Advanced Higher Course Assessment Specification

Advanced Higher Mathematics Course Assessment Specification (C747 77)

Valid from August 2015

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Please refer to the note of changes at the end of this Course Assessment Specification for details of changes from previous version (where applicable).

Course outline

Course title:	Advanced Higher Mathematics
SCQF level:	7 (32 SCQF credit points)
Course code:	C747 77
Course assessment code:	X747 77

The purpose of the Course Assessment Specification is to ensure consistent and transparent assessment year on year. It describes the structure of the Course assessment and the mandatory skills, knowledge and understanding that will be assessed.

Course assessment structure

Component 1 — question paper	100 marks
Total marks	**100 marks**

This Course includes eight SCQF credit points to allow additional time for preparation for Course assessment. The Course assessment covers the added value of the Course.

Equality and inclusion

This Course Assessment Specification has been designed to ensure that there are no unnecessary barriers to assessment. Assessments have been designed to promote equal opportunities while maintaining the integrity of the qualification.

For guidance on assessment arrangements for disabled learners and/or those with additional support needs, please follow the link to the Assessment Arrangements web page: www.sqa.org.uk/sqa/14977.html.

Guidance on inclusive approaches to delivery and assessment of this Course is provided in the *Course/Unit Support Notes*.

Assessment

To gain the award of the Course, the learner must pass all the Units as well as the Course assessment. Course assessment will provide the basis for grading attainment in the Course award.

Course assessment

SQA will produce and give instructions for the production and conduct of Course assessments based on the information provided in this document.

Added value

The purpose of the Course assessment is to assess added value of the Course as well as confirming attainment in the Course and providing a grade. The added value for the Course will address the key purposes and aims of the Course, as defined in the Course Rationale. It will do this by addressing one or more of breadth, challenge or application.

In this Course assessment, added value will focus on the following:

- breadth — drawing on knowledge and skills from across the Course
- challenge — requiring greater depth or extension of knowledge and skills
- application — requiring application of knowledge and skills in practical or theoretical contexts as appropriate

This added value consists of:

- using mathematical reasoning skills to think logically, provide justification and solve problems
- using a range of complex concepts
- selecting and applying complex operational skills
- using reasoning skills to interpret information and to use complex mathematical models
- effectively communicating solutions in a variety of mathematical contexts
- explaining and justifying concepts through the idea of rigorous proof
- thinking creatively

To achieve success in the Course, learners must show that they can apply knowledge and skills acquired across the Course to unseen situations.

The question paper requires learners to demonstrate aspects of breadth, challenge and application in mathematical contexts. The use of a calculator will be permitted.

Grading

Course assessment will provide the basis for grading attainment in the Course award.

The Course assessment is graded A–D. The grade is determined on the basis of the total mark for the Course assessment.

A learner's overall grade will be determined by their performance across the Course assessment.

Grade description for C
For the award of Grade C, learners will have demonstrated successful performance in all of the Units of the Course. In the Course assessment, learners will typically have demonstrated successful performance in relation to the mandatory skills, knowledge and understanding for the Course.

Grade description for A
For the award of Grade A, learners will have demonstrated successful performance in all of the Units of the Course. In the Course assessment, learners will typically have demonstrated a consistently high level of performance in relation to the mandatory skills, knowledge and understanding for the Course.

Credit
To take account of the extended range of learning and teaching approaches, remediation, consolidation of learning and integration needed for preparation for external assessment, six SCQF credit points are available in Courses at National 5 and Higher, and eight SCQF credit points in Courses at Advanced Higher. These points will be awarded when a Grade D or better is achieved.

Structure and coverage of the Course assessment

The Course assessment will consist of one Component: a question paper.

Component 1 — question paper
The purpose of the question paper is to assess mathematical skills. A calculator may be used.

The question paper will sample the skills, knowledge and understanding that are contained in the 'Further mandatory information on Course coverage' section at the end of this Course Assessment Specification.

The question paper will consist of a series of short and extended response questions (some of which may be set in contexts) that require the application of skills developed in the Course. Learners will be expected to communicate responses clearly and to justify solutions. The paper will have 100 marks.

Assessors can give learners access to the formulae contained in the formulae sheet accompanying the Advanced Higher Mathematics Course assessment. Assessors can also give learners access to any other derivative or formula which does not form part of this Course.

For more information about the structure and coverage of the Course assessment, refer to the Question Paper Brief.

Setting, conducting and marking of assessment

Question paper

The question paper will be set and marked by SQA and conducted in centres under conditions specified for external examinations by SQA. Learners will complete this in 3 hours.

Further mandatory information on Course coverage

The following gives details of mandatory skills, knowledge and understanding for the Advanced Higher Mathematics Course. Course assessment will involve sampling the skills, knowledge and understanding. This list of skills, knowledge and understanding also provides the basis for the assessment of Units of the Course.

This includes:

- the ability to use mathematical reasoning skills to think logically, provide justification and solve problems
- knowledge and understanding of a range of complex concepts
- the ability to select and apply complex operational skills
- the ability to use reasoning skills to interpret information and to use complex mathematical models
- the ability to effectively communicate solutions in a variety of contexts
- the ability to explain and justify concepts through the idea of rigorous proof
- the ability to think creatively

These skills will be assessed across the Course, in the context of the mandatory knowledge.

Mathematics: Methods in Algebra and Calculus (Advanced Higher)	
1.1 Applying algebraic skills to partial fractions	
Sub-skill	**Description**
Expressing rational functions as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	Express a proper rational function as a sum of partial fractions where the denominator may contain: distinct linear factors, an irreducible quadratic factor, a repeated linear factor Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise
1.2 Applying calculus skills through techniques of differentiation	
Sub-skill	**Description**
Differentiating exponential and logarithmic functions	Differentiate functions involving e^x, $\ln x$
Differentiating functions using the chain rule	Apply the chain rule to differentiate the composition of at most 3 functions

Differentiating functions given in the form of a product and in the form of a quotient	Differentiate functions of the form $\left(f\left(x \right) g\left(x \right) \right)$ and $\left(\frac{f\left(x \right)}{g\left(x \right)} \right)$ Know the definitions and use the derivatives of $\tan x$ and $\cot x$ Know the definitions of $\sec x$ and $\operatorname{cosec} x$ Learners should be able to derive and use derivatives of $\tan x,\ \cot x,\ \sec x,\ \operatorname{cosec} x$ Differentiating functions which require more than one application or combination of applications of chain rule, product rule and quotient rule Know and use $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ Use logarithmic differentiation; recognise when it is appropriate in extended products, quotients, and in functions where the variable occurs in an index
Differentiating inverse trigonometric functions	Differentiating expressions of the form eg $\sin^{-1} kx$, $\tan^{-1}\left[f(x) \right]$
Finding the derivative of functions defined implicitly	Use differentiation to find the first derivative of a function defined implicitly including in context Use differentiation to find the second derivative of a function defined implicitly
Finding the derivative of functions defined parametrically	Use differentiation to find the first derivative of a function defined parametrically including in context Use differentiation to find the second derivative of a function defined parametrically Solve practical related rates by first establishing a functional relationship between appropriate variables

1.3 Applying calculus skills through techniques of integration	
Sub-skill	**Description**
Integrating expressions using standard results	Use $\int e^x dx,\ \int \frac{dx}{x},\ \int \sec^2 x\, dx$ Use the integrals of $\frac{1}{\sqrt{a^2 - x^2}}, \frac{1}{a^2 + x^2}$ Recognise and integrate expressions of the form $\int g(f(x))\, f'(x) dx$ and $\int \frac{f'(x)}{f(x)} dx$ Use partial fractions to integrate proper rational functions where the denominator may have: i) two separate or repeated linear factors ii) three linear factors with constant numerator iii) three linear factors with non-constant numerator iv) a linear factor and an irreducible quadratic factor of the form $ax^2 + bx + c$
Integrating by substitution	Integrate where the substitution is given
Integrating by parts	Use integration by parts with one application Use integration by parts involving repeated applications

1.4 Applying calculus skills to solving differential equations	
Sub-skill	**Description**
Solving first order differential equations with variables separable	Solve equations that can be written in the form $\frac{dy}{dx} = g(x)h(y)$ or $\frac{dy}{dx} = \frac{g(x)}{h(y)}$ Find general and particular solutions given suitable information
Solving first order linear differential equations using an integrating factor	Solve equations by first writing linear equations in the standard form $\frac{dy}{dx} + P(x)y = f(x)$ Find general and particular solutions given suitable information
Solving second order differential equations	Find the general solution and particular solution of second order homogeneous ordinary differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ with constant coefficients where the roots of the auxiliary equation are real or complex conjugates Solve second order non-homogeneous ordinary differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ with constant coefficients using the auxiliary equation and particular integral method

Mathematics: Applications in Algebra and Calculus (Advanced Higher)

1.1 Applying algebraic skills to the binomial theorem and to complex numbers

Sub-skill	Description
Expanding expressions using the binomial theorem	Use the binomial theorem $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$, for $r, n \in \mathbb{N}$ Expand an expression of the form $(ax^p + by^q)^n$, where $a, b \in \mathbb{Q}$; $p, q \in \mathbb{Z}$; $n \leq 7$. Using the general term for a binomial expansion, find a specific term in an expression
Performing algebraic operations on complex numbers	Perform all of the operations of addition, subtraction, multiplication and division Find the square root Find the roots of a cubic or quartic with real coefficients when one complex root is given Solve equations involving complex numbers

1.2 Applying algebraic skills to sequences and series

Sub-skill	Description
Finding the general term and summing arithmetic and geometric progressions	Apply the rules on sequences and series to find the nth term, sum to n terms, sum to infinity, common difference, and common ratio of arithmetic and geometric sequences respectively Determine the condition for a particular geometric series to converge
Using the Maclaurin expansion to find a stated number of terms of the power series for a simple function	Use the Maclaurin expansion to find a power series for a simple non-standard function Use the Maclaurin expansion to find a power series

1.3 Applying algebraic skills to summation and mathematical proof	
Sub-skill	**Description**
Applying summation formulae	Know and use sums of certain series and other straightforward results and combinations thereof (formulae which appear on the formulae sheet are 'use' only)
Using proof by induction	Use mathematical induction to prove summation formulae Use proof by induction
1.4 Applying algebraic and calculus skills to properties of functions	
Sub-skill	**Description**
Finding the asymptotes to the graphs of rational functions	Find the vertical asymptote to the graph of a rational function Find the non-vertical asymptote to the graph of a rational function
Investigating features of graphs and sketching graphs of functions	Investigate points of inflection Investigate other features: stationary points, domain and range, symmetry (odd/even), continuous/discontinuous, extrema of functions: the maximum and minimum values of a continuous function f defined on a closed interval [a,b] can occur at stationary points, end points or points where f' is not defined Sketch graphs using features given or obtained Sketch related functions: modulus functions inverse functions differentiated functions translations and reflections
1.5 Applying algebraic and calculus skills to problems	
Sub-skill	**Description**
Applying differentiation to problems, in context	Apply differentiation to problems in context Apply differentiation to optimisation
Applying integration to problems, in context	Apply integration to volumes of revolution where the volume generated is by the rotation of the area under a single curve about the x and y axes Use calculus to determine corresponding connected integrals Apply integration to the evaluation of areas including integration with respect to y

Mathematics : Geometry, Proof and Systems of Equations (Advanced Higher)

1.1 Applying algebraic skills to matrices and systems of equations

Sub-skill	Description
Using Gaussian elimination to solve a 3×3 system of linear equations	Find the solution to a system of equations $A\mathbf{x} = \mathbf{b}$, where A is a 3×3 matrix and where the solution is unique Candidates should understand the term augmented matrix Show that a system of equations has no solutions (inconsistency) Show that a system of equations has an infinite number of solutions (redundancy) Compare the solutions of related systems of two equations in two unknowns and recognise ill-conditioning
Understanding and using matrix algebra	Perform matrix operations (at most order 3): addition, subtraction, multiplication by a scalar, multiplication of matrices Know and apply the properties of matrix addition and multiplication: 1. $A+B=B+A$ 2. $AB \neq BA$ (in general) 3. $(A+B)+C=A+(B+C)$ (associativity) 4. $(AB)C=A(BC)$ (associativity) 5. $A(B+C)=AB+AC$ (addition is distributive over multiplication) Know and apply key properties of the transpose, identity matrix and the inverse: 1. $(a_{ij})'_{m\times n}=(a_{ji})_{n\times m}$ ie rows and columns interchange 2. $(A')'=A$ 3. $(A+B)'=A'+B'$ 4. $(AB)'=B'A'$ 5. A matrix A is orthogonal if $A'A=I$ 6. The $n \times n$ identity matrix I_n: for any square matrix A, $AI_n=I_nA=A$

	7. $B = A^{-1}$ if $AB = BA = I$ 8. $(AB)^{-1} = B^{-1}A^{-1}$
Calculating the determinant of a matrix	Find the determinant of a 2×2 matrix and a 3×3 matrix Determine whether a matrix is singular Know and apply $\det(AB) = \det A \det B$
Finding the inverse of a matrix	Know and use the inverse of a 2×2 matrix Find the inverse of a 3×3 matrix, for example by matrix algebra Use 2×2 matrices to carry out geometric transformations in the plane The transformations should include rotations, reflections and dilatations Apply combinations of transformations
1.2 Applying algebraic and geometric skills to vectors	
Sub-skill	**Description**
Calculating a vector product	Use a vector product method in three dimensions to find the vector product Evaluate the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$
Working with lines in three dimensions	Find the equation of a line in parametric, symmetric or vector form, given suitable defining information Find the angle between two lines in three dimensions Determine whether or not two lines intersect and, where possible, find the point of intersection
Working with planes	Find the equation of a plane in vector form, parametric form or Cartesian form, given suitable defining information Find the point of intersection of a plane with a line which is not parallel to the plane Determine the intersection of two or three planes Find the angle between a line and a plane or between two planes

1.3 Applying geometric skills to complex numbers	
Sub-skill	**Description**
Performing geometric operations on complex numbers	Plot complex numbers in the complex plane (an Argand diagram) Know the definition of modulus and argument of a complex number Convert a given complex number from Cartesian to polar form and vice-versa Use de Moivre's theorem with integer and fractional indices Apply de Moivre's theorem to multiple angle trigonometric formulae Apply de Moivre's theorem to find the n th roots of a complex number Interpret geometrically certain equations or inequalities in the complex plane, ie find the loci defined by (in)equalities
1.4 Applying algebraic skills to number theory	
Sub-skill	**Description**
Using Euclid's algorithm to find the greatest common divisor of two positive integers	Use Euclid's algorithm to find the greatest common divisor of two positive integers, ie use the division algorithm repeatedly Express the greatest common divisor (of two positive integers) as a linear combination of the two Express integers in bases other than ten Know and use the Fundamental Theorem of Arithmetic
1.5 Applying algebraic and geometric skills to methods of proof	
Sub-skill	**Description**
Disproving a conjecture by providing a counter-example	Disprove a conjecture by providing a counter-example Know and be able to use the symbols $\exists$ and $\forall$ Write down the negation of a statement
Using indirect or direct proof in straightforward examples	Prove a statement by contradiction Use further proof by contradiction Use proof by contrapositive Use direct proof in straightforward examples

Administrative information

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History of changes to Course Assessment Specification

Version	Description of change	Authorised by	Date
2.0	Pages 7–15: extensive changes to 'Further mandatory information on Course coverage' section.	Qualifications Development Manager	April 2015
2.1	Page 5: 'Structure and coverage of the Course assessment' section — reference to the formulae sheet added. Pages 8, 12, 14 and 15: 'Further mandatory information on Course coverage' section updated to include references to formulae sheet.	Qualifications Development Manager	June 2015
2.2	Pages 11 and 12: 'Further mandatory information on Course coverage' section clarified: amendments to second sub-skill for Assessment Standard 1.2 and description relating to second sub-skill for Assessment Standard 1.5.	Qualifications Development Manager	September 2015
2.3	Page 8: Further mandatory information on Course coverage' section clarified: amendments to second sub-skill for Assessment Standard 1.2 (Applying calculus skills through techniques of differentiation).	Qualifications Development Manager	November 2015
2.4	Page 5: 'Structure and coverage of the Course assessment' section — reference to the Question Paper Brief added. Pages 11–15: Further mandatory information on Course coverage' section clarified: symbols for standard sets ($\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$) and for matrices standardised; minor amendments to wording relating to second sub-skill for Assessment Standard 1.1 of the Applications in Algebra and Calculus	Qualifications Manager	April 2016

	Unit and Assessment Standard 1.3 of the Geometry, Proof and Systems of Equations Unit; wording relating to formulae sheet amended.		