Advanced Higher Course Assessment Specification

Advanced Higher Mathematics of Mechanics Course Assessment Specification (C702 77)

Valid from August 2015

This edition: April 2016, version 2.4

Please refer to the note of changes at the end of this Course Assessment Specification for details of changes from previous version (where applicable).

Course outline

Course title:	Advanced Higher Mathematics of Mechanics
SCQF level:	7 (32 SCQF credit points)
Course code:	C702 77
Course assessment code:	X702 77

The purpose of the Course Assessment Specification is to ensure consistent and transparent assessment year on year. It describes the structure of the Course assessment and the mandatory skills, knowledge and understanding that will be assessed.

Course assessment structure

Component 1 — question paper	100 marks
Total marks	**100 marks**

This Course includes eight SCQF credit points to allow additional time for preparation for Course assessment. The Course assessment covers the added value of the Course.

Equality and inclusion

This Course Assessment Specification has been designed to ensure that there are no unnecessary barriers to assessment. Assessments have been designed to promote equal opportunities while maintaining the integrity of the qualification.

For guidance on assessment arrangements for disabled learners and/or those with additional support needs, please follow the link to the Assessment Arrangements web page: www.sqa.org.uk/sqa/14977.html.

Guidance on inclusive approaches to delivery and assessment of this Course is provided in the *Course/Unit Support Notes*.

Assessment

To gain the award of the Course, the learner must pass all of the Units as well as the Course assessment. Course assessment will provide the basis for grading attainment in the Course award.

Course assessment

SQA will produce and give instructions for the production and conduct of Course assessments based on the information provided in this document.

Added value

The purpose of the Course assessment is to assess added value of the Course as well as confirming attainment in the Course and providing a grade. The added value for the Course will address the key purposes and aims of the Course, as defined in the Course Rationale. It will do this by addressing one or more of breadth, challenge or application.

In this Course assessment, added value will focus on the following:

- breadth — drawing on knowledge and skills from across the Course
- challenge — requiring greater depth or extension of knowledge and skills
- application — requiring application of knowledge and skills in practical or theoretical contexts as appropriate

This added value consists of:

- using a range of complex concepts in mechanics
- identifying and using appropriate techniques in mechanics
- using mathematical reasoning skills to extract and interpret information
- creating and using mathematical models to solve problems
- communicating identified strategies of solution and providing justification for the resulting conclusions
- analysing the problem both as a whole and as its integral parts

To achieve success in the Course, learners must show that they can apply knowledge and skills acquired across the Course to unseen situations.

The question paper requires learners to demonstrate aspects of breadth, challenge and application in contexts appropriate to mechanics. The use of a calculator will be permitted.

Grading

Course assessment will provide the basis for grading attainment in the Course award.

The Course assessment is graded A–D. The grade is determined on the basis of the total mark for the Course assessment.

A learner's overall grade will be determined by their performance across the Course assessment.

Grade description for C

For the award of Grade C, learners will have demonstrated successful performance in all of the Units of the Course. In the Course assessment, learners will typically have demonstrated successful performance in relation to the mandatory skills, knowledge and understanding for the Course.

Grade description for A

For the award of Grade A, learners will have demonstrated successful performance in all of the Units of the Course. In the Course assessment, learners will typically have demonstrated a consistently high level of performance in relation to the mandatory skills, knowledge and understanding for the Course.

Credit

To take account of the extended range of learning and teaching approaches, remediation, consolidation of learning and integration needed for preparation for external assessment, six SCQF credit points are available in Courses at National 5 and Higher, and eight SCQF credit points in Courses at Advanced Higher. These points will be awarded when a Grade D or better is achieved.

Structure and coverage of the Course assessment

The Course assessment will consist of one Component: a question paper.

Component 1 — question paper

The purpose of the question paper is to assess mathematical skills and their application to mechanics. A calculator may be used.

The question paper will sample the skills, knowledge and understanding that are contained in the ‘Further mandatory information on Course coverage’ section at the end of this Course Assessment Specification.

The question paper will consist of a series of short and extended response questions set in contexts that require the application of skills developed in the Course. Learners will be expected to communicate responses clearly and to justify solutions. The paper will have 100 marks.

Assessors can give learners access to the formulae contained in the formulae sheet accompanying the Advanced Higher Mathematics of Mechanics Course assessment. Assessors can also give learners access to any other derivative or formula which does not form part of this Course.

For more information about the structure and coverage of the Course assessment, refer to the Question Paper Brief.

Setting, conducting and marking of assessment

Question paper

The question paper will be set and marked by SQA and conducted in centres under conditions specified for external examinations by SQA. Learners will complete this in 3 hours.

Further mandatory information on Course coverage

The following gives details of mandatory skills, knowledge and understanding for the Advanced Higher Mathematics of Mechanics Course. Course assessment will involve sampling the skills, knowledge and understanding. This list of skills, knowledge and understanding also provides the basis for the assessment of Units of the Course.

This includes:

- knowledge and understanding of a range of straightforward and complex concepts in mechanics
- the ability to identify and use appropriate techniques in mechanics
- the ability to use mathematical reasoning and operational skills to extract and interpret information
- the ability to create and use multifaceted mathematical models
- the ability to communicate identified strategies of solution and provide justification for the resulting conclusions in a logical way
- the ability to comprehend both the problem as a whole and its integral parts
- the ability to select and use numerical skills

These skills will be assessed across the Course, in the context of the mandatory knowledge.

Mathematics of Mechanics: Force, Energy and Periodic Motion (Advanced Higher)

1.1 Applying skills to principles of momentum, impulse, work, power and energy	
Sub-skill	**Description**
Working with impulse as the change in momentum, and/or force as the rate of change of momentum	Use impulse appropriately in a simple situation, making use of the equations: $\mathbf{I} = m\mathbf{v} - m\mathbf{u} = \int \mathbf{F}dt$ and $I = Ft$.
Working with the concept of conservation of linear momentum	Use the concept of the conservation of linear momentum: $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$ or $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = (m_1 + m_2)\mathbf{v}$ for bodies that coalesce.
	Solve problems on linear motion in lifts, recoil of a gun, pile drivers etc.
Determining work done by a constant force in one or two dimensions, or a variable force during rectilinear motion	Evaluate appropriately the work done by a constant force, making use of the equations: $W = Fd$ (one dimension) $W = \mathbf{F}\cdot\mathbf{d}$ (two dimensions).
	Determine the work done in rectilinear motion by a variable force, using integration: $W = \int \mathbf{F}\cdot\mathbf{i}\,dx = \int \mathbf{F}\cdot\mathbf{v}\,dt$ where $\mathbf{v} = \frac{dx}{dt}\mathbf{i}$.
	Apply to practical examples the concept of power as the rate of doing work: $P = \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v}$ (constant force).
Using the concepts of kinetic (E_K) and/or potential (E_P) energy to applying the work–energy principle	$E_K = \frac{1}{2}mv^2$, $E_P = mgh$ for a uniform gravitational field Work done = change in energy $E_P = \frac{\lambda x^2}{2l}$ for elastic strings/springs $E_P = \frac{GMm}{r}$ associated with Newton's Inverse Square Law
Using the concepts of kinetic (E_K) and/or potential (E_P) energy within the concept of conservation of energy	$E_K + E_P$ = constant for simple problems involving motion in a plane
	Use of this within a situation involving vertical circular motion.

1.2 Applying skills to motion in a horizontal circle with uniform angular velocity	
Sub-skill	**Description**
Applying equations to motion in a horizontal circle with uniform angular velocity	Solve problems involving motion in a circle of radius r, with uniform angular velocity ω, making use of the equations: $\theta = \omega t$ $v = r\omega = r\dot{\theta}$ $a = r\omega^2 = r\dot{\theta}^2 = \frac{v^2}{r}$ $\mathbf{a} = -\omega^2\mathbf{r}$ $T = \frac{2\pi}{\omega}$
	Apply these equations to motion including skidding, banking and other applications.
Using equations for horizontal circular motion alongside Newton's Inverse Square Law of Gravitation	Solve a simple problem involving Newton's Inverse Square Law: $F = \frac{GMm}{r^2}$ Identify modelling assumptions made in particular contexts. Examples include applying this to simplified motion of satellites and moons, making use of the equations of motion for horizontal circular motion to find the time for one orbit, the height of the satellite above the planet's surface etc.
1.3 Applying skills to Simple Harmonic Motion	
Sub-skill	**Description**
Working with the concept of Simple Harmonic Motion (SHM)	Understand the concept of SHM and use the basic equation for SHM, $\ddot{x} = -\omega^2 x$, and the following associated equations, knowing when and where they arise in order to solve basic problems involving SHM in a straight line: $v^2 = \omega^2\left(a^2 - x^2\right)$ where $v = \dot{x}$ $T = \frac{2\pi}{\omega}$ $\lvert v\rvert_{\max} = \omega a$ $\lvert\ddot{x}\rvert_{\max} = \omega^2 a$
	Apply the solutions $x = a\sin\left(\omega t + \alpha\right)$ and the special cases $x = a\sin\omega t$ and $x = a\cos\omega t$ to solve problems.
Applying Hooke's Law to problems involving SHM	Make use of the equation for Hooke's Law, $T = \frac{\lambda x}{l}$

	to determine an unknown tension/thrust, modulus of elasticity or extension/compression or natural length. Consider the position of equilibrium and the equation of motion for an oscillating mass, and apply these to the solution of problems involving SHM, including problems involving shock absorbers, small amplitude oscillations of a simple pendulum etc.
1.4 Applying skills to centres of mass	
Sub-skill	**Description**
Determining the turning effect of force	Evaluate the turning effect of a single force or a set of forces acting on a body, considering clockwise and anticlockwise rotation: Moment of force about point *P* = magnitude of force × perpendicular distance from *P*
	Understand that for a body in equilibrium the sum of the moments of the forces about any point is zero Consider the forces on a body or a rod on the point of tipping or turning
Using moments to find the centre of mass of a body	Equate the moments of individual masses which lie on a straight line to that of a single mass acting at a point on the line ie $\sum m_i x_i = \bar{x} \sum m_i$ where $(\bar{x}, 0)$ is the centre of mass of the system.
	Extend this to two perpendicular directions to find the centre of mass of a set of particles arranged in a plane. ie $\sum m_i x_i = \bar{x} \sum m_i$ and $\sum m_i y_i = \bar{y} \sum m_i$ where $(\bar{x}, \bar{y})$ is the centre of mass of the system.
	Find the positions of centres of mass of standard uniform plane laminas, including rectangle, triangle, circle and semicircle. For a triangle, the centre of mass will be $\frac{2}{3}$ along median from vertex. For a semicircle, the centre of mass will be $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.
	Apply integration to find the centre of mass of a uniform composite lamina of area A, bounded by a given curve $y = f(x)$ and the lines $x = a$ and $x = b$ using $A\bar{x} = \int_a^b xy\,dx \quad A\bar{y} = \int_a^b \frac{1}{2} y^2\,dx$

Mathematics of Mechanics: Linear and Parabolic Motion (Advanced Higher)

1.1 Applying skills to motion in a straight line

Sub-skill	Description
Working with time-dependent graphs	Sketch and annotate, interpret and use displacement/time, velocity/time and acceleration/time graphs.
	Determine the distance travelled using the area under a velocity/time graph.
Working with rates of change with respect to time in one dimension	Use calculus to determine corresponding expressions connecting displacement, velocity and acceleration.
Using equations of motion in one dimension under constant acceleration	Using calculus, derive the equations of motion: $v = u + at$ and $s = ut + \frac{1}{2}at^2$. and use these to establish the equations: $v^2 = u^2 + 2as$ $s = \frac{(u+v)t}{2}$ $s = vt - \frac{1}{2}at^2$ Use these equations of motion in relevant contexts. Identify modelling assumptions made in particular contexts.

1.2 Applying skills to vectors associated with motion

Sub-skill	Description
Using vectors to define displacement, velocity and acceleration	Give the displacement, velocity and acceleration of a particle as a vector and understand speed is the magnitude of the velocity vector. If $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ where x and y are functions of t then $\underset{\sim}{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$ and $\underset{\sim}{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$
Finding resultant velocity, relative velocity or relative acceleration of one body with respect to another	Resolve position, velocity and acceleration vectors into two and three dimensions and use these to consider resultant or relative motion.
	Apply position, velocity and acceleration vectors to practical problems, including navigation, the effects of winds and currents and other relevant contexts.
Applying understanding of relative motion	Solve a simple problem involving collision. Consider conditions for nearest approach.

1.3 Applying skills to projectiles moving in a vertical plane	
Sub-skill	**Description**
Establishing the conditions of motion in horizontal and vertical directions involved in parabolic motion	Derive the formulae $T = \frac{2u\sin\alpha}{g}$ $H = \frac{u^2\sin^2\alpha}{2g}$ $R = u\cos\alpha \times T = \frac{u^2\sin 2\alpha}{g}$ Where T refers total time of flight H refers to greatest height R refers to the horizontal range
Using the equations of motion in parabolic flight	Use these formulae to find the time of flight, greatest height reached, or range of a projectile including maximum range of a projectile and the angle of projection to achieve this.
	Derive and use the equation of the trajectory of a projectile: $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$
	Solve problems in two-dimensional motion involving projectiles under a constant gravitational force. Projection will be considered in one vertical plane but point of projection can be from a different horizontal plane than that of landing.

1.4 Applying skills to forces associated with dynamics and equilibrium	
Sub-skill	**Description**
Using Newton's first and third laws of motion to understand equilibrium	Resolve forces in two dimensions to find their components. Consider the equilibrium of connected particles. Combine forces to find the resultant force.
Understanding the concept of static friction, dynamic friction and limiting friction	Know and use the relationships $F = \mu R$ and $\mu = \tan\theta$. For stationary bodies $F \leq \mu R$.
	Solve problems involving a particle or body in equilibrium under the action of certain forces.
Using Newton's Second Law of motion	Use $F = ma$ to form equations of motion to model practical problems of motion in a straight line where acceleration may be considered as a function of time or of displacement.
	Solve problems involving motion on inclined planes, possibly including friction.

Mathematics of Mechanics: Mathematical Techniques for Mechanics (Advanced Higher)	
1.1 Applying algebraic skills to partial fractions	
Sub-skill	**Description**
Expressing rational functions as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	Express a proper rational function as a sum of partial fractions where the denominator may contain: distinct linear factors, an irreducible quadratic factor, a repeated linear factor. Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise.
1.2 Applying calculus skills through techniques of differentiation	
Sub-skill	**Description**
Differentiating, exponential and logarithmic functions	Differentiate functions involving: e^x; $\ln x$
Differentiating functions using the chain rule	Apply the chain rule to differentiate the composition of at most 3 functions.
Differentiating functions given in the form of a product and/or in the form of a quotient	Differentiate functions of the form $f(x)g(x)$ and/or $\frac{f(x)}{g(x)}$ Know the definitions and use the derivatives of $\tan x$ and $\cot x$. Know the definitions of $\sec x$ and $\operatorname{cosec} x$. Candidates should be able to derive and use derivatives of $\tan x$, $\cot x$, $\sec x$, $\operatorname{cosec} x$. Differentiating functions which require more than one application or combination of applications of chain rule, product rule and quotient rule. Know that. $\frac{dy}{dx} = \frac{1}{\frac{dy}{dx}}$
Finding the derivative of functions defined implicitly	Use differentiation to find the first derivative of a function defined implicitly including in context. Use differentiation to find the second derivative of a function defined implicitly.

Finding the derivative of functions defined parametrically	Use differentiation to find the first derivative of a function defined parametrically. Solve practical related rates by first establishing a functional relationship between appropriate variables.
1.3 Applying calculus skills through techniques of integration	
Sub-skill	**Description**
Integrating expressions using standard results	Use $\int e^x dx,\ \int \frac{dx}{x},\ \int \sec^2 x\, dx$. Recognise and integrate expressions of the form $\int g\left(f(x)\right) f'(x) dx$ and $\int \frac{f'(x)}{f(x)} dx$ Use partial fractions to integrate proper rational functions where the denominator may have: i) two separate or repeated linear factors ii) three linear factors with constant numerator iii) three linear factors with a non-constant numerator
Integrating using a substitution when the substitution is given	Integrate where the substitution is given.
Integrating by parts	Use one application of integration by parts. Use integration by parts involving repeated applications.
Applying integration to a range of physical situations	Apply integration to evaluate volumes of revolution about the x-axis. Apply integration to evaluate volumes of revolution about the y-axis. Apply integration to evaluate areas.

1.4 Applying calculus skills to solving differential equations	
Sub-skill	**Description**
Finding a general solution of a first order differential equation with variables separable	Solve equations that can be written in the form $\frac{dy}{dx} = g(x)h(y)$ or $\frac{dy}{dx} = \frac{g(x)}{h(y)}$ Find the particular solution where initial conditions are given. Use differential equations in context.
Solving a first order linear differential equation using an integrating factor	Solve equations by writing linear equations in the standard form $\frac{dy}{dx} + P(x)y = f(x)$ Use differential equations in context.
Solving second order homogeneous equations	Find the general solution of a second order homogeneous ordinary differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ where the roots of the auxiliary equation are real and distinct where the roots of the auxiliary equation are real and equal Use differential equations in context.

Administrative information

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History of changes to Course Assessment Specification

Version	Description of change	Authorised by	Date
2.0	Pages 7–15: extensive changes to 'Further mandatory information on Course coverage' section.	Qualifications Development Manager	April 2015
2.1	Page 5: 'Structure and coverage of the Course assessment' section — reference to the formulae sheet added. 'Further mandatory information on Course coverage' section: page 10 — additional detail about centre of mass included; page 11 — equation amended.	Qualifications Development Manager	June 2015
2.2	Page 13: 'Further mandatory information on Course coverage' section — wording of Assessment Standards 1.1 and 1.2 under 'Mathematics of Mechanics: Mathematical Techniques for Mechanics (Advanced Higher)' heading amended.	Qualifications Development Manager	September 2015
2.3	Page 13: Further mandatory information on Course coverage' section clarified: amendments to second sub-skill for Assessment Standard 1.2 (Applying calculus skills through techniques of differentiation).	Qualifications Development Manager	November 2015
2.4	Page 5: 'Structure and coverage of the Course assessment' section — reference to the Question Paper Brief added. Pages 10 and 12: 'Further mandatory information on Course coverage' section clarified — amendments to: first and second sub-skills from Assessment Standard 1.4 from Force, Energy and Periodic Motion; second sub-skill from Assessment Standard 1.3 from Linear and Parabolic Motion; third sub-skill from Assessment Standard 1.4 from Linear and Parabolic Motion.	Qualifications Manager	April 2016

Note: You are advised to check SQA's website (**www.sqa.org.uk**) to ensure you are using the most up-to-date version of the Course Assessment Specification.