
- a coordinate diagram on a grid drawn on plain paper or
- a scaled graphic calculator display

and used as the sole means of solving a question will be considered as a "scale drawing" under rubric item 3.

2. Illustrative examples

(a) Rubric item 1

<u>Example Question 1</u>

Solve the equation $3\sin\frac{1}{2}x^{\circ} - 1 = 0$ for $0 \leq x \leq 180$

- The answer $x = 38{\cdot}9$, without any appropriate working, would receive no credit.

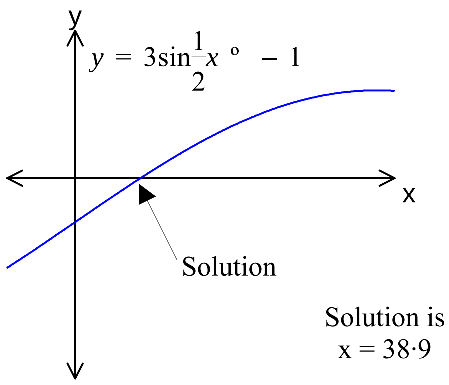
- Candidates who choose to solve this equation graphically, by means of a graphing calculator for example, would need to indicate what they drew the graph of, a sketch (from the screen), where they would find the solution(s) and how they came about the answer "38·9".

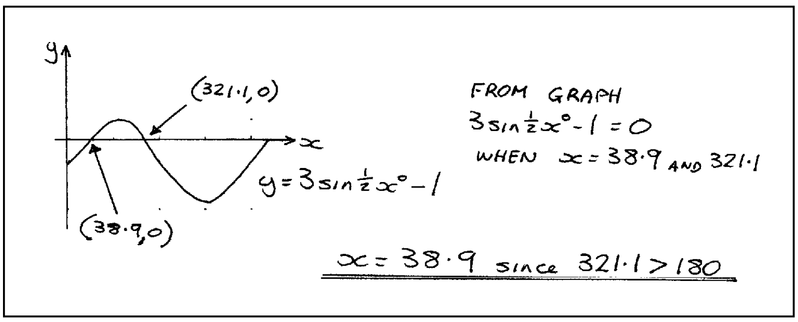
<u>Instances of appropriate working:</u>

1. A "standard" solution

$$\begin{aligned} 3\sin\tfrac{1}{2}x^{\circ} - 1 &= 0 \\ \tfrac{1}{2}x &= \sin^{-1}\tfrac{1}{3} \\ &= 19{\cdot}47, (160{\cdot}53, ...) \\ x &= 38{\cdot}94 \\ x &= 38{\cdot}9 \end{aligned}$$

2. A possible "graphical" solution





Example Question 2

Given that $4\cos x^{\circ} + 5\sin x^{\circ}$ can be expressed in the form $k\sin(x + \alpha)^{\circ}$ where $k > 0$ and $0 \leq x \leq 360$, find the values of k and α giving your answers correct to 2 decimal places.

- The answers $k = 6{\cdot}40$ and $\alpha = 38{\cdot}66$, without any appropriate working, would receive no credit

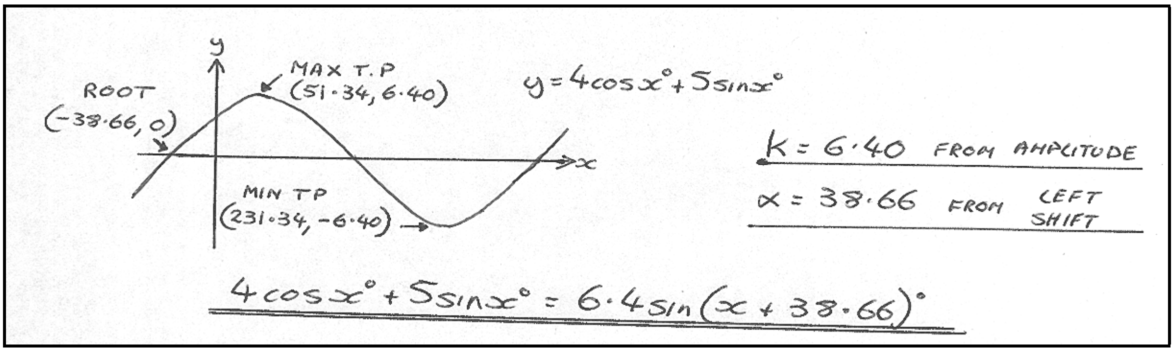
- Candidates who choose to use a graphing calculator for example, would need to indicate what graph was drawn, a sketch (from the screen), where they would find the solution(s) and how they came about the answers $k = 6{\cdot}40$ and $\alpha = 38{\cdot}66$

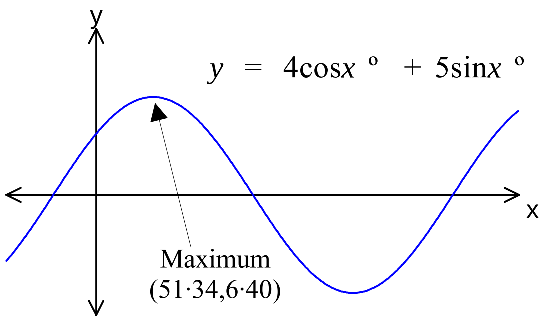
Instances of appropriate working:

1. A "standard" solution

$5\sin x^{\circ} + 4\cos x^{\circ}$	$k = \sqrt{5^2 + 4^2}$
$k\sin x^{\circ}\cos\alpha^{\circ} + k\cos x^{\circ}\sin\alpha^{\circ}$	$= 6{\cdot}40$
$k\sin\alpha^{\circ} = 4,\ \ k\cos\alpha^{\circ} = 5$	$\tan\alpha^{\circ} = \frac{4}{5}, 0 \leq \alpha \leq 9$
	$\alpha = 38{\cdot}66$

3. Two possible "graphical" solutions





This is also the graph of
$y = k\sin(x + \alpha)^{\circ}$ which has a maximum value of k when $\sin(x + \alpha)^{\circ} = 1$
hence
$k = 6{\cdot}40$ when $(51{\cdot}34 + \alpha)^{\circ} = 90^{\circ}$
$k = 6{\cdot}40,\ \ \alpha = 38{\cdot}66$

Example Question 3

The variables x and y are connected by the relationship $y = kx^n$.

A table of values is shown below, with y being given correct to 1 decimal place.

x	1	2	3	4	5
y	3	4·2	5·2	6	6·7

Find the values of k and n

- Some calculators can solve the question without involving any understanding on the part of the candidate. The answers $k = 3$ and $n= 0{\cdot}5$, without any appropriate working, would receive no credit. (See Appendix 1: Higher1998, Paper 2, Qu.11)

Example Question 4

A curve has equation $y = x^4 - 4x^3 + 3$

(a) Find algebraically the coordinates of the stationary points.

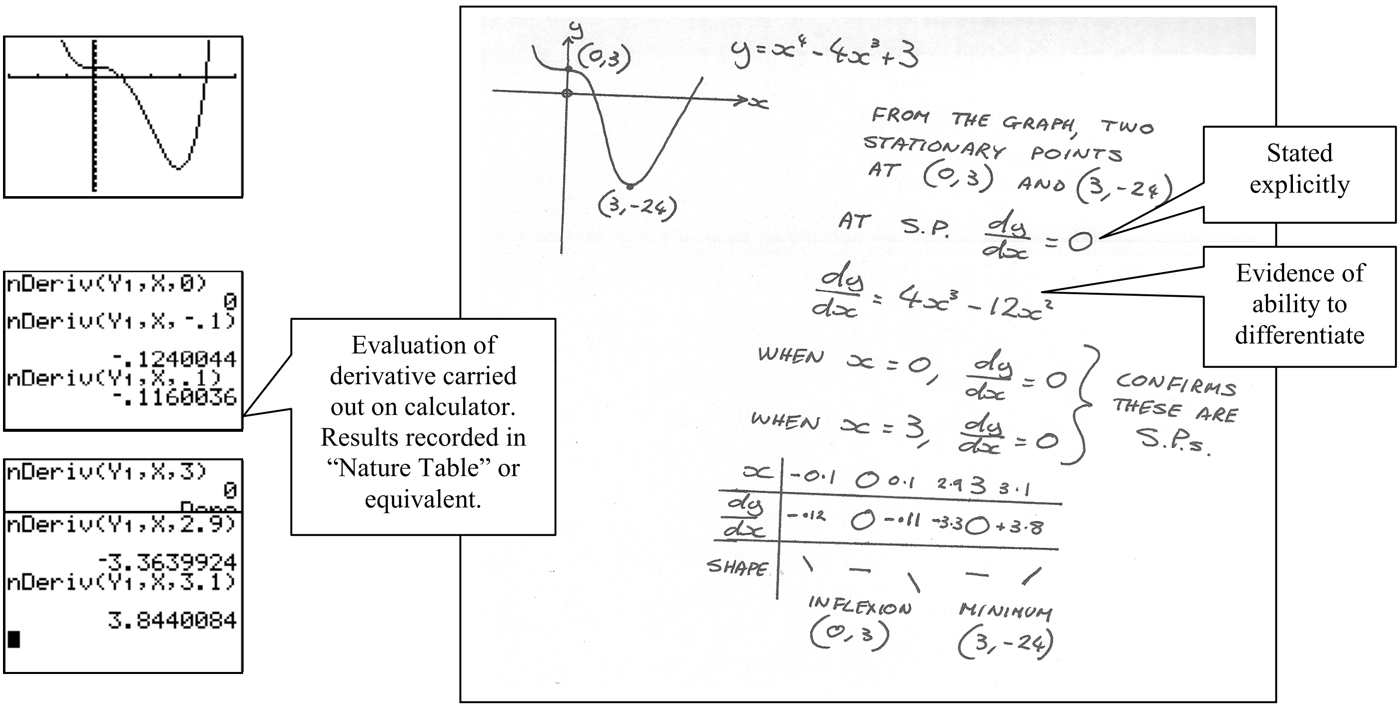
(b) Determine the nature of the stationary points.

- If the candidate is asked to find the stationary point(s) of functions other than quadratic or trigonometric functions, then differentiation is the only acceptable strategy.
- If the candidate is asked to "justify" the nature of the stationary point(s), then differentiation is the only acceptable strategy. (See Appendix 2:Higher1996,Paper 2,Qu.1)

Instances of appropriate working

An acceptable graphing calculator solution

Note : Evidence of the ability to differentiate the function must be demonstrated explicitly.



Example Question 5

Solve algebraically $3\sin 2x° = 2\sin x°$ for $0 \le x \le 360$

- Candidates can expect to have to produce a solution by making use of the double angle formula and the word algebraically will be used to indicate that it would be incorrect for a candidate to solve the equation by a graphical method

Example Question 6

Solve algebraically…….a pair of equations, eg to find the point of intersection of a 'line with a cubic' or 'a line with a circle'.

- As with example question 5, candidates cannot expect to gain any credit for a graphical approach.

Example Question 7

Evaluate $\int_1^2 (3x^2 + 4)dx$

- An answer of 11, without any appropriate working, would receive no credit.

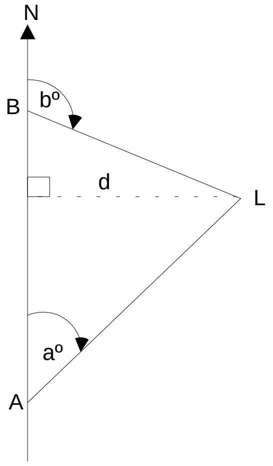
(b) Rubric item 2

Candidates may bring into the examination any approved calculator they choose.
For identification of approved calculators refer to the SQA Handbook for Invigilators.

(c) Rubric item 3

Example Question 8

The following question could be solved by scale drawing.

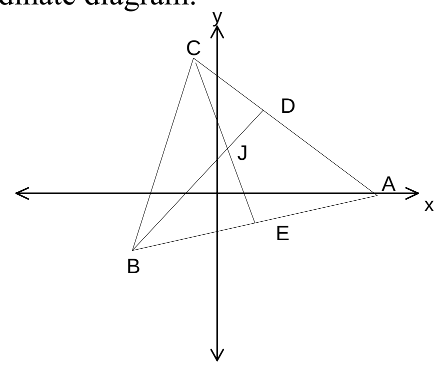


"A ship is sailing due north at a constant speed. When at position A, a lighthouse L is observed on a bearing of a°. One hour later, when the ship is at position B, the lighthouse is on a bearing of b°. Calculate the shortest distance from the ship to the lighthouse when the bearings a° and b° are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour."

- The answer d = 8·9 derived from a scale drawing, would receive no credit.
- Additionally, under rubric item 1, the answer d = 8·9, without any appropriate working, would receive no credit

Example Question 9

The following question could be solved by an accurate coordinate diagram.



"In the diagram, A is the point(7,0), B is the point (-3,-2) and C is (-1,8).
The median CE and the altitude BD intersect at J.
(a) Find the equation of CE and BD
(b) Find the coordinates of J."

- Such a solution from an accurate coordinate diagram would receive no credit.
- Also, under rubric item 1, any solution, without appropriate working would receive no credit.

Appendix 1

Higher:1998 Paper 2 Qu.11

(*a*) The variables x and y are connected by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(*b*) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

x	3.1	3.5	4.1	5.2
y	21 876	72 631	439 392	11 913 076

The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	3.3
(*a*)	3	3.3				3			3.3.7		Source **1998 Paper 2 Qu. 11**
(*b*)	6	3.3				6			3.3.5		

(*a*) •[1] $\log_e y = \log_e ae^{bx}$

•[2] $\log_e y = \log_e a + \log_e e^{bx}$

•[3] $\log_e y = \log_e a + bx$

(*b*) •[4] evidence for strategy being carried out will be appearance of two equations at •[5] stage

•[5] *e.g.* $3.1b + \log a = 9.99$, $5.2b + \log a = 16.29$

•[6] strategy: know to subtract

•[7] $b = 3$

•[8] $a = e^{0.69}$

•[9] $a = 2$

Appendix 2

Higher:1996 Paper 2 Qu.1

A curve has equation $y = x^4 - 4x^3 + 3$.

(a) Find algebraically the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	1.3
			C	A/B	C	A/B	C	A/B	Main Additional	
(a)	6	1.3	6						1.3.12	Source **1996 Paper 2 Qu.1**
(b)	2	1.3	2						1.3.12	

(a)

- •1 $\frac{dy}{dx} =$
- •2 $4x^3 - 12x^2$
- •3 $= 0$ stated explicitly
- •4 e.g. $4x^2(x-3)$
- •5 $x = 0,\ 3$
- •6 $y = 3,\ -24$

(b)

- •7

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$	$-$	0	$-$	0	$+$

- •8 pt of inflection at $x = 0$
  minimum at $x = 3$