

MATHEMATICS
Higher

Sixth edition – published May 2007

**NOTE OF CHANGES TO ARRANGEMENTS
SIXTH EDITION PUBLISHED MAY 2007**

COURSE TITLE: Mathematics (Higher)

**COURSE NUMBERS AND TITLES
FOR ENTRY TO COURSES:** C100 12 Mathematics: Maths 1, 2 and 3

National Course Specification

Course Details: References to C102 12 Mathematics: Maths 1, 2 and Stats have been deleted as this Course has been withdrawn.

Course Assessment details have been updated to reflect changes to the Course Assessment.

National Unit Specification

Unit: Statistics (Higher)
D325 12

This Unit has been removed from these Arrangements as Course C102 12 Mathematics: Maths 1, 2 and Stats deleted as this Course has been withdrawn. Please note that this Unit has been retained and can be offered as a freestanding Unit.

National Course Specification

MATHEMATICS (HIGHER)

COURSE NUMBER **C100 12 Mathematics: Maths 1, 2 and 3**

COURSE STRUCTURE

C100 12 Mathematics: Maths 1, 2 and 3

This course consists of three mandatory units as follows:

<i>D321 12</i>	<i>Mathematics 1 (H)</i>	<i>1 credit (40 hours)</i>
<i>D322 12</i>	<i>Mathematics 2 (H)</i>	<i>1 credit (40 hours)</i>
<i>D323 12</i>	<i>Mathematics 3 (H)</i>	<i>1 credit (40 hours)</i>

In common with all courses, this course includes 40 hours over and above the 120 hours for the component units. This may be used for induction, extending the range of learning and teaching approaches, support, consolidation, integration of learning and preparation for external assessment. This time is an important element of the course and advice on its use is included in the course details.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained one of the following:

- Standard Grade Mathematics Credit award
- Intermediate 2 Mathematics or its component units including *Mathematics 3 (Int 2)*
- equivalent.

Administrative Information

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Additional copies of this specification (including unit specifications) can be purchased from the Scottish Qualifications Authority for £7.50. **Note:** Unit specifications can be purchased individually for £2.50 (minimum order £5).

National Course Specification (cont)

COURSE Mathematics (Higher)

CREDIT VALUE

The Higher Course in Mathematics is allocated 24 SCQF credit points at SCQF level 6.*

**SCQF points are used to allocate credit to qualifications in the Scottish Credit and Qualifications Framework (SCQF). Each qualification is allocated a number of SCQF credit points at an SCQF level. There are 12 SCQF levels, ranging from Access 1 to Doctorates.*

CORE SKILLS

This course gives automatic certification of the following:

Complete core skills for the course Numeracy H

Additional core skills components for the course Critical Thinking H

For information about the automatic certification of core skills for any individual unit in this course, please refer to the general information section at the beginning of the unit.

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Course Specification: course details

COURSE Mathematics (Higher)

RATIONALE

As with all mathematics courses, Higher Mathematics aims to build upon and extend candidates' mathematical skills, knowledge and understanding in a way that recognises problem solving as an essential skill and enables them to integrate their knowledge of different aspects of the subject.

Because of the importance of these features, the grade descriptions for mathematics emphasise the need for candidates to undertake extended thinking and decision making to solve problems and integrate mathematical knowledge. The use of coursework tasks to achieve the course grade descriptions in problem solving is encouraged.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates who complete a Higher Mathematics course successfully are expected to have a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within the course, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

Where appropriate, mathematics should be developed in context and the use of mathematical techniques should be applied in social and vocational contexts related to likely progression routes.

The Higher Mathematics course has the particular objective of meeting the needs of candidates at a stage of their education where career aspirations are particularly important. The course has obvious relevance for candidates with interests in fields such as commerce, engineering and science where the mathematics learned will be put to direct use. For other candidates, the course can be used to gain entry to a Higher Education institution. All candidates taking the Higher Mathematics course, whatever their career aspirations, should acquire an enhanced awareness of the importance of mathematics to technology and to society in general.

COURSE CONTENT

The syllabus is designed to build upon prior learning in the areas of algebra, geometry and trigonometry, and to introduce candidates to elementary calculus.

Mathematics 1 (H), *Mathematics 2 (H)* and *Mathematics 3 (H)* are progressive units, each unit covering the range of mathematical topics described above.

The outcomes and the performance criteria for each unit are statements of basic competence. Additionally, the course makes demands over and above the requirements of individual units. Candidates should be able to integrate their knowledge across the component units of the course. Some of the 40 hours of flexibility time should be used to ensure that candidates satisfy the grade descriptions for mathematics courses which involve solving problems and which require more extended thinking and decision making. Candidates should be exposed to coursework tasks which require them to interpret problems, select appropriate strategies, come to conclusions and communicate intelligibly.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged, throughout the course, to make efficient use of the arithmetical, mathematical and graphical facilities on calculators. At the same time, candidates should also be aware of the limitations of the technology and of the importance of always applying the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, there should be evidence of a checking procedure throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – ‘How sensible is my answer?’
- estimate followed by a repeated calculation
- calculation in a different order.

It is expected that candidates will be able to demonstrate attainment in the algebraic, trigonometric and calculus content of the course without the use of computer software or sophisticated calculators.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

National Course Specification: course details (cont)

DETAILED CONTENT

The content listed below should be covered in teaching the course. All of this content will be subject to sampling in the external assessment. Part of this assessment will be carried out in a question paper where a calculator will not be allowed. Any of the topics may be sampled in this part of the assessment. The external assessment will also assess problem solving skills, see the grade descriptions on pages 25 and 26. Where comment is offered, this is intended to help in the effective teaching of the course.

References shown in this style of print indicate the depth of treatment appropriate to Grades A and B.

CONTENT	COMMENT
<p>Symbols, terms and sets the symbols: \in, \notin, $\{ \}$</p> <p>the terms: set, subset, empty set, member, element</p> <p>the conventions for representing sets, namely: N, the set of natural numbers, $\{1, 2, 3, \dots\}$ W, the set of whole numbers, $\{0, 1, 2, 3, \dots\}$ Z, the set of integers Q, the set of rational numbers R, the set of real numbers</p> <p>The content listed above is not examinable but candidates are expected to be able to understand its use.</p> <p>Mathematics 1 (H) Properties of the straight line know that the gradient of the line determined by the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$ and that the distance between the points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</p>	

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>know that the gradient of a straight line is the tangent of the angle made by the line and the positive direction of the x-axis (scales being equal)</p> <p>recognise the term locus</p> <p>know that the equation of a straight line is of the form $ax + by + c = 0$, and conversely (a, b not both zero)</p> <p>know that the line through (x_1, y_1) with gradient m has equation $y - y_1 = m(x - x_1)$</p> <p>determine the equation of a straight line given two points on the line or one point and the gradient</p> <p>know that the gradients of parallel lines are equal</p> <p>know that the lines with non-zero gradients m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$</p> <p>solve mathematical problems involving the above properties of straight lines</p> <p>know the concurrency properties of medians, altitudes, bisectors of the angles and perpendicular bisectors of the sides of a triangle</p> <p>Functions and graphs</p> <p>know the meaning of the terms: domain and range of a function, inverse of a function and composite function</p> <p>know the meaning of the terms amplitude and period</p> <p>be aware of the general features of the graphs of $f: x \rightarrow \sin(ax + b)$, $x \rightarrow \cos(ax + b)$ for suitable constants a, b</p>	<p>Candidates should meet all of these concurrency facts, but they could be covered by particular numerical cases only, using plane coordinate methods. However, they can also be established by vector methods and it is possible to explore this work through an investigative approach using an interactive geometry package.</p> <p>The main use of ‘inverse’ is to obtain the logarithmic function from the exponential function. An extended treatment is not required.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>given the graph of $f(x)$ draw the graphs of related functions, where $f(x)$ is a simple polynomial or trigonometric function</p> <p>know the general features of the graphs of the functions:</p> <p>$f: x \rightarrow a^x$ ($a > 1$ and $0 < a < 1$, $x \in \mathbf{R}$)</p> <p>$f: x \rightarrow \log_a x$ ($a > 1$, $x > 0$)</p> <p>use the notation $f(g(x))$ for a composite function and find composite functions of the form $f(g(x))$, given $f(x)$ and $g(x)$</p> <p>recognise the probable form of a function from its graph</p>	<p>Candidates should be fluent in associating functions with their graphs and vice versa: simple polynomial functions and trigonometric functions.</p> <p>eg $y = -f(x)$, $f(x) + 2$, $3f(x)$, $f(x + 3)$, $f(x - 1)$, $f'(x)$.</p> <p>eg $y = 3f(x) + 2$, $f(3x + 2)$, $-3f(x)$, $3f(x + \frac{\pi}{2})$. [A/B]</p> <p>Candidates should be fluent in associating functions with their graphs and vice versa: logarithmic and exponential functions.</p> <p>Candidates should recognise that $(\frac{1}{2})^x = 2^{-x}$.</p> <p>Graphs of $y = a^x$ for $a = 2, 3$, etc., and for $a = \frac{1}{2}, \frac{1}{3}$, etc. should be explored, and the inverse function of $f: x \rightarrow a^x$ ($a > 1$, $x \in \mathbf{R}$) should be investigated.</p> <p>Inverse should be used to obtain the logarithmic function from the exponential function.</p> <p>Note: notation $f \circ g$ is not required.</p> <p>Candidates should be able to interpret a diagram containing the sketches of two (or possibly more) relationships, and sketch and annotate the graphs of functions once the critical features, such as points where the graph cuts the coordinate axes and axes of symmetry, as appropriate, have been identified.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
complete the square in a quadratic of the form $x^2 + px + q$	The completed form would be $(x + a)^2 + b$ where a is an integer.
complete the square in a quadratic of the form $ax^2 + bx + c$ [A/B]	eg $2x^2 - x + 1$. [A/B]
interpret formulae and equations	eg $y = 3(x - 1)^2 + 2$ has a minimum value when $x = 1$; $y = 3 \cos x + 2$ has maximum value 5. eg $y = (1 - \sin x)^2 + 2$ has minimum value 2; $y = -2 - 3(2x - 1)^2$ has maximum value -2 when $x = 1/2$. [A/B]
know that π radians = 180°	
know the exact values of sin/cos $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ radians and	
tan $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ radians	

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Basic differentiation know the meaning of the terms limit, differentiable at a point, differentiate, derivative, differentiable over an interval, derived function</p> <p>use the notation $f'(x)$ and $\frac{dy}{dx}$ for a derivative</p> <p>know that $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$</p> <p>know that: if $f(x) = x^n$, then $f'(x) = nx^{n-1}$, $n \in \mathbb{Q}$</p> <p>if $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$</p> <p>if $f(x) = kg(x)$, then $f'(x) = kg'(x)$ where k is a constant</p> <p>know the meaning of the terms rate of change, average gradient, strictly increasing, strictly decreasing, stationary point (value), maximum turning point (value), minimum turning point (value), horizontal point of inflexion</p>	<p>Introduce within a context, such as investigating gradients of tangents at points on a curve. Gradients of a sequence of chords could be found numerically using a calculator. Opportunities for a cooperative approach should be taken.</p> <p>This could be deferred until rules have been consolidated in examples and applications. Note: differentiation from first principles is not included.</p> <p>eg Differentiate, with respect to x, expressions such as $2x^3 + 1 + \frac{1}{x^2}$.</p> <p>Candidates would be expected to simplify product and rational expressions prior to differentiation, eg $\frac{x^3 + x - 1}{x^2}$.</p> <p>Rates of change (equations of motion, for example) are often expressed with respect to time. Work should include other rates, eg rate of change of volume of a sphere with respect to radius.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>know that $f'(a)$ is the rate of change of f at a</p> <p>know that $f'(a)$ is the gradient of the tangent to the curve $y = f(x)$ at $x = a$</p> <p>know that the gradient of the curve $y = f(x)$ at any point on the curve is the gradient of the tangent at that point</p> <p>find the gradient of the tangent to a curve $y = f(x)$ at $x = a$</p> <p>find the points on a curve at which the gradient has a given value</p> <p>know and apply the fact that:</p> <ul style="list-style-type: none">if $f'(x) > 0$ in a given interval then the function f is strictly increasing in that intervalif $f'(x) < 0$ in a given interval then the function f is strictly decreasing in that intervalif $f'(a) = 0$, then the function f has a stationary value at $x = a$ <p>find the stationary point(s) (values) on a curve and determine their nature using differentiation</p> <p>sketch a curve with given equation by finding stationary point(s) and their nature, intersections with the axes, behaviour of y for large positive and negative values of x</p>	<p>Many relationships in science are expressed in terms of rates of change. Candidates should be able to translate from words to symbols, and vice versa.</p> <p>It is important that the recognised convention for scientific symbols is followed to avoid confusion when candidates meet symbols in other subject areas, eg use s (in lower case) for distance and v for speed and velocity.</p> <p>The most common method would be a nature table involving f'.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>determine the greatest/least values of a function on a given interval</p> <p>solve optimisation problems using calculus</p> <p>Recurrence relations</p> <p>know the meaning of the terms: sequence, nth term, limit as n tends to infinity</p> <p>use the notation u_n for the nth term of a sequence</p> <p>define and interpret a recurrence relation of the form $u_{n+1} = mu_n + c$ (m, c constants) in a mathematical model</p> <p>know the condition for the limit of the sequence resulting from a recurrence relation to exist</p> <p>find (where possible) and interpret the limit of the sequence resulting from a recurrence relation in a mathematical model</p>	<p>eg Given $y = f(x)$ on an interval, differentiate and find values of end points, hence deduce maximum value.</p> <p>Candidates should experience a variety of mathematical models of situations involving recurrence relations.</p> <p>'Series' is not included, but could be approached using sequence of partial sums.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Mathematics 2 (H)</p> <p>Factor/Remainder Theorem and quadratic theory use the Remainder Theorem to determine the remainder on dividing a polynomial $f(x)$ by $x - h$</p> <p>determine the roots of a polynomial equation</p> <p>use the Factor Theorem to determine the factors of a polynomial</p> <p>know that the roots of $ax^2 + bx + c = 0$, $a \neq 0$, are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$</p> <p>know that the discriminant of $ax^2 + bx + c$ is $b^2 - 4ac$</p> <p>use the discriminant to: determine whether or not the roots of a quadratic equation are real, and, if real, whether equal or unequal, rational or irrational</p> <p>find the condition that the roots of a quadratic equation are real, and, if real, whether equal or unequal</p>	<p>ie Remainder is $f(h)$.</p> <p>ie If $f(h) = 0$ then $x - h$ is a factor of the polynomial $f(x)$ and conversely.</p> <p>Polynomial $f(x)$ is of the third or higher degree, where $f(x)$ can be expressed as a product of factors of which at most one is quadratic and the remainder are linear. eg $f(x) = (x - 2)(x - 3)(2x + 1)(x - 1)$. ie no need to consider $h \in \mathbb{Q}$ for factor $x - h$. eg $f(x) = (2x - 1)(3x + 2)(2x - 5)$. [A/B]</p> <p>This condition is to be applied in algebraic contexts when the equation is presented in a standard form, and in familiar geometric contexts.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>know the condition for tangency, intersection of a straight line and a parabola</p> <p>solve quadratic inequalities, $ax^2 + bx + c \geq 0$ (or ≤ 0)</p> <p>determine a quadratic equation with given roots</p> <p>prove that an equation has a root between two given values and find that root to a required degree of accuracy</p>	<p>eg For what values of p does the equation $x^2 - 2x + p = 0$ have real roots?</p> <p>eg If $\frac{(x-2)^2}{x^2+2} = k$, $k \in \mathbf{R}$, find values of k such that the given equation has two equal roots. [A/B]</p> <p>The properties of the discriminant will also be required for the intersection of straight lines and circles, tangents to circles.</p> <p>eg From the graph of $y = x^2 + x - 2$, solve $x^2 + x - 2 > 0$.</p> <p>eg Find the real values of x satisfying $x^2 + x - 2 > 0$. [A/B]</p> <p>The formulae for the product and the sum of the roots are not included.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Basic integration</p> <p>know the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve</p> <p>know that if $f(x) = F'(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$ and $\int f(x) dx = F(x) + C$ where C is the constant of integration</p> <p>integrate functions defined by $f(x) = px^n$ for all rational n, except $n = -1$ and the sum or difference of such functions</p> <p>evaluate definite integrals</p> <p>determine the area bounded by the curve $y = f(x)$, the lines $x = a$, $x = b$ and the x-axis using integration</p> <p>determine the area bounded by two curves</p> <p>solve equations of the form $\frac{dy}{dx} = f(x)$ for suitable $f(x)$</p>	<p>Introduce in a context such as investigating areas beneath curves. Calculators could be used to determine approximations to areas. Conjectures made about general results should be investigated. Computer packages are available to assist with this approach.</p> <p>eg integrate, with respect to x, $x^3 + 1 + \frac{1}{x^2}$.</p> <p>Candidates would be expected to simplify product and rational expressions prior to integration, eg $\frac{x^3 + x^2 - 1}{x^2}$.</p> <p>ie Area = $\int_a^b f(x)dx$ if $f(x) \geq 0$ for $a \leq x \leq b$ and area = $-\int_a^b f(x) dx$ if $f(x) \leq 0$ for $a \leq x \leq b$.</p> <p>ie If $f(x) \geq g(x)$ for $a \leq x \leq b$, then the area enclosed by the curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is $\int_a^b (f(x) - g(x))dx$.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Trigonometric formulae</p> <p>solve trigonometric equations in a given interval</p> <p>know and apply the addition formulae and double angle formulae</p> $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\sin 2A = 2 \sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$ <p>apply trigonometric formulae in the solution of geometric problems</p> <p>solve trigonometric equations involving addition formulae and double angle formulae</p>	<p>Solution of trigonometric equations should be introduced graphically. eg $2 \cos 2x = 1, 0 \leq x \leq \pi$.</p> <p>eg $\cos^2 2x = 1, 3 \sin^2 x + 7 \sin x - 6 = 0, 0 \leq x \leq 2\pi$. [A/B]</p> <p>Standard formulae will be given.</p> <p>eg $\sin(P + 2Q) = \sin P \cos 2Q + \cos P \sin 2Q, \sin 2P = 2 \sin P \cos P$.</p> <p>eg $\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$. [A/B]</p> <p>Applications include solution of triangles and three-dimensional situations, such as calculating the size of the angle between a line and a plane or between two planes. Work should extend to problems involving compound angles.</p> <p>Candidates should be given the opportunity to find the general solution to trigonometric equations, although in assessments solutions would be on a given interval, eg $0 \leq \theta \leq 2\pi$.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>The equation of the circle know that the equation of the circle centre (a, b) and radius r is $(x - a)^2 + (y - b)^2 = r^2$</p> <p>know that the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$ provided $g^2 + f^2 - c > 0$</p> <p>determine the equation of a circle</p> <p>solve mathematical problems involving the intersection of a straight line and a circle, a tangent to a circle</p> <p>determine whether two circles touch each other</p>	<p>ie Determine the points at which a given line intersects a given circle, determine whether a given line is a tangent to a given circle, eg Show that the line with equation $y = 2x - 10$ is a tangent to the circle $x^2 + y^2 - 4x + 2y = 0$ and state the coordinates of the point of contact.</p> <p>eg The line with equation $x - 3y = k$ is a tangent to the circle $x^2 + y^2 - 6x + 8y + 15 = 0$. Find the two possible values of k. [A/B]</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Mathematics 3 (H)</p> <p>Vectors in three dimensions know the terms: vector, magnitude (length), direction, scalar multiple, position vector, unit vector, directed line segment, component, scalar product</p> <p>know the properties of vector addition and multiplication of a vector by a scalar</p> <p>determine the distance between two points in three dimensional space</p> <p>know and apply the equality fact $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \Rightarrow a = d, b = e, c = f^*$</p> <p>know and apply the fact that if \mathbf{u} and \mathbf{v} are vectors that can be represented by parallel lines then $\mathbf{u} = k\mathbf{v}$ where k is a constant and the converse</p> <p>know and apply the fact that if A, P and B are collinear points such that $\frac{AP}{PB} = \frac{m}{n}$ then $\vec{AP} = \frac{m}{n} \vec{PB}$</p> <p>determine whether three points with given coordinates are collinear</p> <p>know and apply the basis vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}^*$</p>	<p>Suitable contexts for vectors could be forces and velocity.</p> <p>Triangular concurrency facts can be established by vector methods. Some candidates should be encouraged to explore this work through an investigative approach, which would lead to simple vector proofs.</p> <p>An understanding of both two and three dimensional vectors is expected. The items marked by an asterisk * are quoted for three dimensions but the two dimensional cases are also included.</p> <p>eg Determine the coordinates or position vector of the point which divides the join of two given points in a given ratio.</p> <p>The section formula may be used to find the position vector of P but is not required.</p> <p>Understanding of the concepts should be reinforced by concentrating on numerical examples.</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>know the scalar product facts:</p> $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos \theta$ $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3, \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} *$ $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$ <p>determine whether or not two vectors, in component form, are perpendicular</p> <p>use scalar product to find the angle between two directed line segments</p> <p>Further differentiation and integration</p> <p>know and apply the facts that:</p> <p>if $f(x) = \sin x$, then $f'(x) = \cos x$ and $\int \cos x \, dx = \sin x + C$</p> <p>if $f(x) = \cos x$, then $f'(x) = -\sin x$ and $\int \sin x \, dx = -\cos x + C$</p> <p>if $f(x) = g(h(x))$, then $f'(x) = g'(h(x)) \cdot h'(x)$</p> <p>integrate functions defined by $f(x) = (px + q)^n$ for all rational n, except $n = -1$, $f(x) = p \cos(qx + r)$ and $f(x) = p \sin(qx + r)$ and the sum or difference of such functions, where p, q and r are constants</p>	<p>eg If $\mathbf{a} , \mathbf{b} \neq 0$ then $\mathbf{a} \cdot \mathbf{b} = 0$ if and only if the directions of \mathbf{a} and \mathbf{b} are at right angles.</p> <p>The chain rule could be introduced after candidates are fluent in differentiating simple functions; conjectures about derivatives of $(x + 3)^2$, $(x + 3)^3$, $(2x + 3)^2$, $(2x + 3)^3$, etc., could be made and checked. Candidates should be familiar with other forms of the chain rule such as $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.</p> <p>eg Differentiate $(x + 4)^{1/2}$, $(x - 2)^{-3}$.</p> <p>eg Differentiate $(2x + 5)^3$, $\sin 3x$, $\cos^3 x$. [A/B]</p> <p>eg Integrate $(x + 4)^{1/2}$, $3 \cos x$.</p> <p>eg Integrate $(3x + 1)^3$, $\sin 2x$. [A/B]</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Logarithmic and exponential functions</p> <p>know that $a^y = x \Leftrightarrow \log_a x = y$ ($a > 1, x > 0$)</p> <p>know the laws of logarithms:</p> $\log_a 1 = 0$ $\log_a a = 1$ $\log_a(bc) = \log_a b + \log_a c$ $\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c$ $\log_a(b^n) = n \log_a b$ <p>simplify numerical expressions using the laws of logarithms</p> <p>solve simple logarithmic and exponential equations</p> <p>solve for a and b equations of the following forms, given two pairs of corresponding values of x and y: $\log y = a \log x + b$, $y = ax^b$, $y = ab^x$ [A/B]</p> <p>use a straight line graph to confirm a relationship of the form $y = ax^b$, also $y = ab^x$ [A/B]</p> <p>model mathematically situations involving the logarithmic or exponential function [A/B]</p>	<p>Change of base is not included and when base is understood in a particular context, $\log x$ can be used for $\log_a x$. The base a will normally be 10 or e and the notation $\ln x$ should be introduced to candidates.</p> <p>Numerical uses of logarithms and exponential evaluations are best done by calculators.</p> <p>eg $5 \log_8 2 + \log_8 4 - \log_8 16$.</p> <p>eg $\ln x = 2$, $\log_{10} x = 1.7$, $10^x = 2.5$</p> <p>eg $3.4^x = 5$ [A/B]</p> <p>eg From experimental data draw a graph of $\log y$ against $\log x$ and deduce values of a and b such that $y = ax^b$. [A/B]</p>

National Course Specification: course details (cont)

CONTENT	COMMENT
<p>Further trigonometric relationships express $a \cos \theta + b \sin \theta$ in the form $r \cos(\theta \pm \alpha)$ or $r \sin(\theta \pm \alpha)$</p> <p>solve, by expressing in one of the forms above, equations of the form $a \cos \theta + b \sin \theta = c$</p> <p>find maximum and minimum values of expressions of the form $a \cos \theta + b \sin \theta$ find corresponding values of θ [A/B]</p>	<p>Candidates should be encouraged to show all intermediate working, eg the calculation of r and α, the expansion of trigonometric formulae and the equating of coefficients.</p>

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

ASSESSMENT

To gain the award of the course, the candidate must pass all the unit assessments as well as the external assessment. External assessment will provide the basis for grading attainment in the course award.

When units are taken as component parts of a course, candidates should have the opportunity to achieve at levels beyond that required to attain each of the unit outcomes. This attainment may, where appropriate, be recorded and used to contribute towards course estimates and to provide evidence for appeals.

COURSE ASSESSMENT

The external assessment will take the form of an examination of 2 hours and 40 minutes duration. The external examination will test the candidate's ability to retain and integrate mathematical knowledge across the component units of the course. The examination will consist of two papers. Further information is available in the Course Assessment Specification and in the Specimen Question Paper.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

GRADE DESCRIPTIONS

Higher Mathematics courses should enable candidates to solve problems that integrate mathematical knowledge across performance criteria, outcomes and units, and which require extended thinking and decision making. The award of Grades A, B and C is determined by the candidate's demonstration of the ability to apply knowledge and understanding to problem-solving. To achieve Grades A and B in particular, this demonstration will involve more complex contexts including the depth of treatment indicated in the detailed content tables.

In solving problems, candidates should be able to:

- a) interpret the problem and consider what might be relevant;
- b) decide how to proceed by selecting an appropriate strategy;
- c) implement the strategy through applying mathematical knowledge and understanding and come to a conclusion;
- d) decide on the most appropriate way of communicating the solution to the problem in an intelligible form.

Familiarity and complexity affect the level of difficulty of problems. It is generally easier to interpret and communicate information in contexts where the relevant variables are obvious and where their inter relationships are known. It is usually more straightforward to apply a known strategy than to modify one or devise a new one. Some concepts are harder to grasp and some techniques more difficult to apply, particularly if they have to be used in combination.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

Exemplification of problem solving at Grade C and Grade A

a) **Interpret the problem and consider what might be relevant**

At Grade C candidates should be able to interpret qualitative and quantitative information as it arises within:

- the description of real-life situations
- the context of other subjects
- the context of familiar areas of mathematics.

Grade A performance is demonstrated through coping with the interpretation of more complex contexts requiring a higher degree of reasoning ability in the areas described above.

b) **Decide how to proceed by selecting an appropriate strategy**

At Grade C candidates should be able to tackle problems by selecting an algorithm, or sequence of algorithms, drawn from related areas of mathematics, or a heuristic strategy.

Grade A performance is demonstrated through an ability to decide on and apply a more extended sequence of algorithms to more complex contexts.

c) **Implement the strategy through applying mathematical knowledge and understanding and come to a conclusion**

At Grade C candidates should be able to use their knowledge and understanding to carry through their chosen strategy and come to a conclusion. They should be able to process data in numerical and symbolic form with appropriate regard for accuracy, marshal facts, sustain logical reasoning and appreciate the requirements of proof.

Grade A performance is demonstrated through an ability to cope with processing data in more complex situations, and sustaining logical reasoning, where the situation is less readily identifiable with a standard form.

d) **Decide on the most appropriate way of communicating the solution to the problem in an intelligible form**

At Grade C candidates should be able to communicate qualitative and quantitative mathematical information intelligibly and to express the solution in language appropriate to the situation.

Grade A performance is demonstrated through an ability to communicate intelligibly in more complex situations and unfamiliar contexts.

National Course Specification: course details (cont)

COURSE Mathematics (Higher)

APPROACHES TO LEARNING AND TEACHING

The learning and teaching process should foster positive attitudes to the subject. Exposition to a group or class remains an essential technique at this level, and active candidate involvement in learning should be encouraged through questioning and discussion. However, investigative approaches to learning should also feature prominently in the delivery of the Higher course. Where appropriate, new skills and concepts should be introduced within a context and, when suitable, through an investigative approach, sometimes giving candidates the opportunity to work co-operatively. Coursework tasks can support these approaches, and simultaneously allow the grade descriptions for extended problem solving to be met.

Opportunities should be taken to justify the need for new skills by reference to real problems. While the contexts may include familiar and everyday ones, they should also make use of situations from other subjects, or from areas of interest to candidates in their future study. Contexts which are purely mathematical will also be appropriate, for example, some aspects of quadratic theory might be illustrated in the context of tangency, *Mathematics 2 (H)*.

Wherever possible, candidates should be encouraged to make use of technology. Calculators have great potential provided they are used appropriately and not allowed to provide unnecessary support, nor substitute for personal proficiency. This balance between 'basic skills' and appropriate use of technology is essential. Calculators with mathematical and graphical facilities and those with computer algebra systems (CAS) can be utilised as powerful tools both for processing data, especially in the study of statistics, and for reinforcing mathematical concepts. The use of such calculators should help candidates gain confidence in making conjectures based on numerical or graphical evidence. Candidates should be aware that errors are inevitably introduced in the course of computation or in the limitations of the graphical display.

Computers can also make a significant contribution to learning and teaching. The use of software packages in statistics will enhance the learning and teaching and allow candidates greater flexibility through ease of computation and display.

It is envisaged that increased availability and advances in technology will have a continuing and increasing influence in approaches to learning and teaching at this level.

CANDIDATES WITH DISABILITIES AND/OR ADDITIONAL SUPPORT NEEDS

The additional support needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments, or considering alternative Outcomes for Units. Further advice can be found in the SQA document *Guidance on Assessment Arrangements for Candidates with Disabilities and/or Additional Support Needs* (www.sqa.org.uk).

National Unit Specification: general information

UNIT	Mathematics 1 (Higher)
NUMBER	D321 12
COURSE	Mathematics (Higher)

SUMMARY

Mathematics 1 (H) comprises outcomes in algebra, geometry, graphicacy and elementary calculus. It is a mandatory unit of the Higher Mathematics course and provides a basis for progression to *Mathematics 2 (H)*.

OUTCOMES

- 1 Use the properties of the straight line.
- 2 Associate functions and graphs.
- 3 Use basic differentiation.
- 4 Design and interpret mathematical models of situations involving recurrence relations.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates will normally be expected to have attained one of the following:

- Standard Grade Mathematics Credit award
- Intermediate 2 Mathematics or its component units including *Mathematics 3 (Int 2)*
- equivalent.

Administrative Information

Superclass:	RB
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National Unit Specification: general information (cont)

UNIT Mathematics 1 (Higher)

CREDIT VALUE

1 credit at Higher (6 SCQF credit points at SCQF level 6*).

**SCQF credit points are used to allocate credit to qualifications in the Scottish Credit and Qualifications Framework (SCQF). Each qualification in the Framework is allocated a number of SCQF credit points at an SCQF level. There are 12 SCQF levels, ranging from Access 1 to Doctorates.*

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit Numeracy H

Additional core skills components for the unit Critical Thinking H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT Mathematics 1 (Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use the properties of the straight line.

Performance criteria

- Determine the equation of a straight line given two points on the line or one point and the gradient.
- Find the gradient of a straight line using $m = \tan \theta$.
- Find the equation of a line parallel to and a line perpendicular to a given line.

OUTCOME 2

Associate functions and graphs.

Performance criteria

- Sketch and identify related graphs and functions.
- Identify exponential and logarithmic graphs.
- Find composite functions of the form $f(g(x))$, given $f(x)$ and $g(x)$.

OUTCOME 3

Use basic differentiation.

Performance criteria

- Differentiate a function reducible to a sum of powers of x .
- Determine the gradient of a tangent to a curve by differentiation.
- Determine the coordinates of the stationary points on a curve and justify their nature using differentiation.

OUTCOME 4

Design and interpret mathematical models of situations involving recurrence relations.

Performance criteria

- Define and interpret a recurrence relation in the form $u_{n+1} = mu_n + c$ (m, c constants) in a mathematical model.
- Find and interpret the limit of the sequence generated by a recurrence relation in a mathematical model (where the limit exists).

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessment, candidates are required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 1 (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON THE CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. Within this unit, study of coordinate geometry of the straight line, algebra and trigonometry, with the emphasis on graphicacy, is taken to a greater depth. Previous experience of number patterns is formalised in the context of recurrence relations and differential calculus is introduced.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates working at this level are expected to acquire a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within this unit, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

National Unit Specification: support notes (cont)

UNIT Mathematics 1 (Higher)

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to achieve the algebraic, trigonometric and calculus performance criteria of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates should be required to show their working in carrying out algorithms and processes.

CANDIDATES WITH DISABILITIES AND/OR ADDITIONAL SUPPORT NEEDS

The additional support needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments, or considering alternative Outcomes for Units. Further advice can be found in the SQA document *Guidance on Assessment Arrangements for Candidates with Disabilities and/or Additional Support Needs* (www.sqa.org.uk).

National Unit Specification: general information

UNIT Mathematics 2 (Higher)

NUMBER D322 12

COURSE Mathematics (Higher)

SUMMARY

Mathematics 2 (H) comprises outcomes in algebra, geometry, trigonometry and elementary calculus. It is a mandatory unit of the Higher Mathematics course and provides a basis for progression to *Mathematics 3 (H)*.

OUTCOMES

- 1 Use the Factor/Remainder Theorem and apply quadratic theory.
- 2 Use basic integration.
- 3 Solve trigonometric equations and apply trigonometric formulae.
- 4 Use the equation of the circle.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained one of the following:

- *Mathematics 1 (H)*
- equivalent.

Administrative Information

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National Unit Specification: general information (cont)

UNIT Mathematics 2 (Higher)

CREDIT VALUE

1 credit at Higher (6 SCQF credit points at SCQF level 6*).

**SCQF credit points are used to allocate credit to qualifications in the Scottish Credit and Qualifications Framework (SCQF). Each qualification in the Framework is allocated a number of SCQF credit points at an SCQF level. There are 12 SCQF levels, ranging from Access 1 to Doctorates.*

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	None
Core skills components for the unit	Critical Thinking H Using Number H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT Mathematics 2 (Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use the Factor/Remainder Theorem and apply quadratic theory.

Performance criteria

- Apply the Factor/Remainder Theorem to a polynomial function.
- Determine the nature of the roots of a quadratic equation using the discriminant.

OUTCOME 2

Use basic integration.

Performance criteria

- Integrate functions reducible to the sums of powers of x (definite and indefinite).
- Find the area between a curve and the x -axis using integration.
- Find the area between two curves using integration.

OUTCOME 3

Solve trigonometric equations and apply trigonometric formulae.

Performance criteria

- Solve a trigonometric equation in a given interval.
- Apply a trigonometric formula (addition formula) in the solution of a geometric problem.
- Solve a trigonometric equation involving an addition formula in a given interval.

OUTCOME 4

Use the equation of the circle.

Performance criteria

- Given the centre (a, b) and radius r , find the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.
- Find the radius and centre of a circle given the equation in the form $x^2 + y^2 + 2gx + 2fy + c = 0$.
- Determine whether a given line is a tangent to a given circle.
- Determine the equation of the tangent to a given circle given the point of contact.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 2 (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON THE CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. Within this unit, the coordinate geometry of *Mathematics 1(H)* is extended to include the circle. The Factor/Remainder Theorem and quadratic theory and addition formulae are added to previous experience in algebra and trigonometry respectively. Basic integration is introduced to extend the calculus of *Mathematics 1(H)*.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates working at this level are expected to acquire a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within this unit, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators, as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

National Unit Specification: support notes (cont)

UNIT Mathematics 2 (Higher)

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to achieve the algebraic, trigonometric and calculus performance criteria of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

CANDIDATES WITH DISABILITIES AND/OR ADDITIONAL SUPPORT NEEDS

The additional support needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments, or considering alternative Outcomes for Units. Further advice can be found in the SQA document *Guidance on Assessment Arrangements for Candidates with Disabilities and/or Additional Support Needs* (www.sqa.org.uk).

National Unit Specification: general information

UNIT Mathematics 3 (Higher)

NUMBER D323 12

COURSE Mathematics (Higher)

SUMMARY

Mathematics 3 (H) comprises outcomes in algebra, geometry, trigonometry and elementary calculus at Higher level. It is a mandatory unit of the Higher Mathematics Course

OUTCOMES

- 1 Use vectors in three dimensions.
- 2 Use further differentiation and integration.
- 3 Use properties of logarithmic and exponential functions.
- 4 Apply further trigonometric relationships.

RECOMMENDED ENTRY

While entry is at the discretion of the centre, candidates would normally be expected to have attained:

- *Mathematics 2 (H)*
- equivalent.

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National Unit Specification: general information (cont)

UNIT Mathematics 3 (Higher)

CREDIT VALUE

1 credit at Higher (6 SCQF credit points at SCQF level 6*).

**SCQF credit points are used to allocate credit to qualifications in the Scottish Credit and Qualifications Framework (SCQF). Each qualification in the Framework is allocated a number of SCQF credit points at an SCQF level. There are 12 SCQF levels, ranging from Access 1 to Doctorates.*

CORE SKILLS

This unit gives automatic certification of the following:

Complete core skills for the unit	None
Core skills components for the unit	Critical Thinking H Using Number H

Additional information about core skills is published in *Automatic Certification of Core Skills in National Qualifications* (SQA, 1999).

National Unit Specification: statement of standards

UNIT Mathematics 3 (Higher)

Acceptable performance in this unit will be the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to the Scottish Qualifications Authority.

OUTCOME 1

Use vectors in three dimensions.

Performance criteria

- a) Determine whether three points with given coordinates are collinear.
- b) Determine the coordinates of the point which divides the join of two given points internally in a given numerical ratio.
- c) Use the scalar product.

OUTCOME 2

Use further differentiation and integration.

Performance criteria

- a) Differentiate $k \sin x$, $k \cos x$.
- b) Differentiate using the function of a function rule.
- c) Integrate functions of the form $f(x) = (x + q)^n$, n rational except for -1 , $f(x) = p \cos x$ and $f(x) = p \sin x$.

OUTCOME 3

Use properties of logarithmic and exponential functions.

Performance criteria

- a) Simplify a numerical expression using the laws of logarithms.
- b) Solve simple logarithmic and exponential equations.

OUTCOME 4

Apply further trigonometric relationships.

Performance criteria

- a) Express $a \cos \theta + b \sin \theta$ in the form $r \cos (\theta \pm \alpha)$ or $r \sin (\theta \pm \alpha)$.

Evidence requirements

Although there are various ways of demonstrating achievement of the outcomes, evidence would normally be presented in the form of a closed book test under controlled conditions. Examples of such tests are contained in the National Assessment Bank.

In assessments, candidates are required to show their working in carrying out algorithms and processes.

National Unit Specification: support notes

UNIT Mathematics 3 (Higher)

This part of the unit specification is offered as guidance. The support notes are not mandatory.

While the exact time allocated to this unit is at the discretion of the centre, the notional design length is 40 hours.

GUIDANCE ON THE CONTENT AND CONTEXT FOR THIS UNIT

Each mathematics unit at Higher level aims to build upon and extend candidates' mathematical knowledge and skills. This optional unit is designed to continue the study of mathematics in the areas of algebra, geometry, trigonometry and calculus, and to provide a basis for the study of more advanced mathematics and applied mathematics.

Within this unit, the coordinate geometry of *Mathematics 1(H)* and *Mathematics 2(H)* is broadened into a three dimensional context with the introduction of vector notation and applications. Calculus is further extended to include differentiation and integration of the sine and cosine functions and the rules for integrating and differentiating composite functions. The exponential and logarithmic functions introduced graphically in *Mathematics 1(H)* are used in applications, and further trigonometry is studied in the context of the wave function.

The increasing degree of importance of mathematical rigour and the ability to use precise and concise mathematical language as candidates progress in mathematics assumes a particular importance at this stage. Candidates working at this level are expected to acquire a competence and a confidence in applying mathematical techniques, manipulating symbolic expressions and communicating with mathematical correctness in the solution of problems. It is important, therefore, that, within this unit, appropriate attention is given to the acquisition of such expertise whilst extending the candidate's 'toolkit' of knowledge and skills.

The recommended content for this unit can be found in the course specification. The *detailed content* section provides illustrative examples to indicate the depth of treatment required to achieve a unit pass and advice on teaching approaches.

GUIDANCE ON LEARNING AND TEACHING APPROACHES FOR THIS UNIT

Where appropriate, mathematical topics should be taught and skills in applying mathematics developed through real-life contexts. Candidates should be encouraged throughout this unit to make efficient use of the arithmetical, mathematical and graphical features of calculators as well as basic non-calculator skills. Candidates should be aware of the limitations of the technology and always apply the strategy of checking.

Numerical checking or checking a result against the context in which it is set is an integral part of every mathematical process. In many instances, the checking can be done mentally, but on occasions, to stress its importance, attention should be drawn to relevant checking procedures throughout the mathematical process. There are various checking procedures which could be used:

- relating to a context – 'How sensible is my answer?'
- estimate followed by a repeated calculation
- calculation in a different order.

Further advice on learning and teaching approaches is contained within the Subject Guide for Mathematics.

National Unit Specification: support notes (cont)

UNIT Mathematics 3 (Higher)

GUIDANCE ON APPROACHES TO ASSESSMENT FOR THIS UNIT

The assessment for this unit will normally be in the form of a closed book test. Such tests should be carried out under supervision and it is recommended that candidates attempt an assessment designed to assess all the outcomes within the unit. Successful achievement of the unit is demonstrated by candidates achieving the thresholds of attainment specified for all outcomes in the unit. Candidates who fail to achieve the threshold(s) of attainment need only be retested on the outcome(s) where the outcome threshold score has not been attained. Further advice on assessment and retesting is contained within the National Assessment Bank.

It is expected that candidates will be able to achieve the algebraic, trigonometric and calculus performance criteria of the unit without the use of computer software or sophisticated calculators.

In assessments, candidates are required to show their working in carrying out these algorithms and processes.

CANDIDATES WITH DISABILITIES AND/OR ADDITIONAL SUPPORT NEEDS

The additional support needs of individual candidates should be taken into account when planning learning experiences, selecting assessment instruments, or considering alternative Outcomes for Units. Further advice can be found in the SQA document *Guidance on Assessment Arrangements for Candidates with Disabilities and/or Additional Support Needs* (www.sqa.org.uk).