

X102/304

NATIONAL
QUALIFICATIONS
2007

TUESDAY, 15 MAY
10.30 AM – 12.00 NOON

**MATHEMATICS
HIGHER**
Units 1, 2 and Statistics
Paper 2

Read Carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Statistics:

Sample standard deviation: $s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1} \left(\sum x_i^2 - \frac{1}{n} (\sum x_i)^2 \right)}$ where n is the sample size.

Sums of squares and products: $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{1}{n} (\sum x_i)^2$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{1}{n} (\sum y_i)^2$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{1}{n} \sum x_i \sum y_i$$

Linear regression:

The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β , a and b , are given by:

$$a = \bar{y} - b\bar{x}$$

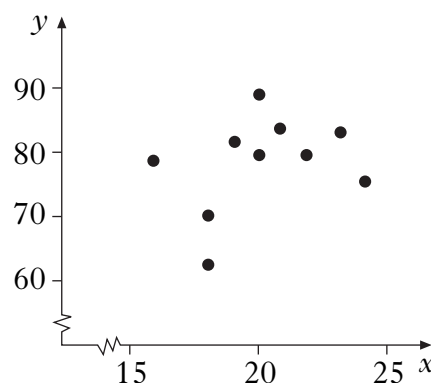
$$b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient: $r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

ALL questions should be attempted.

Marks

1. 10 students, chosen at random, were asked to record their study time for a particular course over a 4-week period. The total study time, x hours, and the corresponding test scores, $y\%$, are shown in the scatter diagram.

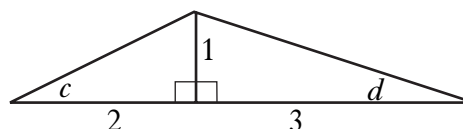


For this sample

$$\sum x = 201, \sum x^2 = 4095, \sum y = 785, \sum y^2 = 62151, \sum xy = 15835.$$

- (a) Find the least squares regression line of y on x . 4
 (b) Estimate the test score of a student who studied for 20 hours. 1

2. The diagram shows two right-angled triangles with angles c and d marked as shown.

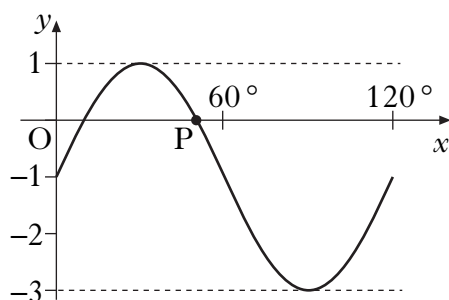


- (a) Find the exact value of $\sin(c + d)$. 4
 (b) (i) Find the exact value of $\sin 2c$. 4
 (ii) Show that $\cos 2d$ has the same exact value. 4

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. 6

4. The diagram shows part of the graph of a function whose equation is of the form $y = a \sin(bx^\circ) + c$.

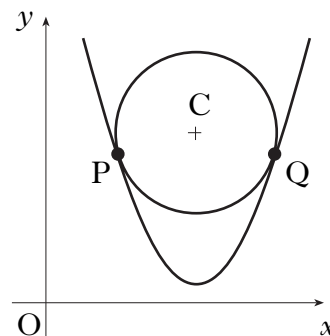
- (a) Write down the values of a , b and c . 3
 (b) Determine the exact value of the x -coordinate of P, the point where the graph intersects the x -axis as shown in the diagram. 3



[Turn over

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q .

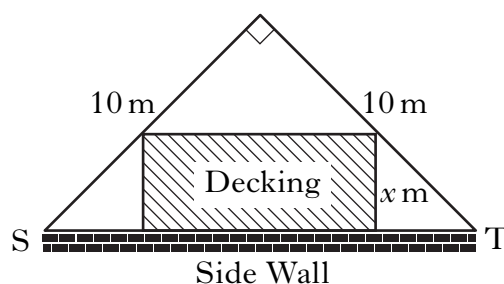
- (a) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q .
- (b) Find the coordinates of P .
- (c) Find the coordinates of C , the centre of the circle.



5
2
2

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



- (a) (i) Find the exact value of ST .
- (ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = (10\sqrt{2})x - 2x^2.$$

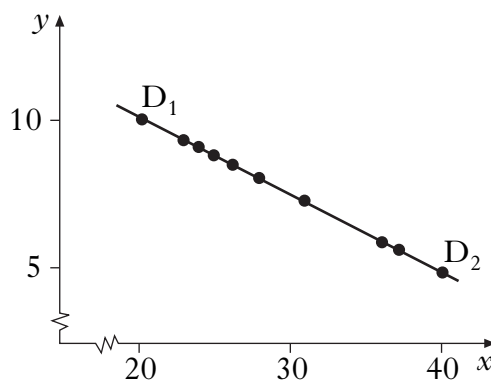
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7. A scatter diagram is drawn for a data set, as shown.

- (a) State the correlation coefficient.
- (b) The two data points shown are $D_1(20, 10)$ and $D_2(40, 5)$.

Find the relationship between x and y .

- (c) Make a prediction for y when $x = 50$ and comment.



1
2
2

8. A random variable X has the following probability distribution.

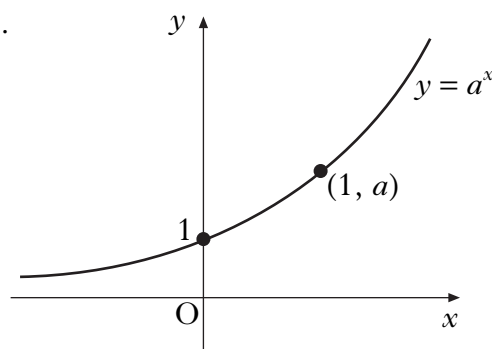
x	1	2	3	4
$P(X = x)$	$\frac{k}{2}$	$\frac{2k}{3}$	$\frac{k}{3}$	$\frac{k}{6}$

- (a) Calculate the value of k . 2
 (b) Find $E(X)$ and $\text{Var}(X)$. 6

9. The diagram shows the graph of $y = a^x$, $a > 1$.

On separate diagrams, sketch the graphs of:

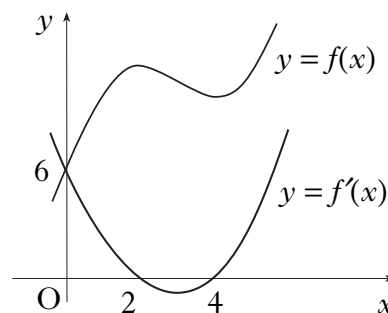
- (a) $y = a^{-x}$; 2
 (b) $y = a^{1-x}$. 2



10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point $(0, 6)$.

The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$.



- (a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:
 (i) write down the values of a and b ;
 (ii) find the value of k . 3
 (b) Find the equation of the graph of the cubic function $y = f(x)$. 4

11. For a certain badminton competition a team has to consist of 4 men and 4 women. In how many ways can the team be chosen from a pool of 6 men and 8 women? 4

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