National
Qualifications
2023 MODIFIED

X847/76/11

Mathematics
Paper 1 (Non-calculator)

THURSDAY, 4 MAY
9:00 AM – 10:15 AM

Total marks — 55

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Total marks — 55

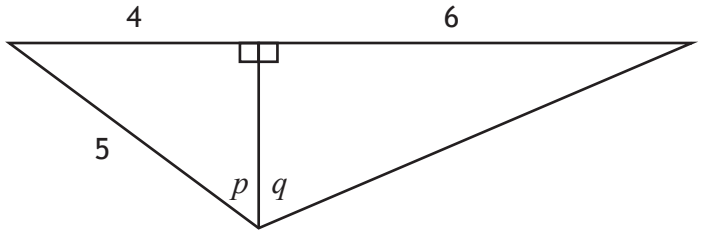
Attempt ALL questions

1. Given that $y = x^{\frac{5}{3}} - \frac{10}{x^4}$, where $x \neq 0$, find $\frac{dy}{dx}$. **3**

2. P and Q are the points $(-2, 6)$ and $(10, 0)$.
Find the equation of the perpendicular bisector of PQ. **4**

3. Solve $\log_5 x - \log_5 3 = 2$. **3**

4. The diagram shows two right-angled triangles with angles p and q as marked.



(a) Determine the value of:

(i) $\cos p$ **1**

(ii) $\cos q$. **1**

(b) Hence determine the value of $\cos(p + q)$. **3**

MARKS

5. The equation $2x^2 + (3p - 2)x + p = 0$ has equal roots.

Determine the possible values of p. **3**

6. Find $\int (2x^5 - 6\sqrt{x})\,dx,\ x \geq 0$. **4**

7. (a) Evaluate $\log_2 5 + \log_2 \frac{1}{40}$. **2**

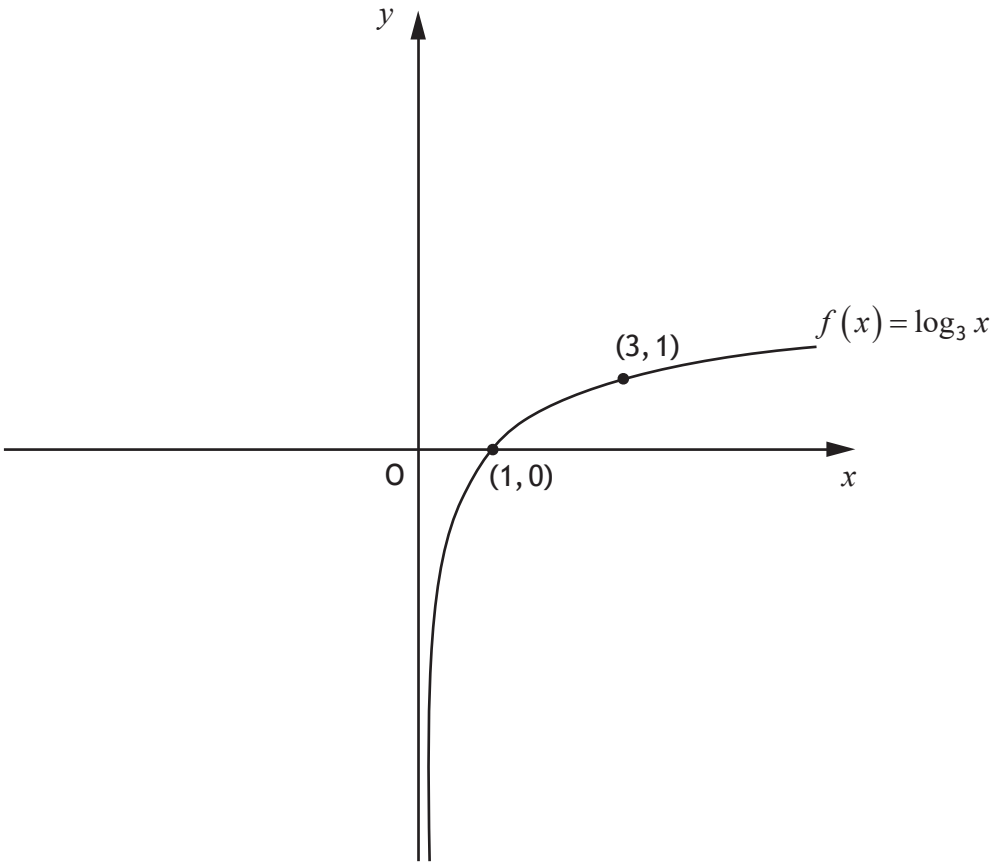
(b) Given that $a \in \mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of a. **1**

8. A function, f, is defined on $\mathbb{R}$, the set of real numbers, by $f(x) = x^3 + 3x^2 - 9x + 5$.

Find the coordinates of the stationary points of f and determine their nature. **6**

MARKS

9. The diagram shows the graph of the function $f(x) = \log_3 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of $y = f^{-1}(x) - 1$. **3**

10. (a) Show that $(x+5)$ is a factor of $x^4 + 3x^3 - 7x^2 + 9x - 30$. **2**

(b) Hence, or otherwise, solve $x^4 + 3x^3 - 7x^2 + 9x - 30 = 0$, $x \in \mathbb{R}$. **5**

[Turn over

MARKS

11. (a) Evaluate $\int_{\frac{\pi}{2}}^{\pi} \left(5\sin x - 3\cos x\right)\, dx$. **3**

The diagram in your answer booklet shows the graphs with equations $y = 5\sin x$ and $y = 3\cos x$, $0 \leq x \leq 2\pi$.

(b) On the diagram in your answer booklet, shade the area represented by the integral in (a). **1**

12. Express $-2x^2 - 12x + 7$ in the form $a\left(x+b\right)^2 + c$. **3**

13. Functions f and g are defined by:

- $f\left(x\right) = 2\sin x$, where $0 < x < \frac{\pi}{2}$
- $g\left(x\right) = 2x$, where $0 < x < \frac{\pi}{4}$

(a) (i) Evaluate $f\left(g\left(\frac{\pi}{6}\right)\right)$. **1**

(ii) Determine an expression for $f\left(g\left(x\right)\right)$. **2**

(b) (i) Given that $f\left(p\right) = \frac{1}{3}$, determine the exact value of $\sin p$. **1**

(ii) Hence, determine the exact value of $f\left(g\left(p\right)\right)$. **3**

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