Research and Information Services

MONITORING STANDARDS REPORT

SCOTTISH QUALIFICATIONS AUTHORITY

Comparison of CSYS Mathematics 1992, 1998 and Advanced Higher Mathematics 2004

1 Syllabus

1.1 General approach

Prior to 1992 there was a menu of five papers at CSYS level:

Paper 1: Algebra and Number Systems
Paper 2: Geometry and Calculus
Paper 3: Probability and Statistics
Paper 4: Numerical Analysis
Paper 5: Mathematical Topics in Mechanics

Many schools could offer only one course -- Paper 2 was the most popular.

Paper 1 had a smaller but still significant uptake. Candidates taking Paper 1 would normally also have taken Paper 2. In 1992 new Revised CSYS courses were introduced, but for that year only schools were given the choice of which syllabuses to offer.

By 1998 the Revised CSYS courses were fully in place, still with five papers:

Paper 1: The General paper
Paper 2: Pure Mathematics
Paper 3: Statistics
Paper 4: Numerical Analysis
Paper 5: Mechanics

As before, many schools could offer only one of the papers, and Paper 1 became the natural choice. It drew together the main elements of the earlier Papers 1 and 2 and formed a natural development of Higher Mathematics.

It also included introductory concepts from the three Applied courses. By 2004 the structure had again changed. There were now two papers at Advanced Higher level:

Mathematics (AH)
Applied Mathematics (AH)

The Mathematics course consisted of three units (Units 1, 2, and 3) containing the essential concepts from the previous General course, together with some earlier topics that were reintroduced. There were no Applied topics in the Mathematics units.

Our sample for 1992 contained scripts from the unrevised Paper 1 and Paper 2, but we have only reviewed the latter because most candidates sat only Paper 2 at that time. Our sample for 1998 contained scripts only from Paper 1. For 2004, our sample contained scripts only from the Mathematics paper. Therefore our comparisons are based on the 3 papers: 1992 Paper 2 (Geometry and Calculus), 1998 Paper 1 (General) and 2004 Mathematics (AH).

1.2 Changes in the syllabus

These are spelled out in the Appendix. The issue of whether or not the syllabus is now more or less demanding is not straightforward because of the changes outlined in 1.1. But in so far as it is possible to answer this, our judgement is that the level of demand is similar.

1.3 Depth and breadth of the syllabus

Again, structural changes make this difficult to answer. But our judgment is that they remain comparable.

2 Assessment instrument

2.1 Trends or gradual changes

In 1992, all of the exams had optional questions, allowing candidates to concentrate on selected parts of the content. All questions in each of two sections, with Section 2 containing longer questions, had to carry equal weight. In 1998, the element of choice in the question papers was removed, allowing setters to use questions varying in difficulty. In 2004, candidates had to pass unit assessments for each of three units (Units 1, 2 and 3) as well as sit the final exam. There was no choice of questions at any stage.

The marking instructions did not seem to be significantly different.

2.2 Repetition of older items in recent question papers

The 2004 Mathematics (AH) paper contains 16 questions to be done in three hours, with all questions to be answered. The questions vary in length and level of difficulty, from 3 marks available (Question 2) to 11 marks (2 parts) for Question 15. About 60 marks are available for short questions and 40 for longer questions.

The corresponding 1998 paper (Paper 1, General) again requires all questions to be answered, but this time in 2.5 hours. The questions are grouped into 2 sections: Section 1 contains 12 relatively short questions (ranging from 3 marks to 7 marks); Section 2 contains 4 questions, 2 with 9 marks, 1 with 10 and 1 with 12.

The corresponding 1992 paper (Paper 2, Geometry and Calculus) requires 8 questions to be done in 2.5 hours, 4 out of 7 from Section A and 4 out of 6 from Section B. The Section A questions are each worth 9 marks and the Section B questions are each worth 16 marks. So the combination of short and long questions in roughly the same proportion has been retained. The major structural difference is the element of choice in the 1992 paper.

Changes to the syllabus have already been identified.

Over the three years being considered, questions on differentiation, partial fractions, integration, some differential equations and an analysis of curves in 2 dimensions are essentially the only things which all these core papers have in common. Plotting curves, which appears in 1992 and 1998, is less prominent in 2004, doubtless due to the availability of graphics calculators. The 1992 paper is focused on geometry and calculus, and in addition to the topics mentioned above, contains a question on partial differentiation (worth 9 marks) and two questions on 3-dimensional geometry (worth 9 and 16 marks respectively).

The 1998 General paper includes quite a few topics that are now restricted to the specialist Statistics, Mechanics and Numerical Analysis material in Applied Mathematics (AH) (questions 2, 4, 5, 10, 11(b)), as well as material on recurrence relations not covered in either the 1992 or 2004 papers. Complex numbers, matrices, sequences and series, and binomial expansions are examined in 1998 and 2004, and in 2004 (only), there are questions on Maclaurin series, the Euclidean algorithm, odd/even functions, volumes and induction. Some material in the 1992 paper makes reappearance in 2004, for example 3-dimensional geometry.

It is easiest to compare the 1998 and 2004 papers because the style is similar, and our conclusion is that there is no great difference between them.

Because the style of the 1992 paper is very different, more can be said. In particular, the presence of longer questions means that these can be constructed in such a way that they end up with quite challenging bits, and quite a depth of knowledge can be needed to get to the end (for example Question 8). There is also some evidence that perhaps a little more was expected of candidates in 1992. For example, comparing directly two questions on integration, Question 4(b) in 1992 requires knowledge of the candidate that partial fractions must be used to simplify the integrand, although this is not made explicit; by contrast, in the 2004 paper, Question 5 directs the candidates to provide the partial fractions as a first step to the solution. However, if it appears as though the questions in the earlier paper can be a little more demanding, this has to be offset by the fact that in 1992, there was a choice of questions. So, while the 2004 paper does not necessarily require the same depth of knowledge as the 1992 paper, a broader knowledge is certainly expected. Our view is that the number of marks available for corresponding tasks does not seem to have changed significantly over the years.

2.2.1 Do candidates in all years perform in the same way on these questions? If no, are there any trends discernible?

The evidence of the scripts (see Section 3 below) suggests that in similar questions, candidates at the same grade perform in a similar way.

2.3 Coverage of the syllabus

The coverage of the syllabus in the question papers seems comparable to previous years.

2.4 Level of demand of questions and marking scheme

Where comparisons were possible (for example in questions on integration, differentiation, partial fractions, differential equations) the standard seemed comparable, and marks available for doing particular tasks seemed not to have changed significantly.

It was possible to identify some 1992 questions which seemed rather more demanding (eg Q4(b) in 1992 compared with Q5 in 2004), but the element of choice was present in 1992. So one trades slightly more demanding (and longer) questions, perhaps requiring a greater depth of knowledge, for the greater breadth required when there is no choice of questions. If one accepts that these effectively compensate for each other, then the current question papers are comparable to earlier ones.

The marking schemes seemed comparable.

3 Scripts

The relevant scripts available to us were:

1992, Paper 2: 2 Candidates gaining each of A, B, C, D
1998, Paper 1: 3 Candidates gaining each of A, B, C, D
2004 Mathematics (AH): 3 Candidates gaining each of A, B, C, D.

3.1 A/B graded scripts

In terms of the marks required, the A/B borderline has not changed much, moving from 75% → 72% → 75% as we go from 1992 to 2004.

Starting with 2004, the A candidates were all borderline candidates. The level of performance seemed appropriate to an A grading, and they generally did well on calculus, and coped less well on topics like induction, or odd/even functions.

Turning to 1998, again these were borderline candidates, and showed evidence of similar abilities to the 2004 cohort. On the sample provided, there was perhaps evidence that the presentation of the more recent candidates is not quite so good.

For the 1992 candidates, both scored very well and were very clear A passes, with marks well clear of the boundary. Borderline cases in all years would have allowed us more readily to compare like with like, but overall, we found no evidence that an A pass was any easier to get in 2004 than in 1998 or 1992.

The movement in the B/C boundary is 61% → 58% → 62%; so again there is little change.

3.2 C/D graded scripts

Again in terms of marks alone, the B/C boundary has not changed much, moving from 47% → 44% → 50% as we move from 1992 to 2004. Our conclusion, based on the evidence available, is also that it is no easier to get a pass in 2004 than in the earlier years.

The two 1992 C grade candidates whose scripts were available appeared better, but they were mid-range examples, with one closer to the B/C borderline than the C/D borderline. As might be expected, the C candidates did not perform so well on the longer questions, although some were able to gain good marks on a small number of these questions.

The performance of the D grade candidates was characterised in all years (as one might expect) by increased patchiness of performance: for example many of the (compulsory) questions in 2004 and 1998 were not attempted or scored zero marks. This was particularly so of the longer questions, again as might be expected.

The two D candidates in 1992 had widely differing marks (unlike most of the other groups), with one being particularly poor. Any conclusions have to be qualified by the observations that (1) in 1992 there was a choice of questions, and (2) there have been quite large changes to the syllabus over all three years of the study.

But where comparability is possible, our conclusion based on the evidence presented is that standards (in particular what is required to obtain an A grade, or to get a C grade) have not changed in any significant way from 1992 through to 2004.

Appendix: 2004, 1998 and 1992 syllabuses

Unit 1

	1998 Paper 1	1992 Paper 2
1.1 Algebra		
Binomial theorem	Yes	Yes
Partial fractions	Yes	Yes
1.2 Differentiation		
Rules	Yes	Yes
Special functions	Yes - other than sec, cosec, cot	Yes
Modelling	Some	
1.3 Integration		
Rules (really H grade)	Yes	
Special functions	Similar	Yes
Substitution	Yes	Yes
Limits of areas	Yes	In H grade
Evaluation of areas	Yes?	In H grade
Evaluation of volumes	No	In H grade
1.4 Properties of functions		
Graph sketching and analysis	Broadly similar	Yes
1.5 Systems of linear equations		
Gaussian elimination	Yes	Covered in Paper 4
Ill-conditioning	Yes	Covered in Paper 4

Unit 2

	1998 Paper 1	1992 Paper 2
2.1 Further differentiation		
Inverse trig. functions	Yes	Yes
General inverse functions	Implied	Yes
Implicit differentiation	No	Yes
Logarithmic differentiation	No	Yes
Parametric differentiation	No	Yes
2.2 Further integration		
Inverse trig. functions	Yes	Yes
Partial fractions	Yes	Yes
Integration by parts	Yes	Yes
Variables separable	Yes	Yes
2.3 Complex numbers		
Elementary properties	Yes	Only in Paper 1
Polar form	Yes	
Equations	Yes	
Locus	No	
de Moivre's theorem	Yes	
Application of de Moivre's	No	
2.4 Sequences and series		
Arithmetic	Yes	In H grade
Geometric	Yes	
Sigma notation	Yes	No
2.5 Elementary number theory and methods of proof		
Proof	In Paper 2	In Paper 1
Mathematical induction	In Paper 2	In Paper 1

Unit 3

	1998 Paper 1	1992 Paper 2
3.1 Vectors		
Vector products	In Paper 2	Yes
Three dimensions	In Paper 2	Yes
3.2 Matrix algebra		
Basic operations	Broadly similar	Paper 1 (probably more)
Inverses	Broadly similar	Paper 1 (probably more)
3.3 Further sequences and series		
Maclaurin series	No	Paper 4 (much more)
Simple iteration	Yes	Paper 4 (much more)
3.4 Further ordinary differential equations		
Use integrating factors	No	Yes
Second order equations	No	Yes (maybe deeper)
3.5 Further number theory and further methods of proof		
$\sum r^2$, $\sum r^3$	Yes	Not clear
Euclidean algorithm	Paper 2	Paper 1

Topics not included in 2004 Mathematics (AH) syllabus

From 1992 Paper 1

Section 2 (Sets relations and mappings)	Not covered at all
Section 5 (Elementary Number Theory)	Sections not covered: 5.2; 5.6; 5.10; 5.13
Section 6 (Inequalities)	Not covered at all (but a very short section)
Section 7 (Complex numbers)	Sections not covered: 7.6; 7.8
Section 8 (Matrices)	Section not covered: 8.5 (but treatment is generally deeper than at present)
Section 9 (Algebraic structures)	Not covered at all

From 1992 Paper 2

Section 2 (Conics)	Not covered at all
Section 5 (Calculus)	Section not covered: 5.10
Section 8 (Calculus)	Section not covered: 8.3

From 1998 Paper 1

4(k) Newton's method
7(i) $\tan(A \pm B)$
8(o) trapezium rule
8(p) numerical differentiation
Section 9 - mechanics
Section 10 - statistics