-SQA- SCOTTISH QUALIFICATIONS AUTHORITY

NATIONAL CERTIFICATE MODULE: UNIT SPECIFICATION

GENERAL INFORMATION

-Module Number- **7180414** **-Session- 1994-95**

-Superclass- **RB**

-Title- **MATHEMATICS: ANALYSIS/ALGEBRA 2**

-DESCRIPTION-

GENERAL COMPETENCE FOR UNIT: Introducing and developing the concept of function and inverse function, applying the concept to the exponential and logarithmic functions, extending the properties of trigonometric functions and vectors and developing skills in practical situations through mathematical investigations.

OUTCOMES

1. use functions and sequences;
2. use exponential and logarithmic functions;
3. use properties of trigonometric relationships;
4. use vectors;
5. carry out a mathematical investigation.

CREDIT VALUE: 1 NC Credit

ACCESS STATEMENT: Access is at the discretion of the centre. However, it would be beneficial if the candidate had competence in mathematics as evidenced by possession of National Certificate Module 7180401 Mathematics: Analysis/Algebra 1 or an equivalent level of experience.

For further information contact: Committee and Administration Unit, SQA, Hanover House, 24 Douglas Street, Glasgow G2 7NQ.

This specification is distributed free to all approved centres. Additional copies may be purchased from SQA (Sales and Despatch section) at a cost of £1.50 (minimum order £5.00).

NATIONAL CERTIFICATE MODULE: UNIT SPECIFICATION

STATEMENT OF STANDARDS

UNIT NUMBER: 7180414

UNIT TITLE: MATHEMATICS: ANALYSIS/ALGEBRA 2

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the specification. All sections of the statement of standards are mandatory and cannot be altered without reference to SQA.

OUTCOME

1. USE FUNCTIONS AND SEQUENCES

PERFORMANCE CRITERIA

(a) Evaluation of simple functions is correct.

(b) Determination of the domain and range of functions, algebraically and graphically, is correct.

(c) Composition of simple functions is correct.

(d) Determination of the inverse of simple functions, algebraically and graphically, is correct.

(e) Determination of the terms of a sequence from a formula and vice versa is correct.

(f) Approximate solution of an equation by an iterative method is correct.

RANGE STATEMENT

Simple functions: polynomials; square root functions.

EVIDENCE REQUIREMENTS

Calculation and graphs.

Oral and/or written evidence of candidate's ability to use functions and sequences.

OUTCOME

2. USE EXPONENTIAL AND LOGARITHMIC FUNCTIONS

PERFORMANCE CRITERIA

(a) Sketching of graphs of a^x and $\log_a x$ is correct.

(b) Evaluation of expressions involving exponential and logarithmic functions is correct.

(c) Simplification of expressions using laws of logarithms is correct.

(d) Solution of exponential equations is correct.

(e) Establishing the relationship between variables from given experimental data, involving the use of logarithms, is correct.

RANGE STATEMENT

The range for this outcome is fully expressed within the performance criteria.

EVIDENCE REQUIREMENTS

Calculations and graphs.

Oral and/or written evidence of the candidate's ability to use exponential and logorithmic functions.

OUTCOME:

3. USE PROPERTIES OF TRIGONOMETRIC RELATIONSHIPS

PERFORMANCE CRITERIA

(a) Solution of equations using the formulae for sin2A, cos2A, sinA ± sinB, cosA ± cosB is correct.

(b) Determination of maxima and minima involving asinx + bcosx by converting to one of the forms: Rsin(x ± c), Rcos(x ± c) is correct.

(c) Solution of equations of the form: asinx + bcosx = K is correct.

RANGE STATEMENT

The range for this outcome is fully expressed within the performance criteria.

EVIDENCE REQUIREMENTS

Calculations.

Oral and/or written evidence of the candidate’s ability to use the properties of trigonometric relationships.

OUTCOME

4. USE VECTORS IN THREE DIMENSIONS

PERFORMANCE CRITERIA

(a) Solution of vector problems in three dimensions is correct.

(b) Use of scalar product is correct.

RANGE STATEMENT

Problems: in context involving component and unit vector notation.

Scalar product: involving component and unit vector notation, magnitude and angle.

EVIDENCE REQUIREMENTS

Diagrams and calculations.

Oral and/or written evidence of the candidate's ability to use vectors in these dimensions.

OUTCOME:

5. CARRY OUT A MATHEMATICAL INVESTIGATION

PERFORMANCE CRITERIA

(a) Identification of key factors of the investigation is correct.

(b) Identification of strategies for undertaking the investigation is appropriate to the situation.

(c) Implementation of appropriate strategies is correct.

(d) Drawing of conclusions is appropriate to the investigation.

(e) Communication of findings is clear and accurate.

RANGE STATEMENT

The range for this outcome is fully expressed within the performance criteria.

EVIDENCE REQUIREMENTS

Evidence should show the structure of the investigation and the processes carried out during the investigation in addition to details of the findings of the investigation.

ASSESSMENT RECORDS

In order to achieve this unit, candidates are required to present sufficient evidence that they have met all the performance criteria for each outcome within the range specified. Details of these requirements are given for each outcome. The assessment instruments used should follow the general guidance offered by the SQA assessment model and an integrative approach to assessment is encouraged. (See references at the end of the support notes).

Accurate records should be made of assessment instruments used showing how evidence is generated for each outcome and giving marking schemes and/or checklists, etc. Records of candidates' achievements should be kept. These records will be available for external verification.

SPECIAL NEEDS

In certain cases, modified outcomes and range statements can be proposed for certification. See references at the end of the support notes.

NATIONAL CERTIFICATE MODULE: UNIT SPECIFICATION

SUPPORT NOTES

UNIT NUMBER: 7180414

UNIT TITLE: MATHEMATICS: ANALYSIS/ALGEBRA 2

SUPPORT NOTES: This part of the unit specification is offered as guidance. None of the sections of the support notes is mandatory.

NOTIONAL DESIGN LENGTH: SQA allocates a notional design length to a unit on the basis of time estimated for achievement of the stated standards by a candidate whose starting point is as described in the access statement. The notional design length for this unit is 40 hours. The use of notional design length for programme design and timetabling is advisory only.

PURPOSE: This module introduces the candidate to the concept of function and inverse function. It will develop the candidate's skills in applying the concept to the exponential and logarithmic functions and extending the properties of trigonometric functions and vectors.

The use of mathematical investigations allows the development of skills in practical situations. "A Guide to Mathematical Investigations: SQA 1991" provides guidance on investigations.

The Appendix gives further guidance on mathematics modules in general and contains a grid showing the relationship between modules.

SQA publishes summaries of NC units for easy reference, publicity purposes, centre handbooks, etc. The summary statement for this unit is as follows:

This module will introduce the candidate to and develop the concept of, function and inverse function. It will help to apply the concept to the exponential and logarithmic functions; extend the properties of trigonometric functions and vectors; and develop the candidate's skills in practical situations through mathematical investigations. The candidate will undertake a series of graphical and calculation exercises and an investigation for problem solving.

CONTENT/CONTEXT:

Corresponding to outcomes:

1. The concept of a function f: x $\rightarrow$ f(x) as a mapping whose domain and range are R or subsets of R. The domain and range of functions such as x $\rightarrow$ 2x, x $\rightarrow$ $2x^2$, x $\rightarrow$ $x^{1/2}$, x $\rightarrow$ $(1 - x^2)^{1/2}$ and of "functions specified by their graph".

   Evaluation of expressions like f(3), f(-2), f(a), f(a+h) for functions such as f(x) = x + 2, f(x) = x^2, and f(x) = 1/x. The composition fog and gof of simple functions such as f(x) = x-1 and g(x) = 2x. The identity function i: x $\rightarrow$ x.

   Inverse of functions such as f(x) = x - 5, f(x) = 4x, f(x) = x^2, f(x) = x^3. A trigonometrical function f(x) = cosx, might be considered as an extensional function. The graph of the inverse of a function is the reflection in the line y = x of the graph of the original function.

   A sequence as a function whose domain is the set of natural numbers. Formula for the nth term of a sequence specified by its first few terms,
   e.g. 1, 4, 9, 16, ...; 11, 8, 5, 2, ...; $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}$ .....

   Calculation of terms of a sequence given the formula, e.g. the 8th term of the sequence u_n = 5n - 3. Informal discussion of convergence.

   Location of a root by the change of sign rule and approximate solution of equations eg. 2sinx = x or x^3 - 5x + 3 = 0 by an iterative method such as bisection, linear interpolation, re-arrangement of equation or Newton-Raphson. Use of a graphics calculator or computer software should be encouraged.

2. Introduction of graphs of the form f(x) = a^x, a = 2,3,4,e.
   Introduction of f(x) = 1nx as the inverse function of f(x) = e^x.

   Sketch graphs such as f(x) = e^{3x}, f(x) = e^{-x}, f(x) = 1n(0.5x).
   Evaluation of expressions such as those involving: biological growth, radioactive decay, growth of a current in an inductive circuit, Newton's law of cooling, atmospheric pressure, standard free energy change of a reaction.

   eg. $N = N_0 \exp(-l\ t), \quad i = I \exp\left(-\frac{Rt}{L}\right)$

   $$R = A \ln\left(\frac{C_1}{C_2}\right), \quad \Delta G = kRT \log_{10} K$$

Laws of logarithms

$\log (pq) = \log p + \log q$
$\log (p/q) = \log p - \log q$

$\log p^n = n \log p$
$\log_a a = 1$

Use $\log x \to \infty$ as $x \to \infty$

Use $\log x \to -\infty$ as $x \to 0^+$

Use other bases in addition to e, eg. base 10.

Solve exponential equations eg
$3.5^x = 70$, $x^{4.7} = 23.5$
Reduce $y = ax^n$, $y = ae^{bx}$ to linear form algebraically and/or graphically. Log/log or log/linear graph paper may be used. Use of computer software should be encouraged.

3. Expansion of expressions such as $\sin(3x+y)$, $\cos(x - 45°)$,

$\sin(a + \pi/3)$, $\cos(2x - 60°)$

Solution of equations such as:

$\sin 2x + \cos x = 0$	$0° \leq x < 360°$
$\cos 2y - 3\cos y + 2 = 0$	$-180° \leq y < 180°$
$\cos 2\theta - 3\sin\theta = 1$	$0 \leq \theta \leq 2\pi$
$\sin x + \sin 3x = 0$	$0° \leq x \leq 180°$

Graphs of expressions such as $\cos x + \sin x$, $\cos x - \sin x$, $2\sin x + 3\cos x$

Determination of maximum and minimum of expressions such as $3\sin x - 4\cos x$, $3 + 5\cos x + 2\sin x$

Solution of equations involving $A\sin x + B\cos x$ e.g.

$4\sin x + 2\cos x = 3 \qquad 0° \leq x \leq 360°$

$5\cos\theta + \sin\theta = 1 \qquad -\pi \le \theta \le \pi$

4. Using vector diagrams in 3 dimensions; using vector components, including scalar product in 3 dimensions. Using vector systems such as force or velocity diagrams, phasor systems.

$x\underline{i} + y\underline{j} + z\underline{k}$ or $\begin{matrix} x \\ y \\ z \end{matrix}$

$$\underline{a}.\underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\underline{a}||\underline{b}|\cos\theta$$

5. For further guidance on mathematical investigations see the reference section at the end of the support notes.

APPROACHES TO GENERATING EVIDENCE: The module descriptor lists discrete outcomes, but the approaches adopted may change the order or integrate the outcomes as appropriate. Several approaches are possible depending on the availability of resources, experience of tutors/ trainers and the type of candidate group. This may involve individualised learning, group work and class work. Multi media approaches should be encouraged where possible: text, practical activities, simulations, computer programs, videos etc.

Problem solving should be encouraged throughout the module as part of the learning and teaching/tutoring process, within the investigations, and as part of the assessment process. Likewise the investigation of mathematical ideas should be encouraged throughout the module. Diagnostic and formative assessment should be used where appropriate. The summative assessment may form an integral part of the whole learning/teaching/tutoring process or may consist of separate tests.

The candidate should be encouraged to keep a log book/workfile. This should form a complete record of the candidate's work throughout the module. The workfile could contain the candidate's notes, class handouts, completed worksheets, exercises, assignments, projects, investigations, log of computer activities and a summary of the important details for later revision purposes.

The sensible use of appropriate technologies, (numeric scientific/graphics programmable calculators or computers etc) should be encouraged. Due account should be taken of estimation, rounding and errors introduced into calculations.

Investigations should allow for divergent mathematical thinking. They may allow for comparisons and contain open ended or closed problems. Situations may occur where no solution is obtainable. The acquisition of mathematical skills may occur within the investigation. A typical investigation used for the purposes of summative assessment may take 4 to 6 hours.

ASSESSMENT PROCEDURES: Centres may use the Instruments of Assessment which are considered by tutors/trainers to be most appropriate. Examples of instruments of assessment which could be used are as follows:

Corresponding to outcomes:

1. Graphical and calculation exercise

   Topics may be assessed on the number of occasions indicated:

(a)	evaluation of a function	2
(b)	both domain and range	
	from a function	2
	from a graph	2
(c)	composition	2
(d)	inversion	
	algebraically	2
	graphically	2
(e)	conversion	
	from formula to terms	2
	from terms to formula	2
(f)	approximate solution	1

   One question may cover more than one topic.

   Evidence may show 3 correct responses from (a) and (c) together, 3 correct responses for each of (b), (d) and (e) and the correct response for (f).

2. Graphical and calculation exercise

   Topics may be assessed on the number of occasions indicated:

(a)	graphs	2
(b)	evaluation of expressions	4
(c)	simplification of expressions	3
(d)	exponential equations	3
(e)	exponential relationship	1

   One question may cover more than one topic.

   Evidence may show 2 correct responses for each of (a), (c) and (d), 3 correct responses for (b) and the correct response for (e).

3. Calculation exercise

Topics may be assessed on the number of occasions indicated.

(a)	solution of equations using sin(A + B) etc	4
(b)	maximum of asinx + bcosx	1
	minimum of csinx + dcosx	1
(c)	solution of equations involving asinx + bcosx	2

One question may cover more than one topic.

Evidence may show 3 correct responses for (a) and 3 correct responses for (b) and (c) together.

4. Graphical and calculation exercise

Topics may be assessed on the number of occasions indicated:

(a)	components	3
(b)	scalar product	1

One question may cover more than one topic.

Evidence may show 2 correct responses for (a) and the correct response for (b).

5. Investigation. The candidate could present evidence which shows the structure of the investigation and the processes carried out during the investigation.

PROGRESSION Refer to the Appendix and module grid following the support notes.

RECOGNITION Many SQA NC units are recognised for entry/recruitment purposes. For up-to-date information see the SQA guide 'Recognised and Recommended Groupings'.

REFERENCES

1. Guidelines for Module Writers.
2. SQA's National Standards for Assessment and Verification.
3. For a fuller discussion on assessment issues, please refer to SQA's Guide to Assessment.
4. Procedures for special needs statements are set out in SQA's guide 'Students with Special Needs'.
5. 'A Guide to Mathematical Investigations: SQA 1991'.

An exemplar assessment pack for this unit is available from SQA. Please call our Sales and Despatch section on 0141 242 2168 to check availability and costs. Quote product code B045.

APPENDIX

FRAMEWORK OF THE MATHEMATICS MODULES 94/95

The module grid summarises the complete structure of the mathematics modules and some of their relationships.

Progression through the grid is to the right.

When considering the suitability of a module, it is important to consider it in relation to others in the grid and not just in isolation.

The first module, Using Numbers in Everyday Situations, relates to the most elementary number concepts and skills.

Numeracy 1, 2, 3 and 4 are core skill units. They can be used in a range of SQA programmes and awards and are currently embedded as a mandatory part of general Scottish Vocational Qualifications. A Core Skills Numeracy Framework has been produced in conjunction with these units.

The modules Using Basic Number Skills, Using Arithmetic Skills, Dealing with Basic Measurements, Dealing With Money, Using Measurement Skills Within Everyday Activities and Small Scale Planning, Estimating and Costing were developed for the BBC Basic Skills Numeracy project.

The modules Core Maths 2, 3 and 4 relate approximately to work done in Standard Grade Mathematics. They are appropriate as National Certificate modules because they allow for consolidation of mathematical skills and they provide candidates with a second opportunity to create a base from which they can develop their mathematical knowledge and skills.

The modules Business Numeracy, Construction Numeracy 1 and 2, Engineering Numeracy and Laboratory Numeracy have a vocational bias and cater for the mathematical needs of candidates on craft, operator, clerical or YT courses.

Craft Technology 1 and 2 are designed to consolidate the mathematical skills at craft level.

The remaining modules meet the needs of candidates requiring further mathematics in support of their other studies.

Modules Analysis/Algebra 1, Analysis/Algebra 2 and Calculus 1 relate approximately to work done in Higher Grade mathematics, but alternative groupings are possible for candidates continuing or intending to continue, with college or university studies in, for example, business studies or engineering.

Specialist modules such as Business Statistics, Boolean Algebra, Numerical Methods, Operational Research and Spherical Trigonometry are available.