-SQA- SCOTTISH QUALIFICATIONS AUTHORITY

NATIONAL CERTIFICATE MODULE: UNIT SPECIFICATION

GENERAL INFORMATION

-Module Number- **7181144** **-Session- 1994-95**

-Superclass- **RB**

-Title- **MATHEMATICS: CALCULUS 1**

-DESCRIPTION-

GENERAL COMPETENCE FOR UNIT: Differentiating and integrating algebraic and trigonometric functions and using calculus in applications.

OUTCOMES

1. use derivatives for algebraic and trigonometric functions;

2. use standard integrals for algebraic and trigonometric functions;

3. carry out a mathematical investigation.

CREDIT VALUE: 1 NC Credit

ACCESS STATEMENT: Access is at the discretion of the centre. However, it would be beneficial if the candidate had competence in mathematics as evidenced by possession of National Certificate Module 7180401 Mathematics: Analysis/Algebra 1 or an equivalent level of experience.

For further information contact: Committee and Administration Unit, SQA, Hanover House, 24 Douglas Street, Glasgow G2 7NQ.

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NATIONAL CERTIFICATE MODULE: UNIT SPECIFICATION

STATEMENT OF STANDARDS

UNIT NUMBER: 7181144

UNIT TITLE: MATHEMATICS: CALCULUS 1

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the specification. All sections of the statement of standards are mandatory and cannot be altered without reference to SQA.

OUTCOME

1. USE DERIVATIVES FOR ALGEBRAIC AND TRIGONOMETRIC FUNCTIONS

PERFORMANCE CRITERIA

(a) Differentiation of functions reducible to the sums of powers of x is correct.

(b) Differentiation of functions of the form $f(x) = a\sin x + b\cos x$ is correct.

(c) Differentiation using the function of a function rule is correct.

(d) Calculation of both gradients and rates of change by differentiation is correct.

(e) Calculation of maxima and minima using differentiation is correct.

RANGE STATEMENT

The range for this outcome is fully expressed within the performance criteria.

EVIDENCE REQUIREMENTS

Calculations.

Oral and/or written evidence of the candidate's ability to use derivatives for algebraic and trigonometric functions.

OUTCOME

2. USE STANDARD INTEGRALS FOR ALGEBRAIC AND TRIGONOMETRIC FUNCTIONS

PERFORMANCE CRITERIA

(a) Integration of functions reducible to sums of powers of x is correct.

(b) Integration of functions of the form $f(x) = a\sin x + b\cos x$ is correct.

(c) Integration of functions of the form $f(x) = (ax + b)^n$ and $f(x) = \sin(ax + b)$ $f(x) = \cos(ax + b)$ is correct.

(d) Calculation of the area between two curves is correct.

RANGE STATEMENT

Integration: definite; indefinite.

EVIDENCE REQUIREMENTS

Calculations.

Oral and/or written evidence of the candidate's ability to use standard integrals for algebraic and trigonometric functions.

OUTCOME

3. CARRY OUT A MATHEMATICAL INVESTIGATION

PERFORMANCE CRITERIA

(a) Identification of key factors of the investigation is correct.

(b) Identification of strategies for undertaking the investigation is appropriate to the situation.

(c) Implementation of appropriate strategies is correct.

(d) Drawing of conclusions is appropriate to the investigation.

(e) Communication of findings is clear and accurate.

RANGE STATEMENT

The range for this outcome is fully expressed within the performance criteria.

EVIDENCE REQUIREMENTS

Evidence should show the structure of the investigation and the processes carried out during the investigation in addition to details of the findings of the investigation.

ASSESSMENT RECORDS

In order to achieve this unit, candidates are required to present sufficient evidence that they have met all the performance criteria for each outcome within the range specified. Details of these requirements are given for each outcome. The assessment instruments used should follow the general guidance offered by the SQA assessment model and an integrative approach to assessment is encouraged. (See references at the end of the support notes).

Accurate records should be made of assessment instruments used showing how evidence is generated for each outcome, and giving marking schemes and/or checklists, etc. Records of candidates achievements should be kept. These records will be available for external verification.

SPECIAL NEEDS

In certain cases, modified outcomes and range statements can be proposed for certification. See references at the end of the support notes.

NATIONAL CERTIFICATE MODULE: UNIT SPECIFICATION

SUPPORT NOTES

UNIT NUMBER: 7181144

UNIT TITLE: MATHEMATICS: CALCULUS 1

SUPPORT NOTES: This part of the unit specification is offered as guidance. None of the sections of the support notes is mandatory.

NOTIONAL DESIGN LENGTH: SQA allocates a notional design length to a unit on the basis of time estimated for achievement of the stated standards by a candidate whose starting point is as described in the access statement. The notional design length for this unit is 40 hours. The use of notional design length for programme design and timetabling is advisory only.

PURPOSE: This module introduces the candidate to calculus within the context of applications and should enable the candidate to understand and apply the concepts of integration and differentiation of algebraic and trigonometric functions.

The use of mathematical investigations allows the development of skills in practical situations. "A Guide to Mathematical Investigations: SQA 1991" provides guidance on investigations.

The Appendix gives further guidance on mathematics modules in general and contains a grid showing the relationship between modules.

SQA publishes summaries of NC units for easy reference, publicity purposes, centre handbooks, etc. The summary statement for this unit is as follows:

The candidate will learn to differentiate and integrate algebraic and trigonometric functions and use calculus in applications. A series of calculation exercises and an investigation will also be undertaken.

CONTENT/CONTEXT

Corresponding to outcomes:

1. Investigate relationships between functions and derivatives. Use rules of differentiation for elementary algebraic functions, eg

   $f(x) = 3x^2 - 5x + 2$

   $f(x) = (2x - 1)(x + 4)$

   $f(t) = \sqrt{t} + \frac{1}{t}$

   $f(x) = (x^3 - 3)/2x^2$

   Use the rules of differentiation for trigonometric functions, eg

   $f(x) = 3\cos x$

   $f(x) = 4\sin x - 1$

   $f(\theta) = 5\sin\theta - 2\cos\theta$

   Use function of a function rule on algebraic and trigonometric functions, eg

   $f(x) = (3x - 1)^4$

   $f(x) = 3/(2x - 1)$

   $f(t) = 3\sin(2t - 1)$

   $f(t) = 5\cos(100\pi t + 0.2)$

   $f(x) = \sin^2 x + \cos^3 x$.

   Calculation of gradients and rates of change using derivatives, eg gradient of $y = 3x^2$ at $x = 2$

   Instantaneous velocity and acceleration. Rate of change of volumes and rate of change of temperature.

   Curve sketching.

2. Investigate the calculation of area of irregular shapes.

Integration as the inverse process of differentiation. Use rules of integration and trigonometric functions, eg:

$f(x) = 4x^3 - 6x^2 + 2$

$f(x) = (3x - 1)(2 - x)$

$f(t) = 4\sin t - 3$

$f(x) = 2.5 - \cos x$

$f(x) = (1 + x)/x^2$

$f(x) = (5x + 2)^3$

$f(x) = \sqrt{(3 - 2x)}$

$f(x) = 8\sin(4x - 1)$

$f(\theta) = 6\cos 2\theta$

Use rules of integration to calculate indefinite and definite integrals eg:

$\int (3x^3 + 4x^2 - 3)dx$ $\int_1^4 (5x^3 - 3x^2 + 1)dx$

$\int (\sin x - 3\cos x)dx$ $\int_4^9 \frac{(x^2 - 1)}{\sqrt{x}}dx$

$\int (\sqrt{x} + \frac{x}{4})dx$ $\int_{-p}^{p} (\sin x - \cos x)dx$

$\int (6t - 1)^3 dt$ $\int_0^3 (8t + 4)^3 dt$

$\int [3\cos(2x + 1)]dx$ $\int_0^{\frac{p}{2}} 6\cos 2q \; dq$

$\int_{\frac{p}{2}}^{p} 4\sin(3x - 1)dx$

Calculation of areas between two curves eg between y = 2x and $y = x^2$.

3. The investigation could be used in the initial development of the concepts, techniques and skills encountered within the module or it could involve the application and extension of the concepts, techniques and skills.

   For further guidance on mathematical investigations see the reference section at the end of the support notes.

APPROACHES TO GENERATING EVIDENCE: The module descriptor lists discrete outcomes, but the approaches adopted may change the order or integrate the outcomes as appropriate. Several approaches are possible depending on the availability of resources, experience of tutors/trainers and the type of candidate group. This may involve individualised learning, group work and class work. Multi media approaches should be encouraged where possible: text, practical activities, simulations, computer programs, videos etc. Problem solving should be encouraged throughout the module as part of the learning and teaching/tutoring process, within the investigations, and as part of the assessment process. Likewise the investigation of mathematical ideas should be encouraged throughout the module. Diagnostic and formative assessment should be used where appropriate. The summative assessment may form an integral part of the whole learning/teaching/ tutoring process or may consist of separate tests.

The candidate should be encouraged to keep a log book/workfile. This should form a complete record of the candidate's work throughout the module. The workfile could contain the candidate's notes, class handouts, completed worksheets, exercises, assignments, projects, investigations, log of computer activities and a summary of the important details for later revision purposes.

The sensible use of appropriate technologies, (numeric scientific/graphics/ programmable calculators or computers etc) should be encouraged. Due account should be taken of estimation, rounding and errors introduced into calculations.

Investigations should allow for divergent mathematical thinking. They may allow for comparisons and contain open ended or closed problems. Situations may occur where no solution is obtainable. The acquisition of mathematical skills may occur within the investigation. A typical investigation used for the purposes of summative assessment may take 4 to 6 hours.

ASSESSMENT PROCEDURES: Centres may use the instruments of assessment which are considered by tutors/trainers to be most appropriate. Examples of instruments of assessment which could be used are as follows:

Corresponding to outcomes:

1. Calculation Exercise. Topics may be assessed on the number of occasions indicated:

(a)	differentiation of algebraic functions		4
(b)	differentiation of trigonometric functions		3
(c)	function of a function rule	4	
(d)	gradients	2	
	rates of change	2	
(e)	maximum	1	
	minimum	1	

One question may cover more than one topic.

Evidence may also show 3 correct responses for each of (a), (c) and (d) and 2 correct responses for each of (b) and (e).

2. Calculation Exercise. Topics may be assessed on the number of occasions indicated:

(a)	indefinite integrals	2
	definite integrals	2
(b)	indefinite integrals	2
	definite integrals	2
(c)	indefinite integrals	2
	definite integrals	2
(d)	area	1

One question may cover more than one topic.

Evidence may show 3 correct responses for (a), (b) and (c) and the correct responses for (d).

3. Investigation. The candidate could present evidence which shows the structure of the investigation and processes carried out during the investigation.

PROGRESSION Refer to the Appendix and module grid following the support notes.

RECOGNITION Many SQA NC units are recognised for entry/recruitment purposes. For up-to-date information see the SQA guide 'Recognised and Recommended Groupings'.

REFERENCES

1. Guidelines for Module Writers.
2. SQA's National Standards for Assessment and Verification.
3. For a fuller discussion on assessment issues, please refer to SQA's Guide to Assessment.
4. Procedures for special needs statements are set out in SQA's guide 'Students with Special Needs'.
5. 'A Guide to Mathematical Investigations: SQA 1991'.

APPENDIX

FRAMEWORK OF THE MATHEMATICS MODULES 93/94

The module grid summarises the complete structure of the mathematics modules and some of their relationships.

Progression through the grid is to the right.

When considering the suitability of a module, it is important to consider it in relation to others in the grid and not just in isolation.

The first module, Using Numbers in Everyday Situations, relates to the most elementary number concepts and skills.

Numeracy 1, 2, 3 and 4 are core skill units. They can be used in a range of SQA programmes and awards and are currently embedded as a mandatory part of general Scottish Vocational Qualifications. A Core Skills Numeracy Framework has been produced in conjunction with these units.

The modules Using Basic Number Skills, Using Arithmetic Skills, Dealing with Basic Measurements, Dealing With Money, Using Measurement Skills Within Everyday Activities and Small Scale Planning, Estimating and Costing were developed for the BBC Basic Skills Numeracy project.

The modules Core Maths 2, 3 and 4 relate approximately to work done in Standard Grade Mathematics. They are appropriate as National Certificate modules because they allow for consolidation of mathematical skills and they provide candidates with a second opportunity to create a base from which they can develop their mathematical knowledge and skills.

The modules Business Numeracy, Construction Numeracy 1 and 2, Engineering Numeracy and Laboratory Numeracy have a vocational bias and cater for the mathematical needs of candidates on craft, operator, clerical or YT courses.

Craft Technology 1 and 2 are designed to consolidate the mathematical skills at craft level.

The remaining modules meet the needs of candidates requiring further mathematics in support of their other studies.

Modules Analysis/Algebra 1, Analysis/Algebra 2 and Calculus 1 relate approximately to work done in Higher Grade mathematics, but alternative groupings are possible for candidates continuing or intending to continue, with college or university studies in, for example, business studies or engineering.

Specialist modules such as Business Statistics, Boolean Algebra, Numerical Methods, Operational Research and Spherical Trigonometry are available.