

# **X102/302**

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NATIONAL  
QUALIFICATIONS  
2007

TUESDAY, 15 MAY  
9.00 AM – 10.10 AM

**MATHEMATICS  
HIGHER**

Units 1, 2 and Statistics  
Paper 1  
(Non-calculator)

**Read Carefully**

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Statistics:

Sample standard deviation:  $s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1} \left( \sum x_i^2 - \frac{1}{n} (\sum x_i)^2 \right)}$  where  $n$  is the sample size.

Sums of squares and products:  $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{1}{n} (\sum x_i)^2$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{1}{n} (\sum y_i)^2$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i$$

Linear regression:

The equation of the least squares regression line of  $y$  on  $x$  is given by  $y = \alpha + \beta x$ , where estimates for  $\alpha$  and  $\beta$ ,  $a$  and  $b$ , are given by:

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient:  $r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

**ALL questions should be attempted.**

*Marks*

1. Find the equation of the line through the point  $(-1, 4)$  which is parallel to the line with equation  $3x - y + 2 = 0$ . **3**

2. The waiting times, to the nearest minute, for 11 patients attending a dental surgery one day were recorded in order as follows.

2   3   3   5   7   8   10   10   13   14   A

It is known that A is an outlier.

What is the smallest value for A? **4**

3. Functions  $f$  and  $g$ , defined on suitable domains, are given by  $f(x) = x^2 + 1$  and  $g(x) = 1 - 2x$ .

Find:

(a)  $g(f(x))$ ; **2**

(b)  $g(g(x))$ . **2**

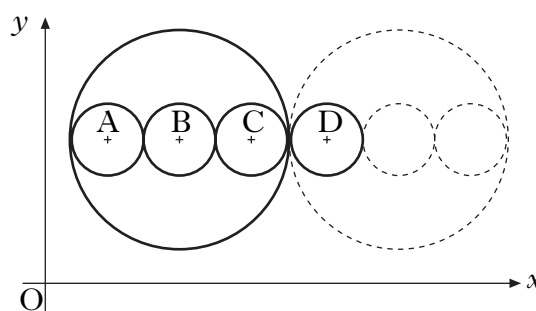
4. Find the range of values of  $k$  such that the equation  $kx^2 - x - 1 = 0$  has no real roots. **4**

5. The large circle has equation  $x^2 + y^2 - 14x - 16y + 77 = 0$ .

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the  $x$ -axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



**5**

**[Turn over**

6. Solve the equation  $\sin 2x^\circ = 6\cos x^\circ$  for  $0 \leq x \leq 360$ . 4

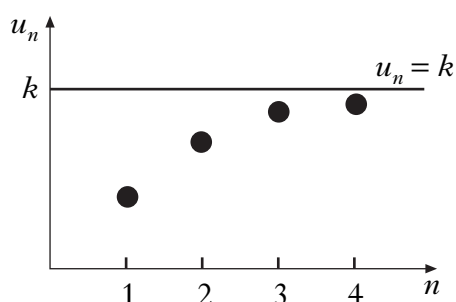
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \quad u_0 = 0.$$

(a) Calculate the values of  $u_1, u_2$  and  $u_3$ . 3

Four terms of this sequence,  $u_1, u_2, u_3$  and  $u_4$  are plotted as shown in the graph.

As  $n \rightarrow \infty$ , the points on the graph approach the line  $u_n = k$ , where  $k$  is the limit of this sequence.



(b) (i) Give a reason why this sequence has a limit.

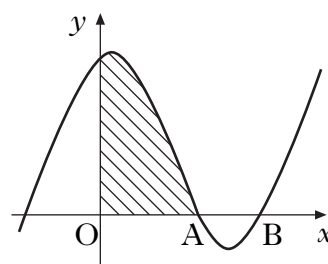
(ii) Find the exact value of  $k$ . 3

8. The diagram shows a sketch of the graph of  $y = x^3 - 4x^2 + x + 6$ .

(a) Show that the graph cuts the  $x$ -axis at  $(3, 0)$ . 1

(b) Hence or otherwise find the coordinates of A. 3

(c) Find the shaded area. 5



9. A function  $f$  is defined by the formula  $f(x) = 3x - x^3$ .  
 (a) Find the exact values where the graph of  $y = f(x)$  meets the  $x$ - and  $y$ -axes. 2

(b) Find the coordinates of the stationary points of the function and determine their nature. 7

(c) Sketch the graph of  $y = f(x)$ . 1

10. Candidate A is campaigning to become a member of the Scottish Parliament. A market research company samples 500 potential voters to determine the effect of candidate A's campaign strategy.

Each voter was asked the question

*Would you vote for candidate A at the forthcoming elections?*

about candidate A at the start of the election campaign.

The same question was asked at the end of the campaign.

The results of the survey are shown in the table below.

		<b>Response at the end of the campaign</b>			
<b>Response at the start of the campaign</b>	<b>Response</b>	<i>Yes</i>	<i>No</i>	<i>Undecided</i>	<i>Total</i>
	<i>Yes</i>	101	25	<i>Y</i>	144
	<i>No</i>	30	<i>X</i>	20	170
	<i>Undecided</i>	50	80	56	186
	<i>Total</i>	181	225	94	500

For this survey:

- (a) determine the values of  $X$  and  $Y$  in the table above; 2
- (b) calculate the probability that the election campaign made no difference to the decision of an individual voter selected at random from this sample. 3
11. The time,  $t$  minutes, between consecutive arrivals in a queue at the ticket office of a museum can be modelled by  $f(t)$  where

$$f(t) = \begin{cases} \frac{1}{2} \left( 1 - \frac{1}{4}t \right) & 0 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases} .$$

Find the mean time between arrivals. 6

[END OF QUESTION PAPER]

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