SQA Advanced Unit Specification

General information for centres

Unit title: Engineering Mathematics 1 (SCQF level 6)

Unit code: HP48 46

Superclass: RB

Publication date: August 2017

Source: Scottish Qualifications Authority

Version: 01

Unit purpose

This Unit is designed to develop or consolidate a basic level of mathematical skills required of learners across a range of Engineering disciplines. The Unit provides learners with opportunities to develop knowledge, understanding and skills to solve mathematical problems including polynomial, trigonometrical, exponential and logarithmic functions. Learners will also be introduced to 3-dimensional vectors and complex numbers.

Outcomes

On successful completion of the Unit the learner will be able to:

1 Solve problems involving functions and trigonometric equations.
2 Solve problems involving exponential and logarithmic equations.
3 Apply mathematical techniques involving vectors and complex numbers.

Credit points and level

1 SQA Credit at SCQF level 6: (8 SCQF credit points at SCQF level 6)

Recommended entry to the Unit

Entry requirements are at the discretion of the centre. However, it would be advantageous if learners had good algebraic skills together with an ability to transpose engineering formulae and solve simple trigonometric equations. This knowledge and understanding may be

evidenced by possession of the NQ Unit *Mathematics: Technician 1* or a pass at National 5 in Mathematics.

Core Skills

Achievement OF this Unit gives automatic certification of the following Core Skills component:

Complete Core Skill	None
Core Skill component	Using Number at SCQF level 6

There are also opportunities to develop aspects of Core Skills which are highlighted in the Support Notes for this Unit specification.

Context for delivery

If this Unit is delivered as part of a Group Award, it is recommended that it should be taught and assessed within the subject area of the Group Award to which it contributes.

The Assessment Support Pack (ASP) for this Unit provides assessment and marking guidelines that exemplify the national standard for achievement. It is a valid, reliable and practicable assessment. Centres wishing to develop their own assessments should refer to the ASP to ensure a comparable standard. A list of existing ASPs is available to download from SQA's website **(http://www.sqa.org.uk/sqa/46233.2769.html)**.

Equality and inclusion

This unit specification has been designed to ensure that there are no unnecessary barriers to learning or assessment. The individual needs of learners should be taken into account when planning learning experiences, selecting assessment methods or considering alternative evidence.

Further advice can be found on our website www.sqa.org.uk/assessmentarrangements.

Unit specification: statement of standards

Unit title: Engineering Mathematics 1 (SCQF level 6)

Acceptable performance in this Unit will be the satisfactory achievement of the standards set out in this part of the Unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to SQA.

Where evidence for Outcomes is assessed on a sample basis, the whole of the content listed in the Knowledge and/or Skills section must be taught and available for assessment. Learners should not know in advance the items on which they will be assessed and different items should be sampled on each assessment occasion.

Outcome 1

Solve problems involving functions and trigonometric equations.

Knowledge and/or Skills

- Properties of functions
- Inverse functions
- Composite Functions
- Trigonometric equations

Outcome 2

Solve problems involving exponential and logarithmic equations.

Knowledge and/or Skills

- Evaluation of logarithmic and exponential expressions
- Transposition from logarithmic to exponential form and vice versa
- Laws of logarithms
- Graphics of logarithmic and exponential functions

Outcome 3

Apply mathematical techniques involving vectors and complex numbers.

Knowledge and/or Skills

- Collinearity
- Addition, subtraction and scalar multiplication of vectors
- Scalar product
- Conversion of complex numbers between rectangular and polar form
- Addition and subtraction of complex numbers
- Multiplication and division of complex numbers
- Representation of complex numbers on an Argand Diagram

Evidence Requirements for this Unit

A sampling approach will be used in the assessment of the Knowledge and/or Skills in this Unit. Learners will need to provide written and/or recorded oral evidence to demonstrate their Knowledge and/or Skills across all Outcomes by showing they can:

Outcome 1

Provide evidence of the following Knowledge and/or Skills each time this Outcome is assessed:

- Solve two problems (one in degrees and one in radians) involving expressions of the form $A\sin(nx \pm a) = b$ or $A\cos(nx \pm a) = b$

Provide evidence of **two out of three** Knowledge and/or Skills remaining in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Identify the domain, range and asymptotes of a given function
- Solve one problem involving an inverse function
- Solve one problem involving a composite function

Outcome 2

Provide evidence of the following Knowledge and/or Skills each time this Outcome is assessed.

Solve three problems to cover the Knowledge and/or Skills as follows, combining them where appropriate:

- Evaluate formulae; one must include exponentials and one must include logarithms
- Transpose expressions from exponential form to logarithmic form and vice versa
- Use two out of the three laws of logarithms:
  - $\log x + \log y = \log xy$
  - $\log x - \log y = \log \frac{x}{y}$
  - $p \log x = \log x^p$
- Sketch or identify a graph of either a logarithmic function (of the form $y = A In x$ or $y = A\log_{10} x$) or an exponential function (of the form Ae^x or $A.10^x$)

Fractional and negative coefficients and indices should be included.

Outcome 3

Provide evidence of **five out of seven** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Verify that three, 3-dimensional coordinates (A,B,C) are collinear and determine the ratio of AB to BC
- Solve two vector problems involving a combination of at least two of the following: addition of vectors; subtraction of vectors and multiplication of a vector by a scalar

- Calculate the angle between two 3-dimensional vectors using the scalar product
- Convert one vector (2-dimensional cases) or complex number in Cartesian form to polar form and one vector or complex number in polar form to Cartesian form showing clearly all working
- Solve one problem involving the addition and subtract of complex numbers (in rectangular form)
- Solve one problem involving the multiplication of two complex numbers (in rectangular form) and one problem involving the division of two complex numbers (in polar form)
- Represent accurately complex numbers in all four quadrants of an Argand Diagram

It is recommended that the assessment for all three Outcomes takes places at a single end of Unit assessment event. Outcomes may also be assessed individually. All re-assessments should be based on a different assessment instrument. This should re-assess all three Outcomes or a full individual Outcome reflecting the format of the original assessment. All re-assessments should be based on a different sample of Knowledge and/or Skills.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions.

Computer algebra must not be used in the assessment of this Unit.

Unit specification: support notes

Unit title: Engineering Mathematics 1 (SCQF level 6)

Unit Support Notes are offered as guidance and are not mandatory.

While the exact time allocated to this Unit is at the discretion of the centre, the notional design length is 40 hours.

Guidance on the content and context for this Unit

This Unit is one of a suite of five Units in Mathematics developed for SQA Advanced Qualifications across a range of engineering disciplines. The five Units are:

Engineering Mathematics 1
Engineering Mathematics 2
Engineering Mathematics 3
Engineering Mathematics 4
Engineering Mathematics 5

In the development of this Unit a list of topics expected to be covered by lecturers has been identified. Recommendations have also been made on how much time lecturers should spend on each Outcome. The use of this list of topics is strongly recommended to ensure continuity of teaching and learning and adequate preparation for the assessment of the Unit. Consideration of this list of topics alongside the Assessment Support Pack developed for this Unit will provide clear indication of the standard expected in this Unit.

Outcome 1 (10 hours)

Solve problems involving functions and trigonometric equations

- Introduce the concept of the function (this may be done in terms of set theory and, in particular, a function or mapping as a rule which relates each element in one set to one and only one element in another set)
- List typical functions that occur in Engineering
- Introduce typical mathematical notation used with functions (eg $f(x)$, $x \epsilon R$, (or N, W, Z, Q etc.)
- Explain what is meant by an independent and dependent variable
- Define the terms domain and range of a function
- Introduce the concept of a pole (s) in a function and explain the impact of a pole (s) on the domain of a function which has one or more poles
- Demonstrate the way in which to determine the range and domains of functions
- Define the term asymptotes
- Demonstrate the way in which to determine the asymptotes to functions
- Introduce the idea of an inverse function $f^1(x)$
- Determine the inverse functions of a range of functions
- Graph functions and their inverse functions
- Introduce composite functions (again this may be done in terms of set theory)
- Derive and evaluate a number of composite functions

- Introduce the concept of the radian

  Convert degrees to radians and vice versa (eg $\text{rad} = \frac{\theta}{180} \times \pi$)
- Determine the solutions of equations of the form $A\sin(nx+\alpha) = \pm B$ or $A\cos(nx+\alpha) = \pm B$

Outcome 2 (8 hours)

Transpose and solve exponential and logarithmic equations

- Introduce the exponential function $y = a^x$
- Plot graphs of this function for different values of a
- Plot a graph of $y = a^{-x}$ to demonstrate that this produces the image of $y = a^x$
- Explain that $x = \log_a$ y is the inverse function of $y = a^x$
- Examine $y = 10^x$ and $x = \log_{10} y$ as an important example of exponential and logarithmic functions
- Plot $y = 10^x$ and $x = \log_{10} y$ for a specified domain
- Examine $y = e^x$ and $x = \log_e y$ as a further important example of exponential and logarithmic functions especially in the context of Engineering
- Plot $y = e^x$ and $x = \log_e y$ for a specified domain
- Evaluate expressions that include $10^x, \log_{10} y, e^x$ and $\log_{10} y$
- Introduce the three laws of logarithms
- Solve problems involving the three laws of logarithms
- Transpose and evaluate expressions from Engineering which include the term Ae^{kt} or Ae^{-kt}
- Solve exponential equations $(5^x = 95, x^{3.5} = 0.1)$
- Solve logarithmic equations (eg $\log_{10}(x+2) + \log_{10}(x+4) = 0.5$; $\log_e(t-3) - \log_e(t+3) = \log_e 6$)

Outcome 3 (14 hours)

Apply mathematical techniques to vectors and complex numbers

2-d and 3-d Vectors

- Introduce the concept of a vector and scalar and give examples of each form from Engineering
- Outline notation commonly used with vectors
- Demonstrate the way in which to add and subtract 2-dimensional vectors graphically
- Introduce 2-dimensional vectors in component form
- Add and subtract vectors using their component form
- Multiply 2-dimensional vectors by a scalar
- Calculate the magnitude of 2-dimensional vectors using the equation

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Introduce 3-dimension coordinates and the representation of 3-dimensional vectors in component form
- Calculate the magnitude of 3-dimensional vectors using the equation

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Apply the concept of collinearity to 2- and 3- dimensional vectors
- Introduce the scalar product for 2-d and 3-d vectors
- Identify the properties of the scalar product (eg $a.b = b.a$, $a.(b+c) = a.b + a.c$ etc)
- Demonstrate the way in which the scalar product can be used to find the angle between two vectors

Complex Numbers

- Introduce complex numbers (different approaches may be taken to this, for example what happens when the determinant $b^2 - 4ac < 0$, or $i = \sqrt{-1}$ etc.)
- Identify normal notation used with complex numbers (engineers tend to use j while mathematicians use i)
- Introduce the Argand Diagram
- Demonstrate the representation of complex numbers in Cartesian and polar form (noting the domain of principal argument, $\arg(z)$)
- Demonstrate the way in which to convert between Cartesian and polar form and vice versa
- Add and subtract complex numbers
- Multiply complex numbers in both Cartesian and polar form
- Introduce the complex conjugate
- Divide complex numbers in both Cartesian and polar form
- Represent complex numbers in all four quadrants of an Argand Diagram

Guidance on approaches to delivery of this Unit

This Unit provides core mathematical principles and processes which underpin much of the studies undertaken in a number of SQA Advanced Qualifications across a range of engineering disciplines. It is recommended that the Unit be delivered towards the beginning of these awards.

Centres may deliver the Outcomes in any order they wish, it is recommended that Outcome 1 is delivered first followed by Outcomes 2 and then Outcome 3.

It is recommended that Unit delivery is principally undertaken using a didactic approach. All teaching input should be supplemented by a significant level of formative assessment in which learners are provided with opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the Unit. Computer software and computer algebra may be used to support learning (eg to confirm the solutions of mathematical problems), but it is strongly recommended that such learning resources are only used in a supportive capacity and not as the principal means of delivering Unit content.

Guidance on approaches to assessment of this Unit

Evidence can be generated using different types of assessment.

A recommended approach is the use of an examination question paper. The question paper should be composed of an appropriate balance of short answer, restricted response and structured questions.

All assessment papers should be unseen by learners prior to the assessment event and at all times, the security, integrity and confidentiality of assessment papers must be ensured. Assessment should be conducted under closed-book, controlled and invigilated conditions.

The questions in the examination should not be grouped by Outcome or be labelled in terms of the Outcomes they relate to when a single end-of-Unit examination is used.

The summative assessment of all three Outcomes — whether individually or at a single assessment event — should not exceed two hours. When assessing a learner's responses to summative assessment lecturers should concentrate principally on the candidate's ability to apply the correct mathematical technique and processes when solving problems. Candidates should not be penalised for making simple numerical errors. An appropriate threshold score may be set for the assessment of this Unit. A threshold score should be used for each assessment if Outcome level assessment is used.

Learners should be provided with a formulae sheet appropriate to the content of this Unit when undertaking their assessment. Computer algebra should not be used in the assessment of this Unit.

It is the learner's responsibility to ensure that any calculator they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- complex number manipulation
- communication with other machines or the internet.

In addition, any calculator used by learners should have no retrievable information stored in them. This includes:

- data banks
- dictionaries
- mathematical formulae

Centres are reminded that prior verification of centre-devised assessments would help to ensure that the national standard is being met. Where learners experience a range of assessment methods, this helps them to develop different skills that should be transferable to work or further and higher education.

Opportunities for e-assessment

E-assessment may be appropriate for some assessments in this Unit. By e-assessment we mean assessment which is supported by Information and Communication Technology (ICT), such as e-testing or the use of e-portfolios or social software. Centres which wish to use e-assessment must ensure that the national standard is applied to all learner evidence and that conditions of assessment as specified in the Evidence Requirements are met, regardless of the mode of gathering evidence. The most up-to-date guidance on the use of e-assessment to support SQA's qualifications is available at **www.sqa.org.uk/e-assessment**.

Opportunities for developing Core and other essential skills

This Unit has the Using Number component of Numeracy embedded in it. This means that when candidates achieve the Unit, their Core Skills profile will also be updated to show that they have achieved Using Number at SCQF level 6.

Administrative information

Version	Description of change	Date

SQA acknowledges the valuable contribution that Scotland's colleges have made to the development of SQA Advanced Qualifications.

FURTHER INFORMATION: Call SQA's Customer Contact Centre on 44 (0) 141 500 5030 or 0345 279 1000. Alternatively, complete our Centre Feedback Form.

General information for candidates

Unit title: Engineering Mathematics 1 (SCQF level 6)

This section will help you decide whether this is the Unit for you by explaining what the Unit is about, what you should know or be able to do before you start, what you will need to do during the Unit and opportunities for further learning and employment.

The *Engineering Mathematics 1* Unit is one of a suite of five Units in Mathematics developed for SQA Advanced Certificates and Diplomas across a range of Engineering disciplines. The five Units help develop the mathematical skills required for workplace roles and for more advanced studies in Engineering, for example, articulation to degree study at university.

The Unit is mandatory in a number of SQA Advanced Certificates in Engineering.

This Unit is designed to develop or consolidate a basic level of mathematical skills required of learners across a range of Engineering disciplines. You will learn about the properties of some mathematical functions commonly used in Engineering including linear, trigonometrical, exponential and logarithmic functions. You will also learn how to solve trigonometrical, exponential and logarithmic equations. You will also study 2 and 3-dimensional vectors and complex numbers which are widely used in mechanical and electrical Engineering.

It is likely that Unit delivery will comprise of a significant teaching input from your lecturer. This will be supplemented by tutorial exercises which will allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes covered in the Unit to a range of Engineering problems.

Depending on which centre you attend, formal assessment may be conducted on an Outcome by Outcome basis or by one single assessment. Assessment will be conducted under closed-book, controlled and invigilated conditions.

Learners considering taking this Unit will normally be expected to have passed the NQ Unit *Mathematics: Technician 1*, or National 5 in Mathematics or National 5 in Mathematics or equivalent.