

# Advanced Higher Mathematics of Mechanics Course/Unit Support Notes



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Please refer to the note of changes at the end of this document for details of changes from previous version (where applicable).

# Contents

Introduction	1
General guidance on the Course/Units	2
Approaches to learning and teaching	4
Approaches to assessment	7
Equality and inclusion	10
Further information on Course/Units	11
Appendix 1: Reference documents	34

# Introduction

These support notes are not mandatory. They provide advice and guidance on approaches to delivering and assessing the Advanced Higher Mathematics of Mechanics Course. They are intended for teachers and lecturers who are delivering the Course and its Units.

These support notes cover both the Advanced Higher Course and the Units in it.

The Advanced Higher Course/Unit Support Notes should be read in conjunction with the relevant:

## **Mandatory Information:**

- ◆ Course Specification
- ◆ Course Assessment Specification
- ◆ Unit Specifications

## **Assessment Support:**

- ◆ Specimen and Exemplar Question Papers and Marking Instructions
- ◆ Exemplar Question Paper Guidance
- ◆ Guidance on the use of past paper questions
- ◆ Unit Assessment Support\*

## **Related information**

Advanced Higher Course Comparison

## **Further information on the Course/Units for Advanced Higher Mathematics of Mechanics**

This information begins on page 11 and both teachers and learners may find it helpful.

# General guidance on the Course/Units

## Aims

The aims of the Course are to enable learners to:

- ◆ use and extend mathematical skills needed to solve problems in mechanics
- ◆ consider the state of equilibrium or the movement of a body and interpret the underlying factors using known mathematical methods
- ◆ analyse the physical factors impacting bodies
- ◆ understand, interpret and apply the effects of both constant and variable forces on a body
- ◆ create mathematical models to simplify and solve problems
- ◆ analyse results in context, and interpret the solution in terms of the real world
- ◆ develop skills in effectively communicating conclusions reached on the basis of physical factors and calculation

## Progression

In order to do this Course, learners should have achieved the Higher Mathematics Course.

Learners who have achieved this Advanced Higher Course may progress to further study, employment and/or training. Opportunities for progression include:

- ◆ Progression to other SQA qualifications
  - Progression to other qualifications at the same level of the Course, eg Mathematics, Statistics or Professional Development Awards (PDAs) or Higher National Certificates (HNCs)
- ◆ Progression to further/higher education
  - For many learners a key transition point will be to further or higher education, for example to Higher National Certificates (HNCs) or Higher National Diplomas (HNDs) or degree programmes.
  - Advanced Higher Courses provide good preparation for learners progressing to further and higher education as learners doing Advanced Higher Courses must be able to work with more independence and less supervision. This eases their transition to further/higher education. Advanced Higher Courses may also allow 'advanced standing' or partial credit towards the first year of study of a degree programme.
  - Advanced Higher Courses are challenging and testing qualifications — learners who have achieved multiple Advanced Higher Courses are regarded as having a proven level of ability which attests to their readiness for education in higher education institutions (HEIs) in other parts of the UK as well as in Scotland.

- ◆ Progression to employment
  - For many learners progression will be directly to employment or work-based training programmes.

This Advanced Higher could be part of the Scottish Baccalaureate in Science. The Scottish Baccalaureates in Expressive Arts, Languages, Science and Social Sciences consist of coherent groups of subjects at Higher and Advanced Higher level. Each award consists of two Advanced Highers, one Higher and an Interdisciplinary Project which adds breadth and value and helps learners to develop generic skills, attitudes and confidence that will help them make the transition into higher education or employment.

## Hierarchies

**Hierarchy** is the term used to describe Courses and Units which form a structured progression involving two or more SCQF levels.

This Advanced Higher Course is not in a hierarchy with the Higher Mathematics Course or its Units.

## Skills, knowledge and understanding covered in this Course

This section provides further advice and guidance about skills, knowledge and understanding that could be included in the Course.

Teachers and lecturers should refer to the *Course Assessment Specification* for mandatory information about the skills, knowledge and understanding to be covered in this Course.

The development of subject-specific and generic skills is central to the Course. Learners should be made aware of the skills they are developing and of the transferability of them. It is the transferability that will help learners with further study and enhance their personal effectiveness.

The skills, knowledge and understanding that will be developed in the Advanced Higher Mathematics of Mechanics Course are:

- ◆ knowledge and understanding of a range of straightforward and complex concepts in mechanics
- ◆ the ability to identify and use appropriate techniques in mechanics
- ◆ the ability to use mathematical reasoning and operational skills to extract and interpret information
- ◆ the ability to create and use multifaceted mathematical models
- ◆ the ability to communicate identified strategies of solution and provide justification for the resulting conclusions in a logical way
- ◆ the ability to comprehend both the problem as a whole and its integral parts
- ◆ the ability to select and use numerical skills

# Approaches to learning and teaching

Advanced Higher Courses place more demands on learners as there will be a higher proportion of independent study and less direct supervision. Some of the approaches to learning and teaching suggested for other levels (in particular, Higher) may also apply at Advanced Higher level but there will be a stronger emphasis on independent learning.

For Advanced Higher Courses, a significant amount of learning may be self-directed and require learners to demonstrate a more mature approach to learning and the ability to work on their own initiative. This can be very challenging for some learners, who may feel isolated at times, and teachers and lecturers should have strategies for addressing this. These could include, for example, planning time for regular feedback sessions/discussions on a one-to-one basis and on a group basis led by the teacher or lecturer (where appropriate).

Teachers and lecturers should encourage learners to use an enquiring, critical and problem-solving approach to their learning. Learners should also be given the opportunity to practise and develop research and investigation skills and higher order evaluation and analytical skills. The use of information and communications technology (ICT) can make a significant contribution to the development of these higher order skills as research and investigation activities become more sophisticated.

Learners will engage in a variety of learning activities as appropriate to the subject, for example:

- ◆ project-based tasks such as investigating the graphs of related functions, which could include using calculators or other technologies
- ◆ a mix of collaborative, co-operative or independent tasks which engage learners
- ◆ using materials available from service providers and authorities
- ◆ problem solving and critical thinking
- ◆ explaining thinking and presenting strategies and solutions to others
- ◆ effective use of questioning and discussion to engage learners in explaining their thinking and checking their understanding of fundamental concepts
- ◆ participating in informed debate and discussion with peers where they can demonstrate skills in constructing and sustaining lines of argument to provide challenge and enjoyment, breadth, and depth, to learning
- ◆ researching information for their subject rather than receiving information from their teacher or lecturer
- ◆ using active and open-ended learning activities such as research, case studies and presentation tasks
- ◆ making use of the internet to draw conclusions about specific issues
- ◆ engaging in wide-ranging independent reading
- ◆ recording in a systematic way the results of research and independent investigation from different sources

- ◆ presenting findings/conclusions of research and investigation activities in a presentation
- ◆ participating in group work with peers and using collaborative learning opportunities to develop teamworking
- ◆ drawing conclusions from complex information
- ◆ using sophisticated written and/or oral communication and presentation skills to present information
- ◆ using appropriate technological resources (eg web-based resources)
- ◆ using appropriate media resources (eg video clips)
- ◆ using real-life contexts and experiences familiar and relevant to young people to meaningfully hone and exemplify skills, knowledge and understanding
- ◆ participating in field trips and visits

Teachers and lecturers should support learners by having regular discussions with them and giving regular feedback. Some learning and teaching activities may be carried out on a group basis and, where this applies, learners could also receive feedback from their peers.

Teachers and lecturers should, where possible, provide opportunities to personalise learning and enable learners to have choices in approaches to learning and teaching. The flexibility in Advanced Higher Courses and the independence with which learners carry out the work lend themselves to this. Teachers and lecturers should also create opportunities for, and use, inclusive approaches to learning and teaching. This can be achieved by encouraging the use of a variety of learning and teaching strategies which suit the needs of all learners. Innovative and creative ways of using technology can also be valuable in creating inclusive learning and teaching approaches.

Centres are free to sequence the teaching of the Outcomes, Units and/or Course in any order they wish. For example:

- ◆ Each Unit could be delivered separately in any sequence.

**And/or:**

- ◆ All Units may be delivered in a combined way as part of the Course. If this approach is used, the Outcomes within Units may either be partially or fully combined.

There may be opportunities to contextualise approaches to learning and teaching to Scottish contexts in this Course. This could be done through mini-projects or case studies.

# Developing skills for learning, skills for life and skills for work

The following skills for learning, skills for life and skills for work should be developed in this Course.

## 2 Numeracy

- 2.1 Number processes
- 2.2 Money, time and measurement
- 2.3 Information handling

## 5 Thinking skills

- 5.3 Applying
- 5.4 Analysing and evaluating

Teachers and lecturers should ensure that learners have opportunities to develop these skills as an integral part of their learning experience.

It is important that learners are aware of the skills for learning, skills for life and skills for work that they are developing in the Course and the activities they are involved in that provide realistic opportunities to practise and/or improve them.

At Advanced Higher level it is expected that learners will be using a range of higher order thinking skills. They will also develop skills in independent and autonomous learning.

# Approaches to assessment

Assessment in Advanced Higher Courses will generally reflect the investigative nature of Courses at this level, together with high-level problem-solving and critical thinking skills and skills of analysis and synthesis.

This emphasis on higher order skills, together with the more independent learning approaches that learners will use, distinguishes the added value at Advanced Higher level from the added value at other levels.

There are different approaches to assessment, and teachers and lecturers should use their professional judgement, subject knowledge and experience, as well as understanding of their learners and their varying needs, to determine the most appropriate ones and, where necessary, to consider workable alternatives.

Assessments must be fit for purpose and should allow for consistent judgements to be made by all teachers and lecturers. They should also be conducted in a supervised manner to ensure that the evidence provided is valid and reliable.

## Unit assessment

Units will be assessed on a pass/fail basis. All Units are internally assessed against the requirements shown in the *Unit Specification*. Each Unit can be assessed on an individual Outcome-by-Outcome basis or via the use of combined assessment for some or all Outcomes.

Assessments must ensure that the evidence generated demonstrates, at the least, the minimum level of competence for each Unit. Teachers and lecturers preparing assessment methods should be clear about what that evidence will look like.

Sources of evidence likely to be suitable for Advanced Higher Units could include:

- ◆ presentation of information to other groups and/or recorded oral evidence
- ◆ exemplification of concepts using (for example) a diagram
- ◆ interpretation of numerical data
- ◆ investigations
- ◆ case studies
- ◆ answers to (multiple choice) questions

Evidence should include the use of appropriate subject-specific terminology as well as the use of real-life examples where appropriate.

Flexibility in the method of assessment provides opportunities for learners to demonstrate attainment in a variety of ways and so reduce barriers to attainment.

The structure of an assessment used by a centre can take a variety of forms, for example:

- ◆ individual pieces of work could be collected in a folio as evidence for Outcomes and Assessment Standards
- ◆ assessment of each complete Outcome
- ◆ assessment that combines the Outcomes of one or more Units
- ◆ assessment that requires more than the minimum competence, which would allow learners to prepare for the Course assessment

Teachers and lecturers should note that learners' day-to-day work may produce evidence which satisfies assessment requirements of a Unit, or Units, either in full or partially. Such naturally-occurring evidence may be used as a contribution towards Unit assessment. However, such naturally-occurring evidence must still be recorded and evidence such as written reports, recording forms, PowerPoint slides, drawings/graphs, video footage or observational checklists provided.

## **Combining assessment across Units**

A combined approach to assessment will enrich the assessment process for the learner, avoid duplication of tasks and allow more emphasis on learning and teaching. Evidence could be drawn from a range of activities for a combined assessment. Care must be taken to ensure that combined assessments provide appropriate evidence for all the Outcomes that they claim to assess.

Combining assessment will also give centres more time to manage the assessment process more efficiently. When combining assessments across Units, teachers/lecturers should use e-assessment wherever possible. Learners can easily update portfolios, electronic or written diaries, and recording sheets.

For some Advanced Higher Courses, it may be that a strand of work which contributes to a Course assessment method is started when a Unit is being delivered and is completed in the Course assessment. In these cases, it is important that the evidence for the Unit assessment is clearly distinguishable from that required for the Course assessment.

## **Preparation for Course assessment**

Each Course has additional time which may be used at the discretion of the teacher or lecturer to enable learners to prepare for Course assessment. This time may be used near the start of the Course and at various points throughout the Course for consolidation and support. It may also be used for preparation for Unit assessment, and, towards the end of the Course, for further integration, revision and preparation and/or gathering evidence for Course assessment.

For this Advanced Higher Course, the assessment method for Course assessment is a question paper. Learners should be given opportunities to practise this method and prepare for it.

## Authenticity

In terms of authenticity, there are a number of techniques and strategies to ensure that learners present work that is their own. Teachers and lecturers should put in place mechanisms to authenticate learner evidence.

In Advanced Higher Courses, because learners will take greater responsibility for their own learning and work more independently, teachers and lecturers need to have measures in place to ensure that work produced is the learner's own work.

For example:

- ◆ regular checkpoint/progress meetings with learners
- ◆ short spot-check personal interviews
- ◆ checklists which record activity/progress
- ◆ photographs, films or audio records

Group work approaches are acceptable as part of the preparation for assessment and also for formal assessment. However, there must be clear evidence for each learner to show that the learner has met the evidence requirements.

For more information, please refer to SQA's [Guide to Assessment](#).

## Added value

Advanced Higher Courses include assessment of added value which is assessed in the Course assessment.

Information given in the *Course Specification* and the *Course Assessment Specification* about the assessment of added value is mandatory.

In Advanced Higher Courses, added value involves the assessment of higher order skills such as high-level and more sophisticated investigation and research skills, critical thinking skills and skills of analysis and synthesis. Learners may be required to analyse and reflect upon their assessment activity by commenting on it and/or drawing conclusions with commentary/justification. These skills contribute to the uniqueness of Advanced Higher Courses and to the overall higher level of performance expected at this level.

In this Course, added value will be assessed by means of a question paper. This is used to assess whether the learner can retain and consolidate the knowledge and skills gained in individual Units. It assesses knowledge and understanding and the various different applications of knowledge such as reasoning, analysing, evaluating and solving problems.

# Equality and inclusion

It is recognised that centres have their own duties under equality and other legislation and policy initiatives. The guidance given in these *Course/Unit Support Notes* is designed to sit alongside these duties but is specific to the delivery and assessment of the Course.

It is important that centres are aware of and understand SQA's assessment arrangements for disabled learners, and those with additional support needs, when making requests for adjustments to published assessment arrangements. Centres will find more guidance on this in the series of publications on Assessment Arrangements on SQA's website: [www.sqa.org.uk/sqa/14977.html](http://www.sqa.org.uk/sqa/14977.html).

The greater flexibility and choice in Advanced Higher Courses provide opportunities to meet a range of learners' needs and may remove the need for learners to have assessment arrangements. However, where a disabled learner needs a reasonable adjustment/assessment arrangements to be made, you should refer to the guidance given in the above link.

# Further information on Course/Units

Unless stated otherwise take  $g = 9.8 \text{ms}^{-2}$  as constant acceleration.

Learners should be able to state modelling assumptions in questions.

Any rounded answer should be accurate to 3 significant figures (or 1 decimal place for angles) unless otherwise stated. If an answer differs due to prior rounding the learner may be penalised. Only penalise 1 mark in any question.

The first column indicates the sub-skills associated with each Assessment Standard.

The second column is the mandatory skills, knowledge and understanding given in the *Course Assessment Specification*. This includes a description of the Unit standard and the added value for the Course assessment. Skills which could be sampled to confirm that learners meet the minimum competence of the Assessment Standards are indicated by a diamond bullet point (◆). Those skills marked by a diamond bullet point (◆) and those marked by an arrow bullet point (➤) can be assessed in the Course assessment.

For Unit assessment when assessing sub-skills assessors should ensure that each ◆ associated with that sub-skill is assessed. Assessors can give learners access to the formulae contained in the formulae sheet accompanying the Advanced Higher Mathematics of Mechanics Course assessment. Assessors can also give learners access to any other derivative or formula which does not form part of this Course.

The third column gives suggested learning and teaching contexts to exemplify possible approaches to learning and teaching. These also provide examples of where the skills could be used in activities.

Mathematics of Mechanics (Advanced Higher) Force, Energy and Periodic Motion		
1.1 Applying skills to principles of momentum, impulse, work, power and energy		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Working with impulse as the change in momentum, and/or force as the rate of change of momentum	<ul style="list-style-type: none"> <li>◆ Use impulse appropriately in a simple situation, making use of the equations  <math display="block">\mathbf{I} = m\mathbf{v} - m\mathbf{u} = \int \mathbf{F}dt \text{ and } I = Ft</math> </li> </ul>	Force has been referred to in the <i>Linear and Parabolic Motion Unit</i> , section 1.4. Examples may include bouncing balls, collisions of objects etc.
Working with the concept of conservation of linear momentum	<ul style="list-style-type: none"> <li>◆ Use the concept of the conservation of linear momentum:  <math display="block">m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2</math> <p>or <math>m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = (m_1 + m_2)\mathbf{v}</math> for bodies that coalesce.</p>           Including use of Newton's second law, <math>F = ma</math> and force as the rate of change of momentum, is essential.</li> <li>➤ Solve problems on linear motion in lifts, recoil of a gun, pile drivers, etc.</li> </ul>	Equations of motion with constant acceleration may occur.
Determining work done by a constant force in one or two dimensions, or a variable force during rectilinear motion	<ul style="list-style-type: none"> <li>◆ Evaluate appropriately the work done by a constant force, making use of the equations  <math display="block">W = Fd \text{ (one dimension)}</math> </li> <li>➤ <math>W = \mathbf{F} \cdot \mathbf{d}</math> (two dimensions)</li> <li>➤ Determine the work done in rectilinear motion by a variable force, using integration:</li> </ul>	Learners should appreciate that work can be done by or against a force.  Examples may be taken from transport, sport, fairgrounds etc.

	$W = \int \mathbf{F} \cdot \mathbf{i} dx = \int \mathbf{F} \cdot \mathbf{v} dt \text{ where } \mathbf{v} = \frac{dx}{dt} \mathbf{i}$ <ul style="list-style-type: none"> <li>◆ Apply to practical examples the concept of power as the rate of doing work:           <math display="block">P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \text{ (constant force)}</math> </li> </ul>	Problems involving inclined planes should be studied.
Using the concepts of kinetic ( $E_K$ ) and/or potential ( $E_P$ ) energy to applying the work-energy principle	<ul style="list-style-type: none"> <li>◆ <math>E_K = \frac{1}{2}mv^2</math>, <math>E_P = mgh</math> for a uniform gravitational field</li> <li>◆ Work done = change in energy           <ul style="list-style-type: none"> <li>➤ <math>E_P = \frac{\lambda x^2}{2l}</math> for elastic strings/springs</li> <li>➤ <math>E_P = \frac{GMm}{r}</math> associated with Newton's Inverse Square Law</li> </ul> </li> </ul>	<p>Learners should be familiar with the difference between kinetic and potential energy, and the meaning of conservative forces such as gravity, and non-conservative forces such as friction.</p> <p>This can be linked with motion along an inclined plane within <i>Linear and Parabolic Motion 1.4</i>. Link with Simple Harmonic Motion from <i>Force, Energy and Periodic Motion 1.3</i> Link with horizontal circular motion from <i>Force, Energy and Periodic Motion 1.2</i>.</p>
Using the concepts of kinetic ( $E_K$ ) and/or potential ( $E_P$ ) energy within the concept of conservation of energy	<ul style="list-style-type: none"> <li>◆ <math>E_K + E_P</math> is constant for simple problems involving motion in a plane</li> <li>➤ Use of this within a situation involving vertical circular motion</li> </ul>	<p>Conditions required to perform full circles should be considered, including cases with a particle attached to an inextensible string, a particle on the end of a light rod, a bead running on the inside or the outside of a cylinder and a bead on a smooth circular wire.</p> <p>Examples should include calculating the initial speed of projection required for each of these cases. For particles of equal mass, describing circles of equal radius, consideration should be given to the requirement for a greater speed of projection in the case of an inextensible string versus a light rod.</p>

1.2 Applying skills to motion in a horizontal circle with uniform angular velocity		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Applying equations to motion in a horizontal circle with uniform angular velocity	<p>◆ Solve problems involving motion in a circle of radius <math>r</math>, with uniform angular velocity <math>\omega</math>, making use of the equations:</p> $\theta = \omega t$ $v = r\omega = r\dot{\theta}$ $a = r\omega^2 = r\dot{\theta}^2 = \frac{v^2}{r}$ $\mathbf{a} = -\omega^2 \mathbf{r}$ $T = \frac{2\pi}{\omega}$ <p>➤ Apply these equations to motion including skidding, banking and other applications.</p>	<p>Terms used will include: angular velocity, angular acceleration, radial and tangential components. Vectors should be used to establish these equations, starting from <math>\mathbf{r} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}</math>, where <math>r</math> is constant and <math>\theta</math> is varying, before considering the special case where <math>\theta = \omega t</math>, <math>\omega</math> being constant:</p> <p>Hence, if <math>\mathbf{e}_r = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}</math> and <math>\mathbf{e}_\theta = -\sin \theta \mathbf{i} + \cos \theta \mathbf{j}</math> are the unit vectors in the radial and tangential directions respectively, it follows that the radial and tangential components of velocity are <math>\mathbf{0}</math> (zero vector) and <math>r\dot{\theta}\mathbf{e}_\theta</math> respectively, and those of acceleration are <math>-r\dot{\theta}^2\mathbf{e}_r</math> and <math>r\ddot{\theta}\mathbf{e}_\theta</math> respectively.</p> <p>Examples could include motion in a horizontal circle around a banked surface, including skidding, the 'wall of death', the conical pendulum.</p>
Using equations for horizontal circular motion alongside Newton's Inverse Square Law of Gravitation	<p>◆ Solve a simple problem using Newton's Inverse Square Law,</p> $F = \frac{GMm}{r^2}$ <p>Identify modelling assumptions made in particular contexts. Examples include applying this to simplified motion of satellites and moons, making use of the equations of motion for horizontal circular motion to find the time for one orbit, the height of the satellite above the planet's surface etc.</p>	<p>Appreciation is needed that the magnitude of the gravitational force of attraction between two particles is inversely proportional to the square of the distance between the two particles.</p> <p>Motion here will consider circular orbits only, and additional effects, such as the rotation of a moon about its own axis whilst orbiting a planet, can be ignored.</p> <p>Link with gravitational potential energy from <i>Force, Energy and Periodic Motion 1.1</i>. Link with escape velocity.</p>

1.3 Applying skills to Simple Harmonic Motion		
Skill	Description of Unit standard and added value	Learning and teaching contexts
Working with the concept of Simple Harmonic Motion (SHM)	<ul style="list-style-type: none"> <li>◆ Understand the concept of SHM and use the basic equation <math>\ddot{x} = -\omega^2 x</math>, and the following associated equations, knowing when and where they arise in order to solve basic problems involving SHM in a straight line:               <math display="block">v^2 = \omega^2 (a^2 - x^2) \text{ where } v = \dot{x}</math> <math display="block">T = \frac{2\pi}{\omega}</math> <math display="block"> v _{\max} = \omega a</math> <math display="block"> \ddot{x} _{\max} = \omega^2 a</math> </li> <li>➤ Apply the solutions <math>x = a \sin(\omega t + \alpha)</math> and the special cases <math>x = a \sin \omega t</math> and <math>x = a \cos \omega t</math> to solve problems</li> </ul>	<p>Terms used will include: oscillation, centre of oscillation, period, amplitude, frequency, maximum velocity and maximum acceleration.</p> <p><math>v^2 = \omega^2 (a^2 - x^2)</math> can be derived from the solution of a separable first order differential equation. Linked with <i>Mathematical Techniques for Mechanics 1.4</i>.</p> <p>At this stage these solutions can be verified or established from <math>\mathbf{r} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}</math> rotating round a circle.</p>
Applying Hooke's Law to problems involving SHM	<ul style="list-style-type: none"> <li>◆ Make use of the equation for Hooke's Law <math>T = \frac{\lambda x}{l}</math>, to determine an unknown tension/thrust, modulus of elasticity or extension/compression of natural length.</li> </ul>	<p>Terms used should include: tension, thrust, natural length, stiffness constant, modulus of elasticity, extension, compression, position of equilibrium, oscillation.</p> <p>Learners should appreciate that the tension in the string or spring is directly proportional to the extension from the natural length:</p> <p>ie <math>T = kx</math>, where the stiffness constant (<math>k</math>) is equivalent to <math>\frac{\lambda}{l}</math>, with <math>\lambda</math> being the modulus of elasticity and <math>l</math> being the natural length of the string/spring.</p> <p>These will include problems involving elastic</p>

	<ul style="list-style-type: none"> <li>➤ Consider the position of equilibrium and the equation of motion for an oscillating mass, and apply these to the solution of problems involving SHM, including problems involving shock absorbers, small amplitude oscillations of a simple pendulum etc</li> </ul>	strings and springs, and a simple pendulum, but not the compound pendulum. Learners should be aware that SHM and linear motion could arise in the same context for a stretched string.
<b>1.4 Applying skills to Centres of Mass</b>		
<b>Skill</b>	<b>Description of Unit standard and added value</b>	<b>Learning and teaching contexts</b>
Determining the turning effect of force	<ul style="list-style-type: none"> <li>◆ Evaluate the turning effect of a single force or a set of forces acting on a body, considering clockwise and anticlockwise rotation: Moment of force about point <math>P = \text{magnitude of force} \times \text{perpendicular distance from } P</math> <i>and/or</i> Understand that for a body in equilibrium the sum of the moments of the forces about any point is zero</li> <li>➤ Consider the forces on a body or a rod on the point of tipping or turning.</li> </ul>	<p>Practical investigations on closing a door, using a spanner, balancing a seesaw etc should provide good opportunity for discussion.</p> <p>The effect of both a single force, in changing its point of application, and the effect of several forces should be considered.</p> <p>Only uniform rods in equilibrium with at most 3 forces acting should be considered.</p> <p>Take moments about a pivot point for a rod on the point of tipping.</p>
Using moments to find the centre of mass of a body	<ul style="list-style-type: none"> <li>◆ Equate the moments of several masses acting along a line to that of a single mass acting at a point on the line <math display="block">\sum m_i x_i = \bar{x} \sum m_i</math> where <math>(\bar{x}, 0)</math> is the centre of mass of the system</li> <li>➤ Extend this to two perpendicular directions to find the centre of mass of a set of particles arranged in a plane.  ie <math>\sum m_i x_i = \bar{x} \sum m_i</math> and <math>\sum m_i y_i = \bar{y} \sum m_i</math> where <math>(\bar{x}, \bar{y})</math> is the centre of mass of the system</li> </ul>	<p>Horizontal or vertical rods with up to three particles placed on them.</p> <p>Take moments about an axis forming a boundary of the body so that all moments are acting in the same sense.</p>

- ◆ Find the positions of centres of mass of standard uniform plane laminas, including rectangle, triangle, circle and semicircle. For a triangle, the centre of mass will be  $\frac{2}{3}$  along median from vertex.

For a semicircle, the centre of mass will be  $\frac{4r}{3\pi}$  along the axis of symmetry from the diameter.

- Apply integration to find the centre of mass of a uniform composite lamina of area  $A$ , bounded by a given curve  $y = f(x)$  and the lines  $x = a$  and  $x = b$  using

$$A\bar{x} = \int_a^b xy dx \quad A\bar{y} = \int_a^b \frac{1}{2} y^2 dx$$

Centres of mass will lie on any axis of symmetry. Learners could investigate centres of mass of solid shapes but these **will not** be assessed in this Course.

Split the composite shape into several standard shapes. Identify the centre of mass of each shape and its position from a fixed point. Replace the lamina by separate particles and consider moments.

Finding the centre of mass of a logo, perforated sheet, loaded plate etc.

## Mathematics of Mechanics (Advanced Higher): Linear and Parabolic Motion

### 1.1 Applying skills to motion in a straight line

Skill	Description of Unit standard and added value	Learning and teaching contexts									
Working with time dependent graphs	<ul style="list-style-type: none"> <li>◆ Sketch and annotate, interpret and use displacement/time, velocity/time and acceleration/time graphs.</li> <li>◆ Determine the distance travelled using the area under a velocity/time graph.</li> </ul>	<p>Velocity/time graphs for both constant and variable acceleration should be considered.</p> <p>Displacement as area under a velocity/time graph may be linked to <i>Mathematical Techniques for Mechanics 1.3</i>.</p> <p>Learners could be encouraged to sketch a displacement/time graph from a velocity/time graph.</p>									
Working with rates of change with respect to time in one dimension	<ul style="list-style-type: none"> <li>◆ Use calculus to determine corresponding expressions connecting displacement, velocity and acceleration eg If <math>s = 2t^3 - 21t^2 + 60t</math> find expressions for velocity and acceleration given specific conditions.  eg If the acceleration of a body is given by <math>a = 4t - t^2</math>, find the velocity and displacement when <math>t = 4</math> seconds, given that the initial velocity is <math>3 \text{ m s}^{-1}</math> when the body is 1 m from the origin.</li> </ul>	<p>The dot notation for differentiation with respect to time <math>\dot{x} = \frac{dx}{dt}</math> and <math>\ddot{x} = \frac{d^2x}{dt^2}</math> may be used.</p> <div style="text-align: center;"> <table style="border: none; margin: auto;"> <tr> <td style="text-align: center;">↓</td> <td style="text-align: center;">Displacement: <math>x</math></td> <td style="text-align: center;">↑</td> </tr> <tr> <td style="text-align: center;">Differentiate</td> <td style="text-align: center;">Velocity: <math>\dot{x} = \frac{dx}{dt}</math></td> <td style="text-align: center;">Integrate</td> </tr> <tr> <td style="text-align: center;">↓</td> <td style="text-align: center;">Acceleration: <math>\ddot{x} = \frac{d^2x}{dt^2}</math></td> <td style="text-align: center;">↑</td> </tr> </table> </div> <p>The solution of differential equations in <b>this</b> unit will only require simple integration.</p>	↓	Displacement: $x$	↑	Differentiate	Velocity: $\dot{x} = \frac{dx}{dt}$	Integrate	↓	Acceleration: $\ddot{x} = \frac{d^2x}{dt^2}$	↑
↓	Displacement: $x$	↑									
Differentiate	Velocity: $\dot{x} = \frac{dx}{dt}$	Integrate									
↓	Acceleration: $\ddot{x} = \frac{d^2x}{dt^2}$	↑									

<p>Using equations of motion in one dimension under constant acceleration</p>	<p>➤ Using calculus derive the equations of motion:</p> $v = u + at \text{ and } s = ut + \frac{1}{2}at^2$ <p>and use these to establish the equations:</p> $v^2 = u^2 + 2as$ $s = \frac{u + v}{2}t$ $s = vt - \frac{1}{2}at^2.$ <p>◆ Use these equations of motion in relevant contexts eg Given the initial and final velocities of a particle moving with constant acceleration, find the time of motion and the displacement.</p> <p>➤ eg A stone is dropped from the top of a tower. In the last second of its motion, it falls one fifth of the height of the tower. Find the height of the tower.</p> <p>◆ Identify modelling assumptions made in particular contexts.</p>	<p>Equations of motion under <b>constant</b> acceleration could be derived from definition of constant acceleration and displacement, as well as deriving these equations using calculus.</p> <p>One-dimensional motion and freefall under gravity must be considered. Stopping distances at traffic lights, speed cameras, etc can be investigated.</p> <p>Air friction (resistance) in vertical motion, constant velocity and interpretation of negative velocity, variation in the value of <math>g</math>, and the need to model all bodies as particles should be discussed.</p>
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1.2 Applying skills to vectors associated with motion		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Using vectors to define displacement, velocity and acceleration	<ul style="list-style-type: none"> <li>Give the displacement, velocity and acceleration of a particle as a vector and understand speed is the magnitude of the velocity vector</li> </ul> <p>If <math>\underline{r} = \begin{pmatrix} x \\ y \end{pmatrix}</math> where <math>x</math> and <math>y</math> are functions of <math>t</math> then</p> $\underline{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \text{ and } \underline{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$ <p>eg If <math>\underline{v} = \begin{pmatrix} \sin 2t \\ t + 1 \\ \cos t + \sin t \end{pmatrix}</math> find expressions for displacement and acceleration given specific conditions.</p>	<p>Vectors can be expressed as column vectors or using <math>\mathbf{i}, \mathbf{j}, \mathbf{k}</math> notation: <math>\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}</math></p> <p>Learners should be familiar with the notation:  <math>\mathbf{r}_P</math> for the position vector of P  <math>\mathbf{v}_P = \dot{\mathbf{r}}_P</math> for the velocity vector of P  <math>\mathbf{a}_P = \dot{\mathbf{v}}_P = \ddot{\mathbf{r}}_P</math> for the acceleration vector of P  <math>\mathbf{i}, \mathbf{j}, \mathbf{k}</math>, as the unit vectors in <math>x, y</math> and <math>z</math> directions  Speed = <math> \mathbf{v}  =  \dot{\mathbf{r}} </math></p> <p>Learners will meet this process when studying implicit differentiation in <i>Mathematical Techniques for Mechanics 1.2</i>. This can be extended to three dimensions.  Differentiation and integration can be used to find displacement, velocity and acceleration given one of these as a function of time.</p>
Finding resultant velocity, relative velocity or relative acceleration of one body with respect to another	<ul style="list-style-type: none"> <li>Resolve position, velocity and acceleration vectors into 2 and 3 dimensions and use these to consider resultant or relative motion</li> </ul> <p>eg A man can travel at <math>3.5 \text{ ms}^{-1}</math> in still water. A river is 80 m wide and its current flows at <math>2 \text{ ms}^{-1}</math>. Find the shortest time taken to cross the river and the distance downstream that the boat is carried.</p>	<p><b>Two dimensions:</b> If a body is travelling in <math>xy</math>-plane with speed <math>v \text{ ms}^{-1}</math> making an angle <math>\theta^\circ</math> with <math>OX</math>, then its velocity vector can be expressed as <math>v \cos \theta \mathbf{i} + v \sin \theta \mathbf{j}</math>.</p> <p><b>Three dimensions:</b> If body is travelling in <math>xyz</math>-plane with speed <math>v \text{ ms}^{-1}</math> making an angle <math>\theta_1^\circ</math> with <math>OX</math>, <math>\theta_2^\circ</math> with <math>OY</math> and</p>

	<p>➤ eg A man can travel at <math>3.5 \text{ m s}^{-1}</math> in still water. A river is 80 m wide and its current flows at <math>2 \text{ m s}^{-1}</math>. Find the course set to cross the river directly and the time taken for such crossing</p> <p>◆ Apply position, velocity and acceleration vectors to practical problems, including navigation, the effects of winds and currents and other relevant contexts.</p> <p>eg To a man driving due North at <math>40 \text{ km h}^{-1}</math> the wind appears to come from <math>\text{N}60^\circ\text{W}</math> with a speed of <math>30 \text{ km h}^{-1}</math>. What is the actual velocity of the wind?</p> <p>➤ When a ship travels due North at <math>40 \text{ km h}^{-1}</math> the wind appears to come from <math>\text{N}40^\circ\text{E}</math>. When it travels South at <math>50 \text{ km h}^{-1}</math> the wind appears to come from South East. Find the true velocity of the wind.</p>	<p><math>\theta_3^\circ</math> with <math>OZ</math>, then its velocity vector can be expressed as <math>v \cos \theta_1 \mathbf{i} + v \cos \theta_2 \mathbf{j} + v \cos \theta_3 \mathbf{k}</math>.</p> <p>Learners should be familiar with the notation:  <math>{}_q \mathbf{r}_p = \overline{PQ} = \mathbf{q} - \mathbf{p}</math> for the position vector of Q relative to P</p> <p><math>{}_q \mathbf{v}_p = \mathbf{v}_q - \mathbf{v}_p = \dot{\mathbf{r}}_q - \dot{\mathbf{r}}_p</math> for the velocity of Q relative to P  <math>{}_q \mathbf{a}_p = \mathbf{a}_q - \mathbf{a}_p = \dot{\mathbf{v}}_q - \dot{\mathbf{v}}_p = \ddot{\mathbf{r}}_q - \ddot{\mathbf{r}}_p</math> for the acceleration of Q relative to P.</p> <p>Contexts such as crossing a river, flying between airports etc can be used explore the effects of currents and winds.</p>
Applying understanding of relative motion	<p>◆ Solve a simple problem involving collision  eg given position and velocity vectors for two bodies prove that they will collide.</p> <p>➤ eg find the course needed for interception of one body by another</p> <p>➤ Consider conditions for nearest approach  eg Find the shortest distance between two moving bodies.  eg Find the time to closest approach.  eg Find the time for which one moving body is within a certain range of another.</p>	<p>For collision, both positions after time <math>t</math> being equal and <math>{}_a v_b</math> to be in the direction of original relative position should be investigated.</p> <p>For nearest approach, both 'least separation' by differentiation and the vector condition <math>{}_p \mathbf{r}_q \cdot {}_p \mathbf{v}_q = 0</math> can be explored.</p> <p>Suitable contexts could include shipping and aircraft movement and sports contexts.</p>

1.3 Applying skills to projectiles moving in a vertical plane		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Establishing the conditions of motion in horizontal and vertical directions involved in parabolic motion	<ul style="list-style-type: none"> <li>Derive the formulae           <math display="block">T = \frac{2u \sin \alpha}{g}</math> <math display="block">H = \frac{u^2 \sin^2 \alpha}{2g}</math> <math display="block">R = u \cos \alpha \times T = \frac{u^2 \sin 2\alpha}{g}</math> </li> </ul> <p>where <math>T</math> refers total time of flight  <math>H</math> refers to greatest height  <math>R</math> refers to the horizontal range</p>	<p>Learners should derive these formulae from equations of motion or by using calculus.</p> <p>Learners should appreciate that gravity only affects motion in the vertical plane and so motion of the projectile will be approached by considering vertical motion and horizontal motion separately.</p> <p>Learners can be reminded of the properties of the parabola, eg relate time of flight, <math>T</math>, with time to greatest height.</p> <p>This can be done by either:          solving the vector equation <math>\ddot{\mathbf{r}} = -g\mathbf{j}</math> to obtain expressions for <math>\dot{x}</math>, <math>\dot{y}</math>, <math>x</math> and <math>y</math> in a particular case or using the equations of motion under constant acceleration.</p>
Using the equations of motion in parabolic flight	<ul style="list-style-type: none"> <li>Use these formulae to find the time of flight, greatest height reached, or range of a projectile including maximum range of a projectile and the angle of projection to achieve this.</li> <li>Derive and use the equation of the trajectory of a projectile:           <math display="block">y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}</math> </li> <li>Solve problems in two-dimensional motion involving projectiles under a constant gravitational force. Projection will be considered in one vertical plane but point of projection can be from a different horizontal plane than that of landing.</li> </ul>	<p>Learners should be able to derive the equation of the trajectory.</p> <p>Sport will provide good context for this work.          Projection from an inclined plane is not required.</p>

1.4 Applying skills to forces associated with dynamics and equilibrium		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Using Newton's first and third laws of motion to understand equilibrium	<ul style="list-style-type: none"> <li>◆ Resolve forces in two dimensions to find their components. eg If a body is in equilibrium under three forces of which one is unknown, resolve vertically and horizontally to find the magnitude of the third force. eg Consider the equilibrium of a body sitting on an inclined plane.</li> <li>➤ Consider the equilibrium of connected particles.</li> <li>◆ Combine forces to find the resultant force, eg by resolving vertically and horizontally to find the resultant</li> </ul>	<p>Learners should understand the concepts of weight, friction, tension, resistance, normal reaction and gravity as expressions of force. When there is more than one force acting on a body, we choose to find the effects of all forces in two mutually perpendicular directions.</p> <p>Tension in the elastic string will be investigated in <i>Force, Energy and Periodic Motion 1.3</i>.</p> <p>Learners could investigate pulley systems but these <b>will not</b> be assessed in this Course.</p> <p>Lami's Theorem may be used but learners should understand that it is of limited application.</p>
Understanding the concept of static friction, dynamic friction and limiting friction	<ul style="list-style-type: none"> <li>◆ Know and use the relationships <math>F = \mu R</math> and <math>\mu = \tan \theta</math> for bodies on a slope and that if <math>\tan \theta &gt; \mu</math> the body will accelerate down the slope. eg A particle is held on a rough slope inclined at <math>20^\circ</math> to the horizontal. Find the coefficient of friction between the particle and the plane.</li> <li>➤ For stationary bodies <math>F \leq \mu R</math></li> <li>◆ Solve problems involving a particle or body in equilibrium under the action of certain forces. eg Bodies in equilibrium on rough planes. eg State modelling assumptions in questions.</li> <li>➤ eg Apply an external force to keep a body in equilibrium on a slope and consider limiting equilibrium for movement along the line of greatest slope.</li> </ul>	<p>Consider a body in equilibrium on a plane and resolved forces will lead to <math>\mu = \tan \theta</math>, where <math>\theta</math> is the angle between the slope and the horizontal.</p> <p>Understand that there is a limiting value of friction, <math>F_{\max} = \mu R</math> during motion and this implies that <math>F \leq \mu R</math> where bodies are stationary.</p>

<p>Using Newton's Second Law of motion</p>	<ul style="list-style-type: none"> <li>◆ Use <math>F = ma</math> to form equations of motion to model practical problems of motion in a straight line, where acceleration may be considered as a function of time or displacement.</li>   <li>◆ Solve problems involving motion on inclined planes, possibly including friction.</li> </ul>	<p>Learners should understand that the acceleration of a body is proportional to the resultant external force and takes place in the direction of the force.</p> <p>When <math>\mathbf{F} = m\mathbf{a}</math> is a vector equation, the acceleration produced is in the direction of the applied or resultant force.</p> <p>Links with parabolic motion of projectiles are encouraged.</p> <p>Equilibrium on inclined planes will have been considered earlier in this Unit. Both smooth and rough planes should be included. These questions can also be solved using energy considerations — <i>Force, Energy and Periodic Motion 1.1</i>.</p>
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## Mathematics of Mechanics: Mathematical Techniques for Mechanics (Advanced Higher)

### 1.1 Applying algebraic skills to partial fractions

Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Expressing rational functions as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	<p>◆ Express a proper rational function as a sum of partial fractions where the denominator may contain: distinct linear factors, an irreducible quadratic factor, a repeated linear factor:</p> <p>eg</p> <p>i) <math>\frac{7x+1}{x^2+x-6}</math> (linear factors)</p> <p>ii) <math>\frac{5x^2-x+6}{x^3+3x}</math> (irreducible quadratic factor)</p> <p>iii) <math>\frac{3x+10}{(x+3)^2}</math> (repeated linear factor)</p> <p>➤ Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise</p> <p>eg <math>\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}</math></p> <p>eg <math>\frac{x^2+3x}{x^2-4}</math></p>	<p>This is also required for integration of rational functions and may be used with differential equations where the solution requires separating the variables.</p> <p>Some discussion of horizontal and vertical asymptotes in relation to graph sketching should occur with this work but will not be assessed.</p>

1.2 Applying calculus skills through techniques of differentiation		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Differentiating exponential and logarithmic functions	<ul style="list-style-type: none"> <li>Differentiate functions involving <math>e^x</math>, <math>\ln x</math></li> </ul> eg $y = e^{x^2} + 4$ eg $f(x) = \ln(x^3 + 2)$	
Differentiating functions using the chain rule	<ul style="list-style-type: none"> <li>Apply the chain rule to differentiate the composition of at most 3 functions</li> </ul> eg $y = \sqrt{e^{x^2} + 4}$ eg $f(x) = \sin^3(2x - 1)$	
Differentiating functions given in the form of a product and/or in the form of a quotient	<ul style="list-style-type: none"> <li>Differentiate functions of the form <math>(f(x)g(x))</math> and/or <math>\left(\frac{f(x)}{g(x)}\right)</math></li> </ul> eg $y = 3x^4 \sin x$ eg $f(x) = x^2 \ln x, x > 0$ eg $y = \frac{2x - 5}{3x^2 + 2}$ eg $f(x) = \frac{\cos x}{e^x}$ <ul style="list-style-type: none"> <li>Know the definitions and use the derivatives of <math>\tan x</math> and <math>\cot x</math></li> </ul>	<p>Learners could be introduced to product and quotient rules with formal proofs but these would not be assessed.</p> <p>When learners have mastered differentiation rules they can be shown how to use computer algebra systems (CAS). These cannot be used in assessment but their suitability for difficult/real examples can be discussed.</p> <p>When software is used for differentiation in difficult cases, learners should understand which rules were needed for solution.</p> <p>Learners should be exposed to deriving <math>\cot x</math>, <math>\sec x</math>, <math>\cos \operatorname{ec} x</math>.</p>

	<ul style="list-style-type: none"> <li>➤ Know the definitions of <math>\sec x</math>, and <math>\operatorname{cosec} x</math>. Learners should be able to use derivatives of <math>\tan x</math>, <math>\cot x</math>, <math>\sec x</math>, <math>\operatorname{cosec} x</math>.</li> <li>➤ Differentiating functions which require more than one application or combination of applications of chain rule, product rule and quotient rule eg <ul style="list-style-type: none"> <li>i) <math>y = e^{2x} \tan 3x</math></li> <li>ii) <math>y = \ln  -3 + \sin 2x </math></li> <li>iii) <math>y = \frac{\sec 2x}{e^{3x}}</math></li> <li>iv) <math>a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v = v \frac{dv}{dx}</math></li> </ul> </li> <li>➤ Know that <math>\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}</math></li> </ul>	<p>Apply differentiation to simple rates of change, eg rectilinear motion and optimisation.</p>
<p>Finding the derivative of functions defined implicitly</p>	<ul style="list-style-type: none"> <li>◆ Use differentiation to find the first derivative of a function defined implicitly including in context eg <math>x^3 y + xy^3 = 4</math> eg Apply differentiation to related rates in problems where the functional relationship is given implicitly. For example, spherical balloon losing air at a given rate.</li> </ul>	<p>Learners should have a clear understanding of an implicit function and see that some can be manipulated to give an explicit function but this method will allow differentiation to be used with all implicit functions. The use of implicit functions to differentiate exponential functions such as <math>f(x) = 5^x</math> by using logs initially should be explored.</p>

	<p>➤ Use differentiation to find the second derivative of a function defined implicitly</p>	<p>Acceleration as an implicit function:</p> $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{1}{2} \times 2v \times \frac{dv}{dx} = v \frac{dv}{dx}$
<p>Finding the derivative of functions defined parametrically</p>	<p>◆ Use differentiation to find the first derivative of a function defined parametrically eg Apply parametric differentiation to motion in a plane If the position is given by <math>x = f(t)</math>, <math>y = g(t)</math> then</p> <p>i) Velocity components are given by <math>v_x = \frac{dx}{dt}</math>    <math>v_y = \frac{dy}{dt}</math></p> <p>ii) Speed = <math>\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}</math></p> <p>eg Apply differentiation to related rates in problems where the functional relationship is given explicitly.</p> <p>➤ Solve practical related rates by first establishing a functional relationship between appropriate variables eg A snowball in the shape of a sphere is rolling down a hill with its radius increasing at a uniform rate of <math>0.5 \text{ cm s}^{-1}</math>. How fast is the volume increasing when the radius is 4 cm?</p>	<p>Parameters should be introduced by using IT to sketch graphs where <math>x</math> and <math>y</math> are different functions of a variable, eg <math>x = 4 \cos \theta</math> <math>y = 4 \sin \theta</math> representing a circle.</p> <p>Another example in context could be calculating the rate at which the depth of coffee in a conical filter is changing.</p>

1.3 Applying calculus skills through techniques of integration		
Sub-skill	Description of Unit standard and added value	Learning and teaching contexts
Integrating expressions using standard results	<ul style="list-style-type: none"> <li>◆ Use <math>\int e^x dx</math>, <math>\int \frac{dx}{x}</math>, <math>\int \sec^2 x dx</math></li> <li>eg <math>\int e^{5x-7} dx</math>, <math>\int \frac{dx}{2x-4}</math>, <math>x \neq 2</math></li> <li>➤ Recognise and integrate expressions of the form  <math>\int g(f(x))f'(x)dx</math> and <math>\int \frac{f'(x)}{f(x)} dx</math></li> <li>eg <math>\int \cos^3 x \sin x dx</math></li> <li>eg <math>\int xe^{x^2} dx</math></li> <li>eg <math>\int_0^2 \frac{2x}{x^2+3} dx</math></li> <li>eg <math>\int \frac{\cos x}{(5+2 \sin x)} dx</math></li> </ul>	

	<p>◆ Use partial fractions to integrate proper rational functions where the denominator may have:</p> <p>i) two separate or repeated linear factors</p> <p>ii) three linear factors with constant numerator</p> <p>eg <math>\int \frac{4x-9}{(x-2)(x-3)} dx</math>, <math>\int \frac{x+3}{(x+5)^2} dx</math>,</p> $\int \frac{6}{(x-1)(x+2)(x+1)} dx$ <p>➤ Use partial fractions to integrate proper rational functions where the denominator may have three linear factors with a non-constant numerator</p> <p>eg <math>\int \frac{x^2-5}{(2-x)(3+x)(1-x)} dx</math></p>	<p>Learners should be competent with the process of expressing a rational function in partial fractions (1.1). Some revision of logarithmic functions might be useful before working with integrals here.</p> <p>This can be linked to solving problems involving motion with resistance in section 1.4.</p>
<p>Integrating using a substitution when the substitution is given</p>	<p>◆ Integrate where the substitution is given</p> <p>eg Use the substitution <math>u = \ln x</math> to obtain <math>\int \frac{1}{x \ln x} dx</math>, where <math>x &gt; 1</math>.</p> <p>eg Use the substitution <math>u = 3x - 2</math> to obtain <math>\int x\sqrt{3x-2} dx</math> where <math>x &gt; \frac{2}{3}</math></p>	

Integrating by parts	<ul style="list-style-type: none"> <li>◆ Use integration by parts with one application eg <math>\int x \sin x dx</math></li> <li>➤ Use integration by parts involving repeated applications eg <math>\int_0^{\pi} x^2 \cos x dx</math></li> <li>eg <math>\int x^2 e^{3x} dx</math></li> </ul>	This may arise again when using the integrating factor to solve first order differential equations.
Applying integration to a range of physical situations	<ul style="list-style-type: none"> <li>◆ Apply integration to evaluate volumes of revolution about the <math>x</math>-axis</li> <li>➤ Apply integration to evaluate volumes of revolution about the <math>y</math>-axis</li> <li>➤ Apply integration to evaluate areas</li> </ul>	This can be linked to finding displacement for velocity/time graphs and finding centres of mass in <i>Linear and Parabolic Motion Unit</i> .
<b>1.4 Applying calculus skills to solving differential equations</b>		
<b>Sub-skill</b>	<b>Description of Unit standard and Added Value</b>	<b>Learning and teaching contexts</b>
Finding a general solution of a first order differential equation with variables separable	<ul style="list-style-type: none"> <li>◆ Solve equations that can be written in the form <math>\frac{dy}{dx} = g(x)h(y)</math> or <math>\frac{dy}{dx} = \frac{g(x)}{h(y)}</math></li> <li>eg <math>\frac{dy}{dx} = y(x-1)</math></li> <li>eg <math>a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v = v \frac{dv}{dx}</math></li> <li>eg <math>v \frac{dv}{dx} = -\omega^2 x</math> associated with SHM</li> </ul>	Learners should be reminded of the use of partial fractions, $\int \frac{1}{x} dx$ and manipulation of logarithmic terms before starting this work. Many of these equations arise naturally in mathematical modelling of physical situations and will be covered again in this section of study.

	<p>➤ Find the particular solution where initial conditions are given</p> <p>eg <math>\frac{1}{x} \frac{dy}{dx} = y \sin x</math> given that when <math>x = \frac{\pi}{2}</math>, <math>y = 1</math></p> <p>➤ Use differential equations in context</p> <p>eg Bacterial growth at a rate proportional to the number of bacteria present at time <math>t</math>: <math>\frac{dB}{dt} = kt</math></p> <p>eg Vertical fall with resistive force: <math>m \frac{dv}{dt} = mg - kv</math></p>	<p>Learners should appreciate the link with differentiation and discuss some physical situations such as: electrical circuits, vibrating systems and motion with resistance where models involving differential equations arise.</p> <p>The differential equation will show how the system will change with time (or other variable). Discussion of initial conditions will help lead to the complete solution. Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good examples and can build on the knowledge and use of logarithms.</p>
<p>Solving a simple first order linear differential equation using an integrating factor</p>	<p>◆ Solve equations written in the standard form</p> $\frac{dy}{dx} + P(x)y = f(x)$ <p>eg <math>\frac{dy}{dx} + \frac{3y}{x} = \frac{e^x}{x^3}</math></p> <p>➤ Solve equations by first writing linear equations in the standard form <math>\frac{dy}{dx} + P(x)y = f(x)</math></p> <p>eg <math>x^2 \frac{dy}{dx} + 3xy = \frac{\sin x}{x}</math></p> <p>➤ Use differential equations in context</p> <p>eg Mixing problems, such as salt water entering a tank of clear water which is then draining at a given rate.</p> $\frac{dM}{dt} = 15 - \frac{1}{100}M$	<p>Learners should be aware of the derivation of the integrating factor method but this will not be assessed.</p> <p>Learners should appreciate the link with differentiation and discuss some physical situations such as: electrical circuits, vibrating systems and motion with resistance where models involving differential equations arise.</p>

	<p>eg Growth and decay problems, an alternative method of solution to separation of variables</p> <p>eg Simple electronic circuits: <math>L \frac{di}{dt} + Ri = V</math> where <math>L, R</math> and <math>V</math> are constant.</p>	<p>The differential equation will show how the system will change with time (or other variable). Discussion of initial conditions will help lead to the complete solution. Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good examples and can build on the knowledge and use of logarithms.</p>
<p>Solving second order homogeneous equations</p>	<p>◆ Find the general solution of a second order homogeneous ordinary differential equation</p> $a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$ <p>where the roots of the auxiliary equation are real and distinct</p> <p>eg <math>\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 3y = 0</math></p> <p>➤ where the roots of the auxiliary equation are real and equal</p> <p>eg <math>\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = 0</math></p> <p>➤ Use differential equations in context</p> <p>eg simple examples of damped simple harmonic motion where the equation of motion is</p> $m\ddot{x} = -m\omega^2 x - mkv$ $\ddot{x} = -\omega^2 x - kv$ $\ddot{x} + k\dot{x} + \omega^2 x = 0 \quad \frac{d^2 y}{dx^2} + k \frac{dy}{dx} + \omega^2 x = 0$	<p>Learners will use second order differential equation when working with Simple Harmonic Motion in Force, Energy and Periodic Motion Unit.</p> <p>Damped SHM should only form discussion. Learners should understand that real distinct roots of the auxiliary equation lead to heavy damping, equal roots to critical damping and unreal roots to light damping. Assessment would only require a statement in explanation.</p>

# Appendix 1: Reference documents

The following reference documents will provide useful information and background.

- ◆ Assessment Arrangements (for disabled candidates and/or those with additional support needs) — various publications are available on SQA's website at: [www.sqa.org.uk/sqa//14977.html](http://www.sqa.org.uk/sqa//14977.html).
- ◆ Building the Curriculum 4: Skills for Learning, Skills for Life and Skills for Work
- ◆ Building the Curriculum 5: A Framework for Assessment
- ◆ [Course Specification](#)
- ◆ [Design Principles for National Courses](#)
- ◆ [Guide to Assessment](#)
- ◆ Principles and practice papers for curriculum area
- ◆ [SCQF Handbook: User Guide](#) and [SCQF level descriptors](#)
- ◆ [SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work](#)
- ◆ [Skills for Learning, Skills for Life and Skills for Work: Using the Curriculum Tool](#)
- ◆ [Coursework Authenticity: A Guide for Teachers and Lecturers](#)

## Administrative information

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**Published:** May 2016 (version 2.2)

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### History of changes to Advanced Higher Course/Unit Support Notes

Version	Description of change	Authorised by	Date
2.0	Extensive changes to 'Further information on Course/Units' section.	Qualifications Development Manager	May 2015
2.1	'Further information on Course/Units' section: Force, Energy and Periodic Motion Unit — amendments to second sub-skill for Assessment Standard 1.1 and first sub-skill for Assessment Standard 1.4; Linear and Parabolic Motion Unit — amendments to second and third sub-skills for Assessment Standard 1.4; Mathematical Techniques for Mechanics Unit — amendments to third sub-skill for Assessment Standard 1.2 and first sub-skill for Assessment Standard 1.4.	Qualifications Development Manager	December 2015
2.2	'Further information on Course/Units' section clarified: amendments to the first and second sub-skills from Assessment Standard 1.4 from Force, Energy and Periodic Motion; the second sub-skill from Assessment Standard 1.3 from Linear and Parabolic Motion; the third sub-skill from Assessment Standard 1.4 from Linear and Parabolic Motion.	Qualifications Manager	May 2016

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