Scottish Qualifications Authority

# SQA Advanced Unit specification

## General information

**Unit title:** Engineering Mathematics 5 (SCQF level 8)

**Unit code:** HT1N 48

**Superclass:** RB

**Publication date:** August 2017

**Source:** Scottish Qualifications Authority

**Version:** 01

## Unit purpose

This Unit is designed to develop the mathematical skills required by learners who wish to use their SQA Advanced Diploma in Engineering to articulate to university degree study. The Unit will provide learners with the opportunities to develop knowledge, understanding and skills to solve second order, constant coefficient differential equations; use partial differentiation and double integration techniques to solve a range of mathematical problems, solve first and second order differential equations using Laplace Transforms and use eigenvalues and eigenvectors to solve linear system equations.

## Outcomes

On successful completion of the Unit the learner will be able to:

1 Solve second order, constant coefficient differential equations.
2 Solve mathematical problems using partial differentiation.
3 Solve mathematical problems using double integration techniques.
4 Solve differential equations using Laplace Transforms.
5 Use eigenvalues and eigenvectors to solve linear system equations.

## Credit points and level

1 SQA Credit at SCQF level 8: (8 SCQF credit points at SCQF level 8)

## Recommended entry to the Unit

Entry requirements are at the discretion of the centre. However, it would be advantageous if learners had a good knowledge and understanding of differential and integral calculus which included techniques for solving first order differential equations; a sound knowledge and understanding of basic matrix techniques together with strong numerical and algebraic skills. This knowledge and understanding may be evidenced by possession of the SQA Advanced Units *Engineering Mathematics 3* and *Engineering Mathematics 4*.

## Core Skills

Achievement OF this Unit gives automatic certification of the following Core Skills component:

| | |
|---|---|
| Complete Core Skill | None |
| Core Skill component | Using Number at SCQF level 6 |

There are also opportunities to develop aspects of Core Skills which are highlighted in the Support Notes for this Unit specification.

## Context for delivery

If this Unit is delivered as part of a Group Award, it is recommended that it should be taught and assessed within the subject area of the Group Award to which it contributes.

The Assessment Support Pack (ASP) for this Unit provides assessment and marking guidelines that exemplify the national standard for achievement. It is a valid, reliable and practicable assessment. Centres wishing to develop their own assessments should refer to the ASP to ensure a comparable standard. A list of existing ASPs is available to download from SQA's website **(http://www.sqa.org.uk/sqa/46233.2769.html)**.

## Equality and inclusion

This Unit specification has been designed to ensure that there are no unnecessary barriers to learning or assessment. The individual needs of learners should be taken into account when planning learning experiences, selecting assessment methods or considering alternative evidence.

Further advice can be found on our website **www.sqa.org.uk/assessmentarrangements**.

## SQA Advanced Unit Specification: Statement of standards

## Unit title: Engineering Mathematics 5 (SCQF level 8)

Acceptable performance in this Unit will be the satisfactory achievement of the standards set out in this part of the Unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to SQA.

Where evidence for Outcomes is assessed on a sample basis, the whole of the content listed in the Knowledge and/or Skills section must be taught and available for assessment. Learners should not know in advance the items on which they will be assessed and different items should be sampled on each assessment occasion.

## Outcome 1

Solve second order, constant coefficient differential equations.

### Knowledge and/or Skills

- Homogenous
- Non-Homogenous
- General solution of second order differential equations
- Linear Approximation Techniques

## Outcome 2

Solve mathematical problems using partial differentiation.

### Knowledge and/or Skills

- Partial Differentiation
- Chain/ Product/Quotient Rules
- Higher Order Partial Derivatives
- Stationary points

## Outcome 3

Solve mathematical problems using double integration techniques.

### Knowledge and/or Skills

- Rectangular Domain
- Polar Domain
- Change order of integration in double integrals
- Volumes or surface areas

## Outcome 4

Solve differential equations using Laplace Transforms.

### Knowledge and/or Skills

- Laplace Transforms
- Inverse Laplace Transforms
- Shift theorems
- Dirac Delta Function
- First and Second Order Differential Equations

## Outcome 5

Use eigenvalues and eigenvectors to solve linear system equations.

### Knowledge and/or Skills

- Eigenvalues and Eigenvectors
- Eigenvalue related problems

### Evidence Requirements for this Unit

A sampling approach will be used in the assessment of the Knowledge and/or Skills in this Unit. Learners will need to provide written and/or recorded oral evidence to demonstrate their Knowledge and/or Skills across all Outcomes by showing they can:

**Outcome 1**

Provide evidence of **three out of four** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Determine the complimentary function of one second order differential equation
- Determine the particular integral of one second order differential equation
- Determine the particular solution of one second order, constant coefficient differential equation given initial conditions
- Use a Taylor linear approximation and solve the resulting second order non-linear differential equation

**Outcome 2**

Provide evidence of **three out of four** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Solve one mathematical problem involving first order partial derivatives
- Solve one partial differentiation problem involving the use of the chain, product or quotient rules

- Solve one problem involving higher order partial differentiation
- Solve one problem which involves finding the location and nature of a stationary point ($s$) for a function of the form $f(x, y)$

**Outcome 3**

Provide evidence of **three out of five** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Define the domain and limits of integration
- Solve a double integral in the rectangular domain
- Solve a double integral after transforming the integrand to the polar domain
- Solve a double integral by changing the order in the double integration
- Use double integration to determine the volume or surface area of a shape or length of a curve

**Outcome 4**

Provide evidence of **three out of five** Knowledge and/or Skills in this Outcome. The following evidence should be provided for the particular Knowledge and/or Skill items sampled:

- Determine the Laplace Transform of a function $f(t)$ from a table of Laplace Transforms
- Determine the inverse Laplace Transform of a function $f(s)$ using the completing the square or partial fraction methods
- Solve one problem which involves the use of the first or second shift theorems
- Solve one problem which involves the use of the Dirac Delta function
- Solve a first or second order differential equation with initial conditions using Laplace Transforms

**Outcome 5**

Provide evidence of the following Knowledge and/or Skills each time this Outcome is assessed:

- Determine eigenvalue and eigenvector for one 3 × 3 matrix
- Determine a diagonalisation transform to solve a problem of $A^n$

The assessment of all five Outcomes should take place at a single end of Unit assessment event. All re-assessments should be based on a different assessment instrument. This should re-assess all five Outcomes. All re-assessments should be based on a different sample of Knowledge and/or Skills.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions.

Computer algebra must not be used in the assessment of this Unit.

## SQA Advanced Unit Specification:Support notes

## Unit title: Engineering Mathematics 5 (SCQF level 8)

Unit Support Notes are offered as guidance and are not mandatory.

While the exact time allocated to this Unit is at the discretion of the centre, the notional design length is 40 hours.

## Guidance on the content and context for this Unit

This Unit is one of a suite of five Units in Mathematics developed for SQA Advanced Qualifications across a range of engineering disciplines. The five Units are:

*Engineering Mathematics 1*
*Engineering Mathematics 2*
*Engineering Mathematics 3*
*Engineering Mathematics 4*
*Engineering Mathematics 5*

In the development of this Unit a list of topics expected to be covered by lecturers has been identified. Recommendations have also been made on how much time lecturers should spend on each Outcome. The use of this list of topics is strongly recommended to ensure continuity of teaching and learning and adequate preparation for the assessment of the Unit. Consideration of this list of topics alongside the Assessment Support Pack developed for this Unit will provide clear indication of the standard expected in this Unit.

**Outcome 1 (6 hours)**

**Solve second order, constant coefficient differential equations.**

- Introduce the general form of a constant coefficient, second order linear differential equation as follows:

  $$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$$

  where $a$, $b$, $c$ are constants
- Provide examples of second order differential equations from engineering

- Explain that solving a second order linear differential equation involves the following three stages:
  - Finding the complementary function, $y_{\text{cf}}$
  - Finding a particular integral, $y_{\text{pi}}$
  - Determining the general solution by adding together the complementary function and the particular integral ($y = y_{\text{cf}} + y_{\text{pi}}$) and applying initial conditions where known
- Explain the difference between homogeneous and non-homogenous in the context of second order differential equations
- Introduce the auxiliary equation (characteristic equation) and explain that when finding roots of the auxiliary equation there are three cases we need to consider
- Explain that to find the complementary function one is finding the solution to the equation

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$$

- Find the complementary functions of second order differential equations using solutions of the form $y=e^{kx}$
- Explore the various forms a complementary function can take
- Explain that the particular integral is any function which satisfies the following equation

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$$

- Explain that for special classes of $f(x)$ we can use the Method of Undetermined Coefficients to find the particular integral
- Find the general solution of second order differential equations including applying initial conditions

**Outcome 2 (5 hours)**

**Solve mathematical problems using partial differentiation.**

- Introduce partial differentiation as the process of differentiating functions of two or more variables
- Identify the notation used in partial differentiation
- For any function $f(x, y)$ differentiate with respect to $x$ treating all terms in $y$ as constant
- For any function $f(x, y)$ differentiate with respect to $y$ treating all terms in $x$ as constant
- Undertake partial differentiation involving the use of the chain, product or quotient rules
- Introduce higher order partial derivatives
- Demonstrate the way in which to determine higher order derivatives
- Introduce examples of partial differential equations (solutions not required) which occur in engineering
- Explain and demonstrate the way in which to find the location and nature of a stationary point for a function of the form $z = f(x, y)$

**Outcome 3 (5 hours)**

**Solve mathematical problems using double integration techniques.**

- Explain that double integration involves integrating a function $f(x, y)$ as follows:

  $$\iint_R f(x, y)\, dy\, dx$$

  where $R$ is the region of integration in the $x - y$ plane
- Explain that the process of double integration normally comprises the following three stages:
  - — Work out the limits of integration if these are not known
  - — Determine the inner integral assuming terms in $x$ are constant
  - — Determine the outer integral

- Perform double integration where the limits are known and where they have to be determined
- Demonstrate that changing the order of integration can sometimes make double integration easier or possible to perform
- Demonstrate the way in which double integration may be performed by transforming variables from the rectangular domain to the polar domain
- Use double integration to determine the volume or surface areas of objects or the length of a curve

**Outcome 4 (6 hours)**

**Solve differential equations using Laplace Transforms.**

- Explain that the Laplace transform method is an integral transform method in which a linear constant coefficient differential equations is transformed into an algebraic equation. The corresponding algebraic equation is then solved and the transform reversed to find the solution of the differential equation.
- Introduce the Laplace Transform of $f(t)$ as:

  $$F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$$ (non-assessable)

- Determine one or two simple Laplace Transforms using the above equation
- Direct learners to tables of Laplace Transforms and Inverse Laplace Transforms
- Demonstrate techniques for finding the inverse of functions of $F(s)$ using completing the square and partial fractions
- Introduce the first and second shift theorems
- Demonstrate the application of the two theorems
- Introduce the Dirac Delta function (simple treatment only) — provide engineering analogies (eg a power supply spike or a hammer striking an object)

- State the Laplace transform of the Dirac Delta function as $L(\delta(t-a)) = \mathrm{e}^{-as}$
- Solve differential equations involving the use of the Dirac Delta function
- Solve first and second order differential equations with initial conditions using Laplace Transforms
- Solve systems of linear differential equations using Laplace Transforms if time permits

**Outcome 5 (6 hours)**

**Use eigenvalues and eigenvectors to solve linear system equations**

- Introduce the idea of a trivial and non-trivial solution in the context of matrix theory
- Define the concept of an eigenvalue (in terms of, for example, the characteristic equation $|A-\lambda| = 0$)
- Determine eigenvalues for 2 × 2 and 3 × 3 matrices
- Introduce the concept of eigenvectors as the non-trivial solutions $X$ of the equation $AX = \lambda X$
- Find the eigenvectors for 2 × 2 and 3 × 3 matrices
- Solve eigenvalue/eigenvector problems (eg diagonalisation matrices/transformation matrices)

## Guidance on approaches to delivery of this Unit

This Unit provides many of the core mathematical principles and processes required when studying engineering at a more advanced level. Given the nature of the subject matter in the Unit it is advisable that the Unit is not delivered until learners have studied *Engineering Mathematics 4*.

Centres may deliver the Outcomes in any order they wish.

The Unit may be delivery using a didactic approach. All teaching input should be supplemented by a significant level of formative assessment in which learners are provided with opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the Unit. Alternatively, as learners taking this Unit may be preparing to enter an degree course at university (possibly at an advanced level) the Unit could be delivered as a series of lectures supported by tutorial sessions to help learners prepare better for future university studies.

Computer software and computer algebra may be used to support learning (eg to confirm the solutions of mathematical problems), but it is strongly recommended that such learning resources are only used in a supportive capacity and not as the principal means of delivering Unit content.

## Guidance on approaches to assessment of this Unit

The recommended approach is the use of an examination question paper. The question paper should be composed of an appropriate balance of short answer, restricted response and structured questions. The questions in the examination should not be grouped by Outcome or be labelled in terms of the Outcomes they relate to.

All assessment papers should be unseen by the learners prior to the assessment event and at all times, the security, integrity and confidentiality of assessment papers must be ensured. Assessment should be conducted under closed-book, controlled and invigilated conditions.

The summative assessment of all four Outcomes should not exceed two hours and thirty minutes. When assessing a learner's responses to summative assessment lecturers should concentrate principally on the learner's ability to apply the correct mathematical technique and processes when solving problems. Learners should not be penalised for making simple numerical errors. An appropriate threshold score may be set for the assessment of this Unit.

Learners should be provided with a formulae sheet appropriate to the content of this Unit when undertaking their assessment. Computer algebra should not be used in the assessment of this Unit.

It is the learners' responsibility to ensure that any calculator they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- symbolic differentiation or integration
- communication with other machines or the internet

In addition, any calculator used by learners should have no retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematic formulae

Centres are reminded that prior verification of centre-devised assessments would help to ensure that the national standard is being met. Where learners experience a range of assessment methods, this helps them to develop different skills that should be transferable to work or further and higher education.

## Opportunities for e-assessment

E-assessment may be appropriate for some assessments in this Unit. By e-assessment we mean assessment which is supported by Information and Communication Technology (ICT), such as e-testing or the use of e-portfolios or social software. Centres which wish to use e-assessment must ensure that the national standard is applied to all learner evidence and that conditions of assessment as specified in the Evidence Requirements are met, regardless of the mode of gathering evidence.

The most up-to-date guidance on the use of e-assessment to support SQA's qualifications is available at **www.sqa.org.uk/e-assessment**.

## Opportunities for developing Core and other essential skills

This Unit has the Using Number component of Numeracy embedded in it. This means that when candidates achieve the Unit, their Core Skills profile will also be updated to show that they have achieved Using Number at SCQF level 6.

## History of changes to Unit

| Version | Description of change | Date |
|---|---|---|
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SQA acknowledges the valuable contribution that Scotland's colleges have made to the development of SQA Advanced Qualifications.

**FURTHER INFORMATION**: Call SQA's Customer Contact Centre on 44 (0) 141 500 5030 or 0345 279 1000. Alternatively, complete our Centre Feedback Form.

## General information for learners

## **Unit title:** Engineering Mathematics 5 (SCQF level 8)

This section will help you decide whether this is the Unit for you by explaining what the Unit is about, what you should know or be able to do before you start, what you will need to do during the Unit and opportunities for further learning and employment.

The *Engineering Mathematics 5* Unit is one of a suite of five Units in Mathematics developed for SQA Advanced Certificates and Diplomas across a range of Engineering disciplines. The five Units help develop the mathematical skills required for workplace roles and for more advanced studies in Engineering, for example, articulation to degree study at university.

The Unit is optional in a number of SQA Advanced Diplomas in Engineering.

This Unit is designed to develop the mathematical skills required by learners who wish to use their SQA Advanced Diploma in Engineering to articulate to university degree study.

You will learn a range of techniques for solving second order differential equations. These equations arise frequently in many areas of engineering. You will develop the knowledge, understanding and skills to perform partial differentiation and double integration. You will also learn about the very powerful Laplace Transforms method for solving a wide range of differential equations. You will also be introduced to eigenvalues and eigenvectors which are used in the solution of linear system equations.

Unit delivery may comprise of a significant teaching input from your lecturer supplemented by tutorial exercises which will allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes covered in the Unit to a range of engineering problems. Alternatively, in order to prepare you for studies at university the centre where you are taking the *Engineering Mathematics 4* Unit may choose to deliver it as a series of lectures supported by tutorials.

Formal assessment will be through an examination. Assessment will be conducted under closed-book, controlled and invigilated conditions.

Learners considering taking this Unit will normally be expected to have passed the *Engineering Mathematics 3* SQA Advanced Unit or equivalent. It also recommended that you have passed the *Engineering Mathematics* 4 SQA Advanced Unit.