SQA Advanced Unit Specification

General information

Unit title: Engineering Mathematics 4 (SCQF level 8)

Unit code: HT03 48

Superclass: RB

Publication date: August 2017

Source: Scottish Qualifications Authority

Version: 01

Unit purpose

This Unit is designed to develop the mathematical skills required by learners who wish to use their SQA Advanced Diploma in Engineering to articulate to university degree study. The Unit will provide learners with the opportunities to develop the knowledge, understanding and skills to solve problems involving the use of complex algebra, matrices and mathematical series. Learners will also learn techniques to solve first order differential equations.

Outcomes

On successful completion of the Unit the learner will be able to:

1 Solve problems involving complex algebra.
2 Solve problems involving the manipulation of matrices.
3 Solve problems requiring the use of series representation of functions.
4 Solve first order differential equations.

Credit points and level

1 SQA Credit at SCQF level 8: (8 SCQF credit points at SCQF level 8)

Recommended entry to the Unit

Entry requirements are at the discretion of the centre. However, it would be advantageous if learners had a good knowledge and understanding of differential and integral calculus together with sound numerical and algebraic skills. This knowledge and understanding may be evidenced by possession of the SQA Advanced Units *Engineering Mathematics 2* and *Mathematics 3*.

Core Skills

Achievement OF this Unit gives automatic certification of the following Core Skills component:

Complete Core Skill	None
Core Skill component	Using Number at SCQF level 6

There are also opportunities to develop aspects of Core Skills which are highlighted in the Support Notes for this Unit specification.

Context for delivery

If this Unit is delivered as part of a Group Award, it is recommended that it should be taught and assessed within the subject area of the Group Award to which it contributes.

The Assessment Support Pack (ASP) for this Unit provides assessment and marking guidelines that exemplify the national standard for achievement. It is a valid, reliable and practicable assessment. Centres wishing to develop their own assessments should refer to the ASP to ensure a comparable standard. A list of existing ASPs is available to download from SQA's website **(http://www.sqa.org.uk/sqa/46233.2769.html)**.

Equality and inclusion

This Unit specification has been designed to ensure that there are no unnecessary barriers to learning or assessment. The individual needs of learners should be taken into account when planning learning experiences, selecting assessment methods or considering alternative evidence.

Further advice can be found on our website **www.sqa.org.uk/assessmentarrangements**.

SQA Advanced Unit Specification: Statement of standards

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Acceptable performance in this Unit will be the satisfactory achievement of the standards set out in this part of the Unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to SQA.

Where evidence for Outcomes is assessed on a sample basis, the whole of the content listed in the Knowledge and/or Skills section must be taught and available for assessment. Learners should not know in advance the items on which they will be assessed and different items should be sampled on each assessment occasion.

Outcome 1

Solve problems involving complex algebra.

Knowledge and/or Skills

- Complex algebra and Argand Diagrams
- Euler's Relation
- DeMoivre's theorem

Outcome 2

Solve problems involving the manipulation of matrices.

Knowledge and/or Skills

- Definitions
- Matrix's operations
- Determinants
- Inverse Matrices
- Gaussian Elimination

Outcome 3

Solve problems requiring the use of series representation of functions.

Knowledge and/or Skills

- MacLaurin Series
- Taylor Series
- Expansion of products of functions using series forms
- Linear approximations of polynomial and non-linear functions

Outcome 4

Solve first order differential equations.

Knowledge and/or Skills

- Separation of variables
- Substitution
- Integrating Factor Method
- Initial value problems using the above methods

Evidence Requirements for this Unit

A sampling approach will be used in the assessment of the Knowledge and/or Skills in this Unit. Learners will need to provide written and/or recorded oral evidence to demonstrate their Knowledge and/or Skills across all Outcomes by showing they can:

Outcome 1

Provide evidence of **two out of three** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Evaluate one problem involving the multiplication of two complex numbers and one problem involving the division of two complex numbers and represent the results on an Argand Diagram
- Solve one problem involving the use of Euler Relation (eg converting a complex number/expression into exponential form or vice versa, representing a trigonometrical function in exponential form, etc)
- Solve one problem using De Moivre's theorem (eg raising a complex number to the n^{th} power, proving a trigonometrical identity, etc)

Outcome 2

Provide evidence of **three out of five** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Define the meaning of the term matrix in the context of Mathematics (this may include specials cases: square, identity, transpose, etc)
- Solve one problem involving the scalar product, addition or subtraction of two matrices and one problem involving the multiplication of two matrices
- Find the determinant of a 3 × 3 matrix
- Find the inverse of a 3 × 3 matrix
- Apply Gaussian Elimination to solve a system of linear equations containing three variables

Outcome 3

Provide evidence of **two out of four** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Derive the Taylor series of a given function
- Derive the MacLaurin series of a given function
- Expand the product of two functions using series forms
- Develop a linear approximation of one polynomial or one non-linear function

Outcome 4

Provide evidence of **two out of four** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Solve a first order differential equation using the method of separation of variables
- Solve a first order differential equation by substitution
- Solve a first order differential equation using the integrating factor method
- Solve a first order differential equation with an initial value using one of the three methods shown above

The assessment of all four Outcomes should take place at a single end of Unit assessment event. All re-assessments should be based on a different assessment instrument. This should re-assess all four Outcomes. All re-assessments should be based on a different sample of Knowledge and/or Skills.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions.

Computer algebra must not be used in the assessment of this Unit.

SQA Advanced Unit Specification:Support notes

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Unit Support Notes are offered as guidance and are not mandatory.

While the exact time allocated to this Unit is at the discretion of the centre, the notional design length is 40 hours.

Guidance on the content and context for this Unit

This Unit is one of a suite of five Units in Mathematics developed for SQA Advanced Qualifications across a range of Engineering disciplines. The five Units are:

Engineering Mathematics 1
Engineering Mathematics 2
Engineering Mathematics 3
Engineering Mathematics 4
Engineering Mathematics 5

In the development of this Unit a list of topics expected to be covered by lecturers has been identified. Recommendations have also been made on how much time lecturers should spend on each Outcome. The use of this list of topics is strongly recommended to ensure continuity of teaching and learning and adequate preparation for the assessment of the Unit. Consideration of this list of topics alongside the Assessment Support Pack developed for this Unit will provide clear indication of the standard expected in this Unit.

Outcome 1 (5 hours)

Solve problems involving complex algebra

- Introduce j in terms of $j^2 = -1$, $j = \sqrt{-1}$
- Complex and polar forms
- Representation of complex numbers on an Argand Diagram (comment on the domain of the principal argument, $\arg(z)$)
- Addition and subtraction of complex numbers
- Multiplication of complex numbers in rectangular and polar forms
- Complex conjugate
- Division of complex numbers in rectangular and polar form
- Introduce Euler's relation $e^{j\theta} = \cos\theta + j\sin\theta$ or $e^{-j\theta} = \cos\theta - j\sin\theta$

- $z = r(\cos\theta + \mathrm{j}\sin\theta) = re^{\mathrm{j}\theta}$ and $z = r(\cos\theta - \mathrm{j}\sin\theta) = re^{-\mathrm{j}\theta}$
- Solve problems involving the use of Euler's relation (eg express the following in exponential form: –10, (– 2 + j5), (1 – j)(1 + 3j): given that $z_1 = 6\mathrm{e}^{\mathrm{j}\pi/6}$ and $z_2 = 9\mathrm{e}^{-\mathrm{j}\pi/3}$ determine z_1z_2 and z_1/z_2 etc)
- Introduce De Moivre's theorem as $(\cos\theta + \mathrm{j}\sin\theta)^n = \cos n\theta + \mathrm{j}\sin n\theta$
- $z^n = r^n \angle n\theta$
- Solve problems involving De Moivre's theorem (eg express $(\sqrt{2} + \mathrm{j})^5$ in the form $a + \mathrm{j}b$, use De Moivre's theorem to express $\sin 3\theta$ as powers of $\sin\theta$ etc)
- Solve complex equations (eg $z^3 + 7z = 0$, $z^2 + 2z + 2 = 0$, $z^3 - 8 = 0$ etc)

Outcome 2 (10 hours)

Solve problems involving the manipulation of matrices

- Define what is meant by the term matrix in the context of Mathematics
- Identify some special matrices (eg square, identity, transpose, augmented)
- Multiply a matrix by a scalar
- Add and subtract matrices
- State the conditions for multiplying matrices
- Multiply matrices
- Explain the technique for calculating the determinant of a 2 × 2 matrix
- Explain what is meant by the minor and the cofactor of an element
- Explain the technique for calculating the determinant of a 3 × 3 matrix
- Calculate the determinants of 3 × 3 matrices
- Explain the technique for calculating the determinant of a 4 × 4 matrix to demonstrate the extension of the method to higher order matrices (if time permits)
- Calculate the determinants of simple 4 × 4 matrices (if time permits)
- Explain the way in which to find the inverse of a 2 × 2 matrix using determinant and cofactor method
- Determine the inverses of 2 × 2 matrices using the determinant and cofactor method
- Explain the determinant and cofactor method for finding the inverse of a 3 × 3 matrix
- Determine the inverses of 3 × 3 matrices using the determinant and cofactor method
- Explain the Gaussian elimination technique for finding the inverse of any matrix and apply it to examples up to and including echelon form
- Use Gaussian elimination techniques to solve Engineering related problems

Outcome 3 (5 hours)

Solve problems requiring the use of series representation of functions.

- Distinguish between a sequence and a series
- Brief treatment of arithmetic and geometrical sequences
- Introduce Taylor's series as

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + (x - x_0)^2 \frac{f''(x_0)}{2!} + \ldots + (x - x_0)^n \frac{f^{(n)}(x_0)}{n!} + \ldots$$

- Determine the series representation of functions using Taylor's series around a variety of x_0 values.
- Introduce the series representation of functions using MacLaurin's series

$$f(x) = f(0) + xf'(0) + x^2 \frac{f''(0)}{2!} + \ldots + x^n \frac{f^{(n)}(0)}{n!} + \ldots$$

- Expand the product of two functions using series forms (eg $e^x \ln(1 + x)$, $e^x \tan x$ etc.) (learners could be provided with a table showing the series expansion of standard functions such as e^x, $\sin x$, $\cos x$, $(1 + x)^n$ etc)
- Develop a linear approximation of one polynomial or one non-linear function (eg $\sin x$, $\cos x$, e^{-x} etc)

Outcome 4 (12 hours)

Solve first order differential equations

- Introduction to differential equations — explain the meaning of dependent and independent variables, order of a differential equation, linearity, initial conditions, etc.
- Solve first order differential equations by direct integration
- Introduce the separation of variables method as applying to equations of the following form

  $\frac{dy}{dx} = f(x)g(y)$
- Apply the method of separation of variables to solve first order differential equations (including first order differential equations that arise in Engineering)
- Introduce the method of solving differential equations by substitution (eg using $y = vx$ or $y = ax + by + c$
- Apply the method of substitution to the solution of first order differential equations
- Introduce the integrating factor method as applying to first order differential equations of the following form:

  $\frac{dy}{dx} + p(x)y = q(x)$

  where the integrating factor $= e^{\int p(x)dx}$

- Apply the integrating factor method to the solution of first order differential equations
- Illustrate how some first order differential equations may be solved by more than one method
- Solve first order differential equations taken from Engineering with an initial value using one of the methods shown above

Guidance on approaches to delivery of this Unit

This Unit provides many of the core mathematical principles and processes required when studying Engineering at a more advanced level. Given the nature of the subject matter in the Unit it is advisable that the Unit is not delivered until learners have studied *Engineering Mathematics 3*.

Centres may deliver the Outcomes in any order they wish.

The Unit may be delivery using a didactic approach. All teaching input should be supplemented by a significant level of formative assessment in which learners are provided with opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the Unit. Alternatively, as learners taking this Unit may be preparing to enter an degree course at university (possibly at an advanced level) the Unit could be delivered as a series of lectures supported by tutorial sessions to help learners prepare better for future university studies.

Computer software and computer algebra may be used to support learning (eg to confirm the solutions of mathematical problems), but it is strongly recommended that such learning resources are only used in a supportive capacity and not as the principal means of delivering Unit content.

Guidance on approaches to assessment of this Unit

The recommended approach is the use of an examination question paper. The question paper should be composed of an appropriate balance of short answer, restricted response and structured questions. The questions in the examination should not be grouped by Outcome or be labelled in terms of the Outcomes they relate to.

All assessment papers should be unseen by the learners prior to the assessment event and at all times, the security, integrity and confidentiality of assessment papers must be ensured. Assessment should be conducted under closed-book, controlled and invigilated conditions.

The summative assessment of all four Outcomes should not exceed two hours. When assessing a learner's responses to summative assessment lecturers should concentrate principally on the learner's ability to apply the correct mathematical technique and processes when solving problems. Learners should not be penalised for making simple numerical errors. An appropriate threshold score may be set for the assessment of this Unit.

Learners should be provided with a formulae sheet appropriate to the content of this Unit when undertaking their assessment. Computer algebra should not be used in the assessment of this Unit.

SQA Advanced Unit Specification: Support notes

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It is the learners' responsibility to ensure that any calculator they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- symbolic differentiation or integration
- communication with other machines or the internet

In addition, any calculator used by learners should have no retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematic formulae

Centres are reminded that prior verification of centre-devised assessments would help to ensure that the national standard is being met. Where learners experience a range of assessment methods, this helps them to develop different skills that should be transferable to work or further and higher education.

Opportunities for e-assessment

E-assessment may be appropriate for some assessments in this Unit. By e-assessment we mean assessment which is supported by Information and Communication Technology (ICT), such as e-testing or the use of e-portfolios or social software. Centres which wish to use e-assessment must ensure that the national standard is applied to all learner evidence and that conditions of assessment as specified in the Evidence Requirements are met, regardless of the mode of gathering evidence. The most up-to-date guidance on the use of e-assessment to support SQA's qualifications is available at **www.sqa.org.uk/e-assessment**.

Opportunities for developing Core and other essential skills

This Unit has the Using Number component of Numeracy embedded in it. This means that when candidates achieve the Unit, their Core Skills profile will also be updated to show that they have achieved Using Number at SCQF level 6.

History of changes to Unit

Version	Description of change	Date

SQA acknowledges the valuable contribution that Scotland's colleges have made to the development of SQA Advanced Qualifications.

FURTHER INFORMATION: Call SQA's Customer Contact Centre on 44 (0) 141 500 5030 or 0345 279 1000. Alternatively, complete our Centre Feedback Form.

General information for learners

Unit title: Engineering Mathematics 4 (SCQF level 8)

This section will help you decide whether this is the Unit for you by explaining what the Unit is about, what you should know or be able to do before you start, what you will need to do during the Unit and opportunities for further learning and employment.

The *Engineering Mathematics 4* Unit is one of a suite of five Units in Mathematics developed for SQA Advanced Certificates and Diplomas across a range of Engineering disciplines. The five Units help develop the mathematical skills required for workplace roles and for more advanced studies in Engineering, for example, articulation to degree study at university.

The Unit is optional in a number of SQA Advanced Diplomas in Engineering.

This Unit is designed to develop the mathematical skills required by learners who wish to use their SQA Advanced Diploma in Engineering to articulate to university degree study. You will learn about complex numbers (otherwise known as imaginary numbers) and the algebra that underpins the use of these numbers. Complex numbers arise in many branches of Engineering (eg ac electrical circuits). You will also learn to apply matrix techniques which are particularly useful when, for example, solving systems of linear equations containing several variables. You will develop the knowledge, understanding and skills to use Taylor's and MacLaurin's series to represent functions, and finally you will be introduced to a range of techniques for solving an important class of equations known as first order differential equations which arise extensively in many areas of Engineering.

Unit delivery may comprise of a significant teaching input from your lecturer supplemented by tutorial exercises which will allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes covered in the Unit to a range of Engineering problems. Alternatively, in order to prepare you for studies at university the centre where you are taking this Unit may choose to deliver it as a series of lectures supported by tutorials.

Formal assessment will be through an examination. Assessment will be conducted under closed-book, controlled and invigilated conditions.

Learners considering taking this Unit will normally be expected to have passed the *Engineering Mathematics 3* SQA Advanced Unit or equivalent.