



# Advanced Higher Mathematics of Mechanics

<b>Course code:</b>	C802 77
<b>Course assessment code:</b>	X802 77
<b>SCQF:</b>	level 7 (32 SCQF credit points)
<b>Valid from:</b>	session 2019–20

This document provides detailed information about the course and course assessment to ensure consistent and transparent assessment year on year. It describes the structure of the course and the course assessment in terms of the skills, knowledge and understanding that are assessed.

This document is for teachers and lecturers and contains all the mandatory information required to deliver the course.

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# Course overview

This course consists of 32 SCQF credit points, which includes time for preparation for course assessment. The notional length of time for candidates to complete the course is 160 hours.

The course assessment has one component.

Component	Marks	Duration
Component 1: question paper	100	3 hours

Recommended entry	Progression
<p>Entry to this course is at the discretion of the centre.</p> <p>Candidates should have achieved the Higher Mathematics course or equivalent qualifications and/or experience prior to starting this course.</p>	<ul style="list-style-type: none"><li>◆ other qualifications in mathematics or related areas</li><li>◆ further study, employment and/or training</li></ul>

## Conditions of award

The grade awarded is based on the total mark achieved in the course assessment component.

## Course rationale

National Courses reflect Curriculum for Excellence values, purposes and principles. They offer flexibility, provide time for learning, focus on skills and applying learning, and provide scope for personalisation and choice.

Every course provides opportunities for candidates to develop breadth, challenge and application. The focus and balance of assessment is tailored to each subject area.

Mechanics encourages independent thinking and an enquiring approach. Learning mechanics develops questioning skills, logical reasoning, analysis, problem-solving skills, creativity and the ability to communicate explanations concisely. It uses a universal language of numbers and symbols, which allows us to communicate ideas in a concise, unambiguous and rigorous way.

The course develops important mathematical techniques, which are critical to successful progression beyond Advanced Higher level in Mathematics of Mechanics and many other curriculum areas. The skills, knowledge and understanding in the course also support learning in technology, health and wellbeing, science, and social studies.

## Purpose and aims

Mathematics is important in everyday life. It helps us to make sense of the world we live in and to manage our lives. Mechanics uses mathematics to enable us to model real-life situations and to equip us with the skills we need to interpret and understand how things work, simplify and solve problems, identify limitations, and draw conclusions.

The course aims to:

- ◆ use and extend mathematical skills needed to solve problems in mechanics
- ◆ consider the state of equilibrium or the movement of a body and interpret the underlying factors using known mathematical methods
- ◆ analyse the physical factors impacting bodies
- ◆ understand, interpret and apply the effects of both constant and variable forces on a body
- ◆ create mathematical models to simplify and solve problems
- ◆ analyse results in context and interpret the solution in terms of the real world
- ◆ develop skills in effectively communicating conclusions reached on the basis of physical factors and calculation

## Who is this course for?

This course is particularly suitable for candidates who:

- ◆ have demonstrated an aptitude for Higher Mathematics
- ◆ are interested in developing mathematical techniques to use in further study or in the workplace

# Course content

The Advanced Higher Mathematics of Mechanics course develops, deepens, and extends the mathematical skills necessary at this level and beyond.

Throughout this course, candidates gain and apply operational skills necessary for exploring ideas in mechanics through symbolic representation and mathematical modelling.

Candidates develop mathematical reasoning skills and gain experience in making informed decisions.

## Skills, knowledge and understanding

### Skills, knowledge and understanding for the course

The following provides a broad overview of the subject skills, knowledge and understanding developed in the course:

- ◆ knowledge and understanding of a range of straightforward and complex concepts in mechanics
- ◆ identifying and using appropriate techniques in mechanics
- ◆ using mathematical reasoning and operational skills to extract and interpret information
- ◆ creating and using multifaceted mathematical models
- ◆ communicating identified strategies of solution and providing justification for the resulting conclusions in a logical way
- ◆ comprehending both the problem as a whole and its integral parts
- ◆ selecting and using numerical skills

## Skills, knowledge and understanding for the course assessment

The following provides details of skills, knowledge and understanding sampled in the course assessment.

Forces, energy and momentum	
Skills	Explanation
Applying impulse, change in momentum and conservation of momentum	<ul style="list-style-type: none"> <li>◆ using impulse appropriately in a simple situation, making use of the equations:  <math display="block">\mathbf{I} = m\mathbf{v} - m\mathbf{u} = \int \mathbf{F}dt \text{ and } I = Ft</math> </li> <li>◆ using the concept of the conservation of linear momentum:  <math display="block">m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2</math> </li> </ul>
Determining and applying work done by a constant or variable force	<ul style="list-style-type: none"> <li>◆ evaluating appropriately the work done by a constant force, making use of the equations:  <math display="block">W = Fd \text{ (one dimension)}</math> <math display="block">W = \mathbf{F} \cdot \mathbf{d} \text{ (two dimensions)}</math> </li> <li>◆ determining the work done in rectilinear motion by a variable force, using integration:  <math display="block">W = \int \mathbf{F} \cdot \mathbf{i} dx = \int \mathbf{F} \cdot \mathbf{v} dt \text{ where } \mathbf{v} = \frac{dx}{dt} \mathbf{i}</math> </li> <li>◆ applying the concept of power as the rate of doing work:  <math display="block">P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \text{ (constant force)}</math> </li> </ul>
Using the concepts of kinetic and potential energy; applying the work–energy principle; and applying the principle of the conservation of energy	<ul style="list-style-type: none"> <li>◆ <math>E_K = \frac{1}{2}mv^2</math></li> <li>◆ <math>E_p = mgh</math> for a uniform gravitational field</li> <li>◆ <math>E_p = \frac{\lambda x^2}{2l}</math> for elastic strings or springs</li> <li>◆ <math>E_p = \frac{GMm}{r}</math> associated with Newton's Inverse Square Law</li> <li>◆ work done = change in energy, where external forces other than gravity are involved</li> <li>◆ <math>E_K + E_p = \text{constant}</math>, for simple problems where external forces other than gravity are not involved</li> </ul>
Determining the turning effect of force	<ul style="list-style-type: none"> <li>◆ evaluating the turning effect of a single force or a set of forces acting on a body, considering clockwise and anticlockwise rotation using  moment of force = magnitude of force <math>\times</math> perpendicular distance </li> <li>◆ applying the principle that, for a body in equilibrium, the sum of the moments of the forces about any point is zero</li> <li>◆ considering the forces on a body or a rod on the point of tipping or turning</li> </ul>

Forces, energy and momentum	
Skills	Explanation
Using moments to find the centre of mass of a body	<ul style="list-style-type: none"> <li>◆ equating the moments of individual masses that lie on a straight line to that of a single mass acting at a point on the line:               <math display="block">\sum m_i x_i = \bar{x} \sum m_i</math>               where <math>(\bar{x}, 0)</math> is the centre of mass of the system             </li> <li>◆ extending this to two perpendicular directions to find the centre of mass of a set of particles arranged in a plane               <math display="block">\sum m_i x_i = \bar{x} \sum m_i \text{ and } \sum m_i y_i = \bar{y} \sum m_i</math>               where <math>(\bar{x}, \bar{y})</math> is the centre of mass of the system             </li> <li>◆ finding the positions of centres of mass of standard uniform plane laminas, including rectangle, triangle, circle, and semicircle:               <ul style="list-style-type: none"> <li>▪ For a triangle, the centre of mass will be <math>\frac{2}{3}</math> along median from vertex.</li> <li>▪ For a semicircle, the centre of mass will be <math>\frac{4r}{3\pi}</math> along the axis of symmetry from the diameter.</li> </ul> </li> <li>◆ applying integration to find the centre of mass of a uniform composite lamina of area <math>A</math>, bounded by a given curve <math>y = f(x)</math> and the lines <math>x = a</math>, <math>x = b</math> and the <math>x</math>-axis using               <math display="block">\bar{x} = \frac{1}{A} \int_a^b xy \, dx, \quad \bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} y^2 \, dx</math> </li> </ul>
Using Newton's First and Third laws of motion to understand equilibrium	<ul style="list-style-type: none"> <li>◆ resolving forces in two dimensions to find their components</li> <li>◆ considering the equilibrium of connected particles</li> <li>◆ combining forces to find the resultant force</li> <li>◆ making use of the equation for Hooke's Law, <math>T = \frac{\lambda x}{l}</math>, to determine an unknown tension/thrust, modulus of elasticity or extension/compression or natural length</li> </ul>
Using the concepts of static friction and limiting friction on bodies in equilibrium	<ul style="list-style-type: none"> <li>◆ knowing and using the relationships <math>F = \mu R</math>, <math>\mu = \tan \lambda</math>, and for stationary bodies <math>F \leq \mu R</math></li> <li>◆ solving problems involving a particle or body in equilibrium under the action of two or more forces, including friction</li> </ul>

Straight line, periodic and parabolic motion	
Skills	Explanation
Applying graphs, calculus and equations of motion in one dimension to problems involving displacement, velocity and acceleration	<ul style="list-style-type: none"> <li>◆ sketching and annotating, interpreting and using displacement–time, velocity–time and acceleration–time graphs</li> <li>◆ determining the distance travelled using the area under a velocity–time graph</li> <li>◆ using calculus to determine corresponding expressions connecting displacement, velocity and acceleration</li> <li>◆ using calculus to derive the equations of motion:               <math display="block">v = u + at \text{ and } s = ut + \frac{1}{2}at^2</math>               and using these to establish the equations:               <math display="block">v^2 = u^2 + 2as, \quad s = \frac{(u+v)t}{2} \text{ and } s = vt - \frac{1}{2}at^2</math> </li> <li>◆ using these equations of motion in relevant contexts</li> </ul>
Applying displacement, velocity and acceleration vectors to resultant and relative motion	<ul style="list-style-type: none"> <li>◆ giving the displacement, velocity, and acceleration of a particle as a vector, and understanding speed is the magnitude of the velocity vector               <p style="margin-left: 20px;">If <math>\underline{r} = \begin{pmatrix} x \\ y \end{pmatrix}</math> where <math>x</math> and <math>y</math> are functions of <math>t</math>, then</p> <math display="block">\underline{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \text{ and } \underline{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}</math> </li> <li>◆ resolving position, velocity, and acceleration vectors into two and three dimensions and using these to consider resultant or relative motion</li> <li>◆ applying position, velocity, and acceleration vectors to practical problems, including navigation, the effects of winds and currents, and other relevant contexts</li> <li>◆ solving a simple problem involving collision</li> <li>◆ considering conditions for nearest approach</li> </ul>
Applying Newton's Second Law of motion, including situations involving dynamic friction	<ul style="list-style-type: none"> <li>◆ using <math>F = ma</math> to form equations of motion to model practical problems of motion in a straight line, where acceleration may be considered as a function of time or of displacement</li> <li>◆ solving problems involving motion on both smooth and rough inclined planes</li> </ul>
Establishing and applying equations of motion to projectiles in horizontal and vertical directions moving with parabolic motion	<ul style="list-style-type: none"> <li>◆ deriving the formulae               <math display="block">T = \frac{2u \sin \alpha}{g}, \quad H = \frac{u^2 \sin^2 \alpha}{2g} \text{ and } R = uT \cos \alpha = \frac{u^2 \sin 2\alpha}{g}</math>               where <math>T</math> refers to total time of flight, <math>H</math> refers to greatest height and <math>R</math> refers to the horizontal range             </li> </ul>



Straight line, periodic and parabolic motion	
Skills	Explanation
	<ul style="list-style-type: none"> <li>♦ using these formulae to find the time of flight, greatest height reached, or range of a projectile, including maximum range of a projectile and the angle of projection to achieve this</li> <li>♦ deriving and using the equation of the trajectory of a projectile:  <math display="block">y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}</math> </li> <li>♦ solving problems in two-dimensional motion involving projectiles under a constant gravitational force — projection can be considered in one vertical plane, but point of projection can be from a different horizontal plane than that of landing</li> </ul>
Applying equations to motion in horizontal and vertical circles with uniform angular velocity, including gravitational motion	<ul style="list-style-type: none"> <li>♦ solving problems involving motion in a circle of radius <math>r</math>, with uniform angular velocity <math>\omega</math>, making use of the equations:  <math display="block">\theta = \omega t, v = r\omega = r\dot{\theta}, a = r\omega^2 = r\dot{\theta}^2 = \frac{v^2}{r}, \mathbf{a} = -\omega^2 \mathbf{r}</math>   and <math>T = \frac{2\pi}{\omega}</math>  and applying these equations to motion including skidding, banking, and other applications</li> <li>♦ solving a simple problem involving Newton's Inverse Square Law, <math>F = \frac{GMm}{r^2}</math>, applied to simplified motion of satellites and moons, making use of the equations of motion for horizontal circular motion above</li> <li>♦ using conservation of energy within a situation involving vertical circular motion</li> </ul>
Applying the concept of simple harmonic motion (SHM), including problems involving Hooke's Law	<ul style="list-style-type: none"> <li>♦ using the equation for SHM, <math>\ddot{x} = -\omega^2 x</math>, and the following associated equations:  <math display="block">v^2 = \omega^2 (a^2 - x^2), T = \frac{2\pi}{\omega},  v _{\max} = a\omega,  \ddot{x} _{\max} = a\omega^2</math>   <math display="block">x = a \sin(\omega t + \alpha)</math> </li> <li>♦ considering SHM starting from points other than the centre of oscillation</li> <li>♦ applying Hooke's Law to SHM problems involving extensible strings and springs</li> </ul>

Mathematical techniques for mechanics	
Skills	Explanation
Decomposing a rational function into a sum of partial fractions (denominator of degree at most three)	<ul style="list-style-type: none"> <li>◆ decomposing a proper rational function as a sum of partial fractions where the denominator may contain distinct linear factors, an irreducible quadratic factor, or a repeated linear factor</li> <li>◆ reducing an improper rational function to a polynomial and a proper rational function by division or otherwise</li> </ul>
Differentiating, exponential, natural logarithmic, and trigonometric functions	<ul style="list-style-type: none"> <li>◆ differentiating functions involving <math>e^x</math> and <math>\ln x</math></li> <li>◆ knowing the definitions and applying the derivatives of <math>\tan x</math>, <math>\cot x</math>, <math>\sec x</math> and <math>\operatorname{cosec} x</math></li> <li>◆ deriving the derivatives of <math>\tan x</math>, <math>\sec x</math>, <math>\operatorname{cosec} x</math> and <math>\cot x</math></li> </ul>
Differentiating functions using the chain rule, and functions given in the form of a product and in the form of a quotient	<ul style="list-style-type: none"> <li>◆ applying the chain rule to differentiate the composition of at most three functions</li> <li>◆ differentiating functions of the form <math>f(x)g(x)</math> and <math>\frac{f(x)}{g(x)}</math></li> <li>◆ differentiating functions that require more than one application or combination of applications of the chain rule, product rule, and quotient rule</li> </ul>
Finding the derivative where relationships are defined implicitly or parametrically	<ul style="list-style-type: none"> <li>◆ using differentiation to find the first derivative of a function defined implicitly, including in context</li> <li>◆ using differentiation to find the second derivative of a function defined implicitly, including in context</li> <li>◆ knowing that <math>\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}</math></li> <li>◆ using differentiation to find the first derivative of a function defined parametrically</li> <li>◆ solving practical related rates by first establishing a functional relationship between appropriate variables</li> </ul>
Integrating expressions using standard results	<ul style="list-style-type: none"> <li>◆ using <math>\int e^x dx</math>, <math>\int \frac{dx}{x}</math>, and <math>\int \sec^2 x dx</math></li> <li>◆ recognising and integrating expressions of the form <math>\int g(f(x))f'(x)dx</math> and <math>\int \frac{f'(x)}{f(x)} dx</math></li> <li>◆ using partial fractions to integrate proper or improper rational functions</li> </ul>
Integrating using a substitution	<ul style="list-style-type: none"> <li>◆ integrating where the substitution is given</li> </ul>

<b>Mathematical techniques for mechanics</b>	
<b>Skills</b>	<b>Explanation</b>
Integrating by parts	<ul style="list-style-type: none"> <li>◆ using integration by parts with one or more applications</li> </ul>
Applying integration to a range of physical situations	<ul style="list-style-type: none"> <li>◆ applying integration to volumes of revolution, where the volume generated is by the rotation of the area under a single curve about the <math>x</math>-axis or <math>y</math>-axis</li> <li>◆ applying integration to evaluate areas</li> </ul>
Solving a first-order linear differential equation with variables separable	<ul style="list-style-type: none"> <li>◆ finding the general and particular solutions to equations that can be written in the form  <math display="block">\frac{dy}{dx} = g(x)h(y) \text{ or } \frac{dy}{dx} = \frac{g(x)}{h(y)}</math> </li> </ul>
Solving a first-order linear differential equation using an integrating factor	<ul style="list-style-type: none"> <li>◆ finding the general or particular solution by writing equations in the standard form <math>\frac{dy}{dx} + P(x)y = f(x)</math></li> </ul>
Solving a second-order homogeneous differential equation	<ul style="list-style-type: none"> <li>◆ finding the general or particular solution of a second-order homogeneous ordinary differential equation  <math display="block">a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0</math> , where the roots of the auxiliary equation may be: <ul style="list-style-type: none"> <li>▪ real and distinct</li> <li>▪ real and equal</li> </ul> </li> </ul>
Applying mathematical techniques to problems	<ul style="list-style-type: none"> <li>◆ using all the above techniques in isolation or within mechanics contexts from all parts of the course</li> </ul>

Skills, knowledge and understanding included in the course are appropriate to the SCQF level of the course. The SCQF level descriptors give further information on characteristics and expected performance at each SCQF level, and are available on the SCQF website.

# Skills for learning, skills for life and skills for work

This course helps candidates to develop broad, generic skills. These skills are based on [SQA's Skills Framework: Skills for Learning, Skills for Life and Skills for Work](#) and draw from the following main skills areas:

## **2 Numeracy**

- 2.1 Number processes
- 2.2 Money, time and measurement
- 2.3 Information handling

## **5 Thinking skills**

- 5.3 Applying
- 5.4 Analysing and evaluating

You must build these skills into the course at an appropriate level, where there are suitable opportunities.

# Course assessment

Course assessment is based on the information in this course specification.

The course assessment meets the purposes and aims of the course by addressing:

- ◆ breadth — drawing on knowledge and skills from across the course
- ◆ challenge — requiring greater depth or extension of knowledge and/or skills
- ◆ application — requiring application of knowledge and/or skills in practical or theoretical contexts as appropriate

This enables candidates to:

- ◆ use a range of complex concepts in mechanics
- ◆ identify and use appropriate techniques in mechanics
- ◆ use mathematical reasoning skills to extract and interpret information
- ◆ create and use mathematical models to solve problems
- ◆ communicate identified strategies of solution and provide justification for the resulting conclusions
- ◆ analyse the problem both as a whole and as its integral parts

## Course assessment structure: question paper

### Question paper

**100 marks**

The question paper allows candidates to demonstrate knowledge of mathematical skills and their application to mechanics. Candidates can use a calculator.

The question paper gives candidates an opportunity to apply algebraic, geometric, trigonometric, calculus, and reasoning skills specified in the 'skills, knowledge and understanding for the course assessment' section.

The question paper has 100 marks out of a total of 100 marks for the course assessment. It consists of short-answer and extended-response questions.

### Setting, conducting and marking the question paper

The question paper is set and marked by SQA, and conducted in centres under conditions specified for external examinations by SQA.

Candidates have 3 hours to complete the question paper.

Specimen question papers for Advanced Higher courses are published on SQA's website. These illustrate the standard, structure and requirements of the question papers. The specimen papers also include marking instructions.

# Grading

Candidates' overall grades are determined by their performance across the course assessment. The course assessment is graded A–D on the basis of the total mark for the course assessment component.

## **Grade description for C**

For the award of grade C, candidates will typically have demonstrated successful performance in relation to the skills, knowledge and understanding for the course.

## **Grade description for A**

For the award of grade A, candidates will typically have demonstrated a consistently high level of performance in relation to the skills, knowledge and understanding for the course.

# Equality and inclusion

This course is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

Guidance on assessment arrangements for disabled candidates and/or those with additional support needs is available on the assessment arrangements web page:

[www.sqa.org.uk/assessmentarrangements](http://www.sqa.org.uk/assessmentarrangements).

# Further information

- ◆ [Advanced Higher Mathematics of Mechanics subject page](#)
- ◆ [Assessment arrangements web page](#)
- ◆ [Building the Curriculum 3–5](#)
- ◆ [Guide to Assessment](#)
- ◆ [Guidance on conditions of assessment for coursework](#)
- ◆ [SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work](#)
- ◆ [Coursework Authenticity: A Guide for Teachers and Lecturers](#)
- ◆ [Educational Research Reports](#)
- ◆ [SQA Guidelines on e-assessment for Schools](#)
- ◆ [SQA e-assessment web page](#)
- ◆ [SCQF website: framework, level descriptors and SCQF Handbook](#)



# Appendix 1: course support notes

## Introduction

These support notes are not mandatory. They provide advice and guidance to teachers and lecturers on approaches to delivering the course. Please read these course support notes in conjunction with the course specification and the specimen question paper.

## Approaches to learning and teaching

Approaches to learning and teaching should be engaging, with opportunities for personalisation and choice built in where possible. These could include:

- ◆ project-based tasks such as investigating the graphs of related functions, which could include using calculators or other technologies
- ◆ a mix of collaborative, co-operative or independent tasks that engage candidates
- ◆ solving problems and thinking critically
- ◆ explaining thinking and presenting strategies and solutions to others
- ◆ using questioning and discussion to encourage candidates to explain their thinking and to check their understanding of fundamental concepts
- ◆ debating and discussing topics and concepts so that candidates can demonstrate skills in constructing and sustaining lines of argument to provide challenge, enjoyment, breadth, and depth in their learning
- ◆ using active and open-ended learning activities such as research, case studies and presentation tasks
- ◆ encouraging candidates to engage in independent reading from a range of sources, including the internet
- ◆ demonstrating how candidates should record the results of their research and independent investigation from different sources
- ◆ asking candidates to share the findings and conclusions of their research and investigation activities in a presentation
- ◆ using collaborative learning opportunities and group work to develop team working
- ◆ using technological and media resources, for example web-based resources and video clips
- ◆ using real-life contexts and experiences familiar and relevant to candidates to hone and exemplify skills, knowledge and understanding
- ◆ field trips and visits

You should support candidates by having regular discussions with them and giving them regular feedback. For group activities, candidates could also receive feedback from their peers.

You should, where possible, provide opportunities for candidates to personalise their learning and give them choices about learning and teaching approaches. The flexibility in Advanced

Higher courses and the independence with which candidates carry out the work lend themselves to this.

You should use inclusive approaches to learning and teaching. There may be opportunities to contextualise approaches to learning and teaching to Scottish contexts in this course. You could do this through mini-projects or case studies.

## Preparing for course assessment

The course assessment focuses on breadth, challenge and application. Candidates draw on and extend the skills they have learned during the course. These are assessed through a question paper in which candidates can use a calculator.

To help candidates prepare for the course assessment, they should have the opportunity to:

- ◆ analyse a range of real-life problems and situations involving mathematics
- ◆ select and adapt appropriate mathematical skills
- ◆ apply mathematical skills with and without the aid of a calculator
- ◆ determine solutions
- ◆ explain solutions and/or relate them to context
- ◆ present mathematical information appropriately

The question paper assesses a selection of knowledge and skills acquired during the course and provides opportunities for candidates to apply skills in a wide range of situations, some of which may be new.

Before the course assessment, candidates may benefit from responding to short-answer questions and extended-response questions.

## Developing skills for learning, skills for life and skills for work

You should identify opportunities throughout the course for candidates to develop skills for learning, skills for life and skills for work.

Candidates should be aware of the skills they are developing and you can provide advice on opportunities to practise and improve them.

SQA does not formally assess skills for learning, skills for life and skills for work.

There may also be opportunities to develop additional skills depending on the approach centres use to deliver the course. This is for individual teachers and lecturers to manage.

Some examples of potential opportunities to practise or improve these skills are provided in the following table.

<b>SQA skills for learning, skills for life and skills for work framework definition</b>	<b>Suggested approaches for learning and teaching</b>
<p><b>Numeracy</b> is the ability to use numbers to solve problems by counting, doing calculations, measuring, and understanding graphs and charts. It is also the ability to understand the results.</p>	<p>Candidates could:</p> <ul style="list-style-type: none"> <li>◆ develop their numerical skills throughout the course, for example by using surds in differential and integral calculus, and using exact trigonometric values in context</li> <li>◆ use numbers to solve contextualised problems involving other STEM subjects</li> <li>◆ manage problems, tasks and case studies involving numeracy by analysing the context, carrying out calculations, drawing conclusions, and making deductions and informed decisions</li> </ul>
<p><b>Applying</b> is the ability to use existing information to solve a problem in a different context, and to plan, organise and complete a task.</p>	<p>Candidates could:</p> <ul style="list-style-type: none"> <li>◆ apply the skills, knowledge and understanding they have developed to solve mathematical problems in a range of real-life contexts</li> <li>◆ think creatively to adapt strategies to suit the given problem or situation</li> <li>◆ show and explain their thinking to determine their level of understanding</li> <li>◆ think about how they are going to tackle problems or situations, decide which skills to use and then carry out the calculations necessary to complete the task, for example using conservation of energy</li> </ul>
<p><b>Analysing and evaluating</b> is the ability to identify and weigh up the features of a situation or issue and to use judgement to come to a conclusion. It includes reviewing and considering any potential solutions.</p>	<p>Candidates could:</p> <ul style="list-style-type: none"> <li>◆ identify which real-life tasks or situations require the use of mathematics</li> <li>◆ interpret the results of their calculations and draw conclusions; conclusions drawn could be used to form the basis of making choices or decisions</li> <li>◆ identify and analyse situations involving mathematics that are of personal interest</li> </ul>

During the course, candidates have opportunities to develop their literacy skills and employability skills.

**Literacy skills** are particularly important, as these skills allow candidates to access, engage in and understand their learning, and to communicate their thoughts, ideas and opinions. The

course provides candidates with the opportunity to develop their literacy skills by analysing real-life contexts and communicating their thinking by presenting mathematical information in a variety of ways. This could include the use of numbers, formulae, diagrams, graphs, symbols and words.

**Employability skills** are the personal qualities, skills, knowledge, understanding and attitudes required in changing economic environments. Candidates can apply the mathematical operational and reasoning skills developed in this course in the workplace. The course provides them with the opportunity to analyse a situation, decide which mathematical strategies to apply, work through those strategies effectively, and make informed decisions based on the results.

## Appendix 2: skills, knowledge and understanding with suggested learning and teaching contexts

The first two columns are identical to the tables of 'Skills, knowledge and understanding for the course assessment' in the course specification.

The third column gives suggested learning and teaching contexts. These provide examples of where the skills could be used in individual activities or pieces of work.

Forces, energy and momentum		
Skills	Explanation	Suggested learning and teaching contexts
Applying impulse, change in momentum and conservation of momentum	<ul style="list-style-type: none"> <li>◆ using impulse appropriately in a simple situation, making use of the equations:  <math display="block">\mathbf{I} = m\mathbf{v} - m\mathbf{u} = \int \mathbf{F}dt \text{ and } I = Ft</math> </li> <li>◆ using the concept of the conservation of linear momentum: <math>m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2</math></li> </ul>	Learning and teaching contexts could include: <ul style="list-style-type: none"> <li>◆ bouncing balls, collisions of objects</li> <li>◆ equations of motion with constant acceleration</li> </ul>

Forces, energy and momentum		
Skills	Explanation	Suggested learning and teaching contexts
Determining and applying work done by a constant or variable force	<ul style="list-style-type: none"> <li>◆ evaluating appropriately the work done by a constant force, making use of the equations:  <math>W = Fd</math> (one dimension)  <math>W = \mathbf{F} \cdot \mathbf{d}</math> (two dimensions)</li> <li>◆ determining the work done in rectilinear motion by a variable force, using integration:  <math display="block">W = \int \mathbf{F} \cdot \mathbf{i} dx = \int \mathbf{F} \cdot \mathbf{v} dt \text{ where } \mathbf{v} = \frac{dx}{dt} \mathbf{i}</math></li> <li>◆ applying the concept of power as the rate of doing work:  <math display="block">P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \text{ (constant force)}</math></li> </ul>	<p>Candidates should appreciate that work can be done by or against a force.</p> <p>Examples could come from transport, sport, or fairgrounds.</p> <p>Candidates should study problems involving inclined planes.</p>
Using the concepts of kinetic and potential energy; applying the work–energy principle; and applying the principle of the conservation of energy	<ul style="list-style-type: none"> <li>◆ <math>E_K = \frac{1}{2}mv^2</math></li> <li>◆ <math>E_p = mgh</math> for a uniform gravitational field</li> <li>◆ <math>E_p = \frac{\lambda x^2}{2l}</math> for elastic strings or springs</li> <li>◆ <math>E_p = \frac{GMm}{r}</math> associated with Newton’s Inverse Square Law</li> <li>◆ work done = change in energy, where external forces other than gravity are involved</li> <li>◆ <math>E_K + E_p = \text{constant}</math>, for simple problems where external forces other than gravity are not involved</li> </ul>	<p>Candidates should be familiar with the difference between kinetic and potential energy, and the meaning of conservative forces such as gravity, and non-conservative forces such as friction.</p> <p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ motion along an inclined plane</li> <li>◆ simple harmonic motion</li> <li>◆ horizontal circular motion</li> </ul>

Forces, energy and momentum		
Skills	Explanation	Suggested learning and teaching contexts
Determining the turning effect of force	<ul style="list-style-type: none"> <li>◆ evaluating the turning effect of a single force or a set of forces acting on a body, considering clockwise and anticlockwise rotation using moment of force = magnitude of force × perpendicular distance</li> <li>◆ applying the principle that, for a body in equilibrium, the sum of the moments of the forces about any point is zero</li> <li>◆ considering the forces on a body or a rod on the point of tipping or turning</li> </ul>	<p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ practical investigations and discussion around closing a door, using a spanner, or balancing a seesaw</li> <li>◆ considering the effect of both a single force in changing its point of application, and the effect of several forces</li> <li>◆ taking moments about a pivot point for a rod on the point of tipping</li> </ul>
Using moments to find the centre of mass of a body	<ul style="list-style-type: none"> <li>◆ equating the moments of individual masses that lie on a straight line to that of a single mass acting at a point on the line: <math display="block">\sum m_i x_i = \bar{x} \sum m_i</math> where <math>(\bar{x}, 0)</math> is the centre of mass of the system</li> <li>◆ extending this to two perpendicular directions to find the centre of mass of a set of particles arranged in a plane <math display="block">\sum m_i x_i = \bar{x} \sum m_i \text{ and } \sum m_i y_i = \bar{y} \sum m_i</math> where <math>(\bar{x}, \bar{y})</math> is the centre of mass of the system</li> <li>◆ finding the positions of centres of mass of standard uniform plane laminas, including rectangle, triangle, circle, and semicircle:</li> </ul>	<p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ horizontal or vertical rods with particles placed on them</li> <li>◆ splitting the composite shape into several standard shapes; identifying the centre of mass of each shape and its position from a fixed point; and replacing the lamina with separate particles and considering moments</li> <li>◆ finding the centre of mass of a logo, perforated sheet, and loaded plate</li> </ul>

Forces, energy and momentum		
Skills	Explanation	Suggested learning and teaching contexts
	<ul style="list-style-type: none"> <li>▪ For a triangle, the centre of mass will be <math>\frac{2}{3}</math> along median from vertex.</li> <li>▪ For a semicircle, the centre of mass will be <math>\frac{4r}{3\pi}</math> along the axis of symmetry from the diameter.</li> <li>◆ applying integration to find the centre of mass of a uniform composite lamina of area <math>A</math>, bounded by a given curve <math>y = f(x)</math> and the lines <math>x = a</math>, <math>x = b</math> and the <math>x</math>-axis using           <math display="block">\bar{x} = \frac{1}{A} \int_a^b xy \, dx, \quad \bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} y^2 \, dx</math> </li> </ul>	
Using Newton's First and Third laws of motion to understand equilibrium	<ul style="list-style-type: none"> <li>◆ resolving forces in two dimensions to find their components</li> <li>◆ considering the equilibrium of connected particles</li> <li>◆ combining forces to find the resultant force</li> <li>◆ making use of the equation for Hooke's Law, <math>T = \frac{\lambda x}{l}</math>, to determine an unknown tension/thrust, modulus of elasticity or extension/compression or natural length</li> </ul>	<p>Candidates should understand the concepts of weight, friction, tension, resistance, normal reaction, and gravity as expressions of force. When there is more than one force acting on a body, we choose to find the effects of all forces in two mutually perpendicular directions.</p> <p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ Lami's Theorem, when exactly three forces are involved</li> </ul>



Forces, energy and momentum		
Skills	Explanation	Suggested learning and teaching contexts
Using the concepts of static friction and limiting friction on bodies in equilibrium	<ul style="list-style-type: none"> <li>◆ knowing and using the relationships <math>F = \mu R</math>, <math>\mu = \tan \lambda</math>, and for stationary bodies <math>F \leq \mu R</math></li> <li>◆ solving problems involving a particle or body in equilibrium under the action of two or more forces, including friction</li> </ul>	<p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ considering a body in equilibrium on a plane and resolve forces that lead to <math>\mu = \tan \theta</math>, where <math>\theta</math> is the angle between the slope and the horizontal</li> <li>◆ understanding that there is a limiting value of friction, <math>F_{\max} = \mu R</math> during motion and this implies that <math>F \leq \mu R</math> where bodies are stationary</li> </ul>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
Applying graphs, calculus and equations of motion in one dimension to problems involving displacement, velocity and acceleration	<ul style="list-style-type: none"> <li>sketching and annotating, interpreting and using displacement–time, velocity–time and acceleration–time graphs</li> <li>determining the distance travelled using the area under a velocity–time graph</li> <li>using calculus to determine corresponding expressions connecting displacement, velocity and acceleration</li> <li>using calculus to derive the equations of motion:           <math display="block">v = u + at \text{ and } s = ut + \frac{1}{2}at^2</math>           and using these to establish the equations:           <math display="block">v^2 = u^2 + 2as, \quad s = \frac{(u+v)t}{2} \text{ and } s = vt - \frac{1}{2}at^2</math> </li> <li>using these equations of motion in relevant contexts</li> </ul>	<p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>velocity–time graphs for both constant and variable acceleration</li> <li>linking the area under a velocity–time graph with displacement</li> <li>the dot notation for differentiation with respect to time:           <math display="block">\dot{x} = \frac{dx}{dt} \text{ and } \ddot{x} = \frac{d^2x}{dt^2}</math> <div style="text-align: center;"> <math>\downarrow</math> </div> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">           Displacement: <math>x</math>  <math>\downarrow</math>            Differentiate         </div> <div style="text-align: center;">           Velocity: <math>\dot{x} = \frac{dx}{dt}</math> </div> <div style="text-align: center;"> <math>\uparrow</math>            Integrate         </div> </div> <div style="text-align: center; margin-top: 10px;"> <math>\downarrow</math> </div> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">           Acceleration: <math>\ddot{x} = \frac{d^2x}{dt^2}</math>  <math>\downarrow</math> </div> <div style="text-align: center;"> <math>\uparrow</math> </div> </div> </li> <li>deriving equations of motion under <b>constant</b> acceleration from the definition of constant acceleration and displacement, as well as deriving these equations using calculus</li> <li>one-dimensional motion and freefall under gravity</li> <li>stopping distances at traffic lights and speed cameras</li> <li>air friction (resistance) in vertical motion, constant velocity and interpretation of negative velocity, variation in the value of <math>g</math>, and the need to model all bodies as particles</li> </ul>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
Applying displacement, velocity and acceleration vectors to resultant and relative motion	<ul style="list-style-type: none"> <li>giving the displacement, velocity, and acceleration of a particle as a vector, and understanding speed is the magnitude of the velocity vector</li> </ul> <p>If <math>\underline{r} = \begin{pmatrix} x \\ y \end{pmatrix}</math> where <math>x</math> and <math>y</math> are functions of <math>t</math>, then</p> $\underline{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \text{ and } \underline{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$ <ul style="list-style-type: none"> <li>resolving position, velocity, and acceleration vectors into two and three dimensions and using these to consider resultant or relative motion</li> </ul>	<p>Vectors can be expressed as column vectors, or using <math>\mathbf{i}</math>, <math>\mathbf{j}</math>, <math>\mathbf{k}</math></p> <p>notation: <math>\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}</math></p> <p>Candidates should be familiar with the notation:</p> <ul style="list-style-type: none"> <li><math>\mathbf{r}_p</math> for the position vector of P</li> <li><math>\mathbf{v}_p = \dot{\mathbf{r}}_p</math> for the velocity vector of P</li> <li><math>\mathbf{a}_p = \dot{\mathbf{v}}_p = \ddot{\mathbf{r}}_p</math> for the acceleration vector of P</li> <li><math>\mathbf{i}</math>, <math>\mathbf{j}</math>, <math>\mathbf{k}</math> as the unit vectors in <math>x</math>, <math>y</math> and <math>z</math> directions</li> <li>speed = <math> \mathbf{v}  =  \dot{\mathbf{r}} </math></li> </ul> <p>This can be extended to three dimensions.</p> <p>Differentiation and integration can be used to find displacement, velocity and acceleration, given one of these as a function of time.</p> <p><b>Two dimensions:</b> if a body is travelling in <math>xy</math>-plane with speed <math>v \text{ ms}^{-1}</math>, making an angle <math>\theta^\circ</math> with <math>OX</math>, then its velocity vector can be expressed as <math>v \cos \theta^\circ \mathbf{i} + v \sin \theta^\circ \mathbf{j}</math>.</p> <p><b>Three dimensions:</b> if body is travelling in <math>xyz</math>-plane with speed <math>v \text{ ms}^{-1}</math>, making an angle <math>\theta_1^\circ</math> with <math>OX</math>, <math>\theta_2^\circ</math> with <math>OY</math>, and <math>\theta_3^\circ</math> with <math>OZ</math>, then its velocity vector can be expressed as <math>v \cos \theta_1^\circ \mathbf{i} + v \cos \theta_2^\circ \mathbf{j} + v \cos \theta_3^\circ \mathbf{k}</math>.</p>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
	<ul style="list-style-type: none"> <li>◆ applying position, velocity, and acceleration vectors to practical problems, including navigation, the effects of winds and currents, and other relevant contexts</li>   <li>◆ solving a simple problem involving collision</li>   <li>◆ considering conditions for nearest approach</li> </ul>	<p>Candidates should be familiar with the notation:</p> <ul style="list-style-type: none"> <li>◆ <math>{}_q\mathbf{r}_p = \overline{PQ} = \mathbf{q} - \mathbf{p}</math> for the position vector of Q relative to P</li> <li>◆ <math>{}_q\mathbf{v}_p = \mathbf{v}_q - \mathbf{v}_p = \dot{\mathbf{r}}_q - \dot{\mathbf{r}}_p</math> for the velocity of Q relative to P</li> <li>◆ <math>{}_q\mathbf{a}_p = \mathbf{a}_q - \mathbf{a}_p = \dot{\mathbf{v}}_q - \dot{\mathbf{v}}_p = \ddot{\mathbf{r}}_q - \ddot{\mathbf{r}}_p</math> for the acceleration of Q relative to P</li> </ul> <p>Learning and teaching contexts that explore the effects of currents and winds could include crossing a river or flying between airports.</p> <p>For collision, candidates should understand that both positions after time <math>t</math> should be equal, and <math>{}_A\mathbf{v}_B</math> should be in the direction of the original relative position vector.</p> <p>For nearest approach, candidates can explore both 'least separation' by differentiation, and the vector condition <math>{}_P\mathbf{r}_Q \cdot {}_P\mathbf{v}_Q = 0</math>.</p> <p>Learning and teaching contexts could include shipping and aircraft movement and sports.</p>
Applying Newton's Second Law of motion, including	<ul style="list-style-type: none"> <li>◆ using <math>F = ma</math> to form equations of motion to model practical problems of motion in a straight line, where acceleration may be considered as a function of time or of displacement</li> </ul>	<p>Candidates should understand that the acceleration of a body is proportional to the resultant external force, and takes place in the direction of the force.</p>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
situations involving dynamic friction	<ul style="list-style-type: none"> <li>♦ solving problems involving motion on both smooth and rough inclined planes</li> </ul>	<p>When <math>\mathbf{F} = m\mathbf{a}</math> is a vector equation, the acceleration produced is in the direction of the applied or resultant force.</p> <p>Learning and teaching contexts could include parabolic motion of projectiles and the escape velocity of a rocket.</p> <p>Candidates should be able to solve problems involving smooth and rough planes. (Candidates can also solve these problems using energy considerations.)</p>
Establishing and applying equations of motion to projectiles in horizontal and vertical directions moving with parabolic motion	<ul style="list-style-type: none"> <li>♦ deriving the formulae  <math display="block">T = \frac{2u \sin \alpha}{g}, \quad H = \frac{u^2 \sin^2 \alpha}{2g} \quad \text{and}</math> <math display="block">R = uT \cos \alpha = \frac{u^2 \sin 2\alpha}{g}</math>           where <math>T</math> refers to total time of flight, <math>H</math> refers to greatest height and <math>R</math> refers to the horizontal range</li> </ul>	<p>Candidates should be able to derive these formulae from equations of motion or by using calculus.</p> <p>Candidates should appreciate that gravity only affects motion in the vertical plane, and so projectile motion considers vertical motion and horizontal motion separately.</p> <p>Learning and teaching contexts could include reminding candidates of the properties of the parabola, for example relating the time of flight, <math>T</math>, with the time to reach the greatest height. This can be done by:</p> <ul style="list-style-type: none"> <li>♦ solving the vector equation <math>\ddot{\mathbf{r}} = -g\mathbf{j}</math> to obtain expressions for <math>\dot{x}, \dot{y}, x</math> and <math>y</math> in a particular case</li> <li>♦ using the equations of motion under constant acceleration</li> </ul>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
	<ul style="list-style-type: none"> <li>◆ using these formulae to find the time of flight, greatest height reached, or range of a projectile, including maximum range of a projectile and the angle of projection to achieve this</li> <li>◆ deriving and using the equation of the trajectory of a projectile:  <math display="block">y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}</math> </li> <li>◆ solving problems in two-dimensional motion involving projectiles under a constant gravitational force — projection can be considered in one vertical plane, but point of projection can be from a different horizontal plane than that of landing</li> </ul>	<p>Candidates should be able to derive the equation of the trajectory.</p> <p>Learning and teaching contexts could include sport.</p> <p>(Candidates do not have to study projection from an inclined plane.)</p>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
Applying equations to motion in horizontal and vertical circles with uniform angular velocity, including gravitational motion	<p>♦ solving problems involving motion in a circle of radius <math>r</math>, with uniform angular velocity <math>\omega</math>, making use of the equations:</p> $\theta = \omega t, v = r\omega = r\dot{\theta}, a = r\omega^2 = r\dot{\theta}^2 = \frac{v^2}{r},$ $\mathbf{a} = -\omega^2 \mathbf{r} \text{ and } T = \frac{2\pi}{\omega}$ <p>and applying these equations to motion including skidding, banking, and other applications</p>	<p>Candidates should understand the terms angular velocity, angular acceleration, radial and tangential components.</p> <p>Candidates should be able to use vectors to establish these equations, starting from <math>\mathbf{r} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}</math>, where <math>r</math> is constant and <math>\theta</math> is varying, before considering the special case, where <math>\theta = \omega t</math>, <math>\omega</math> being constant.</p> <p>Hence, if <math>\mathbf{e}_r = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}</math> and <math>\mathbf{e}_\theta = -\sin \theta \mathbf{i} + \cos \theta \mathbf{j}</math> are the unit vectors in the radial and tangential directions respectively, it follows that the radial and tangential components of velocity are <math>\mathbf{0}</math> (zero vector) and <math>r\dot{\theta}\mathbf{e}_\theta</math> respectively, and those of acceleration are <math>-r\dot{\theta}^2\mathbf{e}_r</math> and <math>r\ddot{\theta}\mathbf{e}_\theta</math> respectively.</p> <p>Learning and teaching contexts could include motion in a horizontal circle around a banked surface, such as skidding, the 'wall of death', and the conical pendulum.</p>

Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
	<ul style="list-style-type: none"> <li>◆ solving a simple problem involving Newton's Inverse Square Law, <math>F = \frac{GMm}{r^2}</math>, applied to simplified motion of satellites and moons, making use of the equations of motion for horizontal circular motion above</li> <li>◆ using conservation of energy within a situation involving vertical circular motion</li> </ul>	<p>Candidates should appreciate that the magnitude of the gravitational force of attraction between two particles is inversely proportional to the square of the distance between the two particles.</p> <p>Motion here involves circular orbits only, and candidates do not have to consider additional effects, such as the rotation of a moon about its own axis while orbiting a planet.</p> <p>Learning and teaching contexts could include linking gravitational circular motion to gravitational potential energy.</p> <p>Candidates should understand the conditions required to perform full circles, including cases with a particle attached to an inextensible string, a particle on the end of a light rod, a bead running on the inside or the outside of a cylinder, and a bead on a smooth circular wire.</p> <p>Learning and teaching contexts could include calculating the initial speed of projection required for each of these cases. For particles of equal mass describing circles of equal radius, candidates should understand the requirement for a greater speed of projection in the case of an inextensible string versus a light rod.</p>



Straight line, periodic and parabolic motion		
Skills	Explanation	Suggested learning and teaching contexts
Applying the concept of simple harmonic motion (SHM), including problems involving Hooke's Law	<ul style="list-style-type: none"> <li>◆ using the equation for SHM, <math>\ddot{x} = -\omega^2 x</math>, and the following associated equations:  <math display="block">v^2 = \omega^2 (a^2 - x^2), \quad T = \frac{2\pi}{\omega}, \quad  v _{\max} = a\omega,</math> <math display="block"> \ddot{x} _{\max} = a\omega^2 \quad \text{and} \quad x = a \sin(\omega t + \alpha)</math> </li> <li>◆ considering SHM starting from points other than the centre of oscillation</li> <li>◆ applying Hooke's Law to SHM problems involving extensible strings and springs</li> </ul>	<p>Candidates should understand the terms tension, thrust, natural length, stiffness constant, modulus of elasticity, extension, compression, position of equilibrium, and oscillation.</p> <p>Candidates should appreciate that the tension in the string or spring is directly proportional to the extension from the natural length: that is <math>T = kx</math>, where the stiffness constant (<math>k</math>) is equivalent to <math>\frac{\lambda}{l}</math>, with <math>\lambda</math> being the modulus of elasticity and <math>l</math> being the natural length of the string or spring.</p> <p>Learning and teaching contexts could include problems involving elastic strings and springs, and a simple pendulum, but not the compound pendulum. Candidates should be aware that simple harmonic motion and linear motion could arise in the same context for a stretched string.</p> <p>Candidates should understand the terms oscillation, centre of oscillation, period, amplitude, frequency, maximum velocity and maximum acceleration.</p> <p>Candidates should understand that <math>v^2 = \omega^2 (a^2 - x^2)</math> can be derived from the solution of a separable first-order differential equation.</p>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
Decomposing a rational function into a sum of partial fractions (denominator of degree at most three)	<ul style="list-style-type: none"> <li>◆ decomposing a proper rational function as a sum of partial fractions where the denominator may contain distinct linear factors, an irreducible quadratic factor, or a repeated linear factor</li> <li>◆ reducing an improper rational function to a polynomial and a proper rational function by division or otherwise</li> </ul>	<p>For example:</p> <ul style="list-style-type: none"> <li>◆ <math>\frac{7x+1}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}</math></li> <li>◆ <math>\frac{5x^2-x+6}{x^3+3x} = \frac{A}{x} + \frac{Bx+C}{x^2+3}</math></li> <li>◆ <math>\frac{3x+10}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}</math></li> <li>◆ <math>\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}</math></li> <li>◆ <math>\frac{x^2+3x}{x^2-4}</math></li> </ul>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
Differentiating, exponential, natural logarithmic, and trigonometric functions	<ul style="list-style-type: none"> <li>◆ differentiating functions involving <math>e^x</math> and <math>\ln x</math></li> <li>◆ knowing the definitions and applying the derivatives of <math>\tan x</math>, <math>\cot x</math>, <math>\sec x</math> and <math>\operatorname{cosec} x</math></li> <li>◆ deriving the derivatives of <math>\tan x</math>, <math>\sec x</math>, <math>\operatorname{cosec} x</math> and <math>\cot x</math></li> </ul>	<p>For example:</p> <ul style="list-style-type: none"> <li>◆ <math>y = e^{3x}</math></li> <li>◆ <math>f(x) = \ln(x^3 + 2)</math></li> </ul>
Differentiating functions using the chain rule, and functions given in the form of a product and in the form of a quotient	<ul style="list-style-type: none"> <li>◆ applying the chain rule to differentiate the composition of at most three functions</li> <li>◆ differentiating functions of the form <math>f(x)g(x)</math> and <math>\frac{f(x)}{g(x)}</math></li> <li>◆ differentiating functions that require more than one application or combination of applications of the chain rule, product rule, and quotient rule</li> </ul>	<p>For example:</p> <ul style="list-style-type: none"> <li>◆ <math>y = \sqrt{e^{x^2} + 4}</math></li> <li>◆ <math>f(x) = \sin^3(2x - 1)</math></li> </ul> <p>For example:</p> <ul style="list-style-type: none"> <li>◆ <math>y = 3x^4 \sin x</math></li> <li>◆ <math>f(x) = x^2 \ln x, x &gt; 0</math></li> <li>◆ <math>y = \frac{2x - 5}{3x^2 + 2}</math></li> <li>◆ <math>f(x) = \frac{\cos x}{e^x}</math></li> </ul>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
		<p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ encouraging candidates to consider different ways of expressing their answers</li> <li>◆ demonstrating formal proofs of differentiation to candidates</li> <li>◆ introducing candidates to product and quotient rules with formal proofs, although these are not assessed in this course</li> <li>◆ once they have mastered differentiation rules, showing candidates how to use computer algebra systems (CAS) for difficult and real-life examples (CAS cannot be used in assessment. If candidates use CAS for differentiation in difficult cases, they should understand which rules were needed for the solution.)</li> <li>◆ applying differentiation to simple rates of change, for example rectilinear motion and optimisation</li> </ul>
Finding the derivative where relationships are defined implicitly or parametrically	<ul style="list-style-type: none"> <li>◆ using differentiation to find the first derivative of a function defined implicitly, including in context</li> <li>◆ using differentiation to find the second derivative of a function defined implicitly, including in context</li> <li>◆ knowing that <math>\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}</math></li> </ul>	<p>For example:</p> <ul style="list-style-type: none"> <li>◆ <math>x^3y + xy^3 = 4</math></li> <li>◆ <math>y = \frac{x^2\sqrt{7x-3}}{1+x}</math></li> </ul>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
	<ul style="list-style-type: none"> <li>◆ using differentiation to find the first derivative of a function defined parametrically</li> <li>◆ solving practical related rates by first establishing a functional relationship between appropriate variables</li> </ul>	<p>Candidates should appreciate the geometrical importance of parametric equations.</p> <p>If the position is given by <math>x = f(t)</math>, <math>y = g(t)</math>, then:</p> <ul style="list-style-type: none"> <li>◆ velocity components are given by <math>v_x = \frac{dx}{dt}</math>, <math>v_y = \frac{dy}{dt}</math></li> <li>◆ speed = <math>\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}</math></li> </ul> <p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>◆ differentiating velocity as an implicit function:  <math display="block">\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{1}{2} \times 2v \times \frac{dv}{dx} = v \frac{dv}{dx}</math> </li> <li>◆ using IT to sketch graphs, where <math>x</math> and <math>y</math> are different functions of a variable, for example <math>x = 4 \cos \theta</math> <math>y = 4 \sin \theta</math> representing a circle</li> <li>◆ calculating the rate at which the depth of coffee in a conical filter is changing</li> </ul>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
Integrating expressions using standard results	<ul style="list-style-type: none"> <li>◆ using <math>\int e^x dx</math>, <math>\int \frac{dx}{x}</math>, and <math>\int \sec^2 x dx</math></li> <li>◆ recognising and integrating expressions of the form <math>\int g(f(x))f'(x)dx</math> and <math>\int \frac{f'(x)}{f(x)} dx</math></li> <li>◆ using partial fractions to integrate proper or improper rational functions</li> </ul>	<p>For example:</p> <ul style="list-style-type: none"> <li>◆ <math>\int \cos^3 x \sin x dx</math></li> <li>◆ <math>\int xe^{x^2} dx</math></li> <li>◆ <math>\int_0^2 \frac{2x}{x^2 + 3} dx</math></li> <li>◆ <math>\int \frac{\cos x}{(5 + 2 \sin x)} dx</math></li> </ul> <p>Candidates should understand the process of expressing a rational function in partial fractions. Some revision of logarithmic functions might be useful before working with integrals here.</p> <p>Learning and teaching contexts could include solving problems involving motion with resistance.</p>
Integrating using a substitution	<ul style="list-style-type: none"> <li>◆ integrating where the substitution is given</li> </ul>	<p>For example, use the substitution <math>u = \ln x</math> to obtain <math>\int \frac{1}{x \ln x} dx</math>, where <math>x &gt; 1</math>.</p>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
Integrating by parts	<ul style="list-style-type: none"> <li>◆ using integration by parts with one or more applications</li> </ul>	<p>Candidates should be able to derive from the product rule, for example:</p> <ul style="list-style-type: none"> <li>◆ <math>\int x \sin x dx</math> (single application)</li> <li>◆ <math>\int x^2 e^{3x} dx</math> (repeated applications)</li> <li>◆ <math>\int e^x \sin x dx</math> (cyclic integration)</li> <li>◆ <math>\int \ln x dx</math> (by considering <math>\ln x</math> as <math>1 \cdot \ln x</math>)</li> </ul> <p>Candidates should be able to deal with definite or indefinite integrals, as required.</p> <p>Integration by parts may appear in a question, in combination with using the integrating factor to solve first-order differential equations.</p>
Applying integration to a range of physical situations	<ul style="list-style-type: none"> <li>◆ applying integration to volumes of revolution, where the volume generated is by the rotation of the area under a single curve about the <math>x</math>-axis or <math>y</math>-axis</li> <li>◆ applying integration to evaluate areas</li> </ul>	<p>For example, given velocity, candidates should be able to use integration to find displacement.</p> <p>Learning and teaching contexts could include finding displacement for velocity–time graphs and finding centres of mass.</p>

Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
Solving a first-order linear differential equation with variables separable	<ul style="list-style-type: none"> <li>finding the general and particular solutions to equations that can be written in the form <math>\frac{dy}{dx} = g(x)h(y)</math> or <math>\frac{dy}{dx} = \frac{g(x)}{h(y)}</math></li> </ul>	<p>Candidates should understand how to use partial fractions, <math>\int \frac{1}{x} dx</math>, and manipulation of logarithmic terms before starting this work. Many of these equations arise naturally in mathematical modelling of physical situations, and feature in this section of study.</p> <p>Learning and teaching contexts could include:</p> <ul style="list-style-type: none"> <li>physical situations, such as electrical circuits, vibrating systems, and motion with resistance, where models involving differential equations arise</li> <li>scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, and bacterial growth and decay, where models involving logarithms arise (The differential equation shows how the system changes with time — or another variable.)</li> </ul>
Solving a first-order linear differential equation using an integrating factor	<ul style="list-style-type: none"> <li>finding the general or particular solution by writing equations in the standard form <math>\frac{dy}{dx} + P(x)y = f(x)</math></li> </ul>	<p>Candidates should practise rearranging equations into standard form.</p> <p>Candidates should be aware of the derivation of the integrating factor method, although this is not assessed in this course.</p>



Mathematical techniques for mechanics		
Skills	Explanation	Suggested learning and teaching contexts
Solving a second-order homogeneous differential equation	<p>♦ finding the general or particular solution of a second-order homogeneous ordinary differential equation <math>a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0</math>, where the roots of the auxiliary equation may be:</p> <ul style="list-style-type: none"> <li>▪ real and distinct</li> <li>▪ real and equal</li> </ul>	<p>Candidates should understand how to use a second-order differential equation when working with simple harmonic motion.</p> <p>Learning and teaching contexts could include discussing damped simple harmonic motion.</p> <p>Candidates should understand that real distinct roots of the auxiliary equation lead to heavy damping, equal roots to critical damping, and unreal roots to light damping.</p>
Applying mathematical techniques to problems	<p>♦ using all the above techniques in isolation or within mechanics contexts from all parts of the course</p>	

# Appendix 3: question paper brief

The course assessment consists of one question paper, which assesses the:

- ◆ knowledge and understanding of a range of straightforward and complex concepts in mechanics
- ◆ ability to identify and use appropriate techniques in mechanics
- ◆ ability to use mathematical reasoning and operational skills to extract and interpret information
- ◆ ability to create and use multifaceted mathematical models
- ◆ ability to communicate identified strategies of solution and provide justification for the resulting conclusions in a logical way
- ◆ ability to comprehend both the problem as a whole and its integral parts
- ◆ ability to select and use numerical skills

The question paper samples the 'Skills, knowledge and understanding' section of the course specification.

This sample draws on all of the skills, knowledge and understanding from each of the following areas:

- ◆ principles of momentum, impulse, work, power and energy
- ◆ motion in a horizontal circle with uniform angular velocity
- ◆ simple harmonic motion
- ◆ centre of mass
- ◆ motion in a straight line
- ◆ vectors associated with motion
- ◆ projectiles moving in a vertical plane
- ◆ forces associated with dynamics and equilibrium
- ◆ partial fractions
- ◆ calculus skills through techniques of differentiation
- ◆ calculus skills through techniques of integration
- ◆ solving differential equations

Command words are the verbs or verbal phrases used in questions and tasks to ask candidates to demonstrate specific skills, knowledge or understanding. For examples of some of the command words used in this assessment, refer to the [past papers and specimen question paper](#) on SQA's website.

The course assessment consists of one question paper:

	<b>Question paper</b>
<b>Time</b>	3 hours
<b>Marks</b>	100
<b>Skills</b>	<p>This question paper gives candidates an opportunity to apply mathematical techniques and skills to:</p> <ul style="list-style-type: none"> <li>◆ principles of momentum, impulse, work, power and energy</li> <li>◆ motion in a horizontal circle with uniform angular velocity</li> <li>◆ simple harmonic motion</li> <li>◆ centre of mass</li> <li>◆ motion in a straight line</li> <li>◆ vectors associated with motion</li> <li>◆ projectiles moving in a vertical plane</li> <li>◆ forces associated with dynamics and equilibrium</li> <li>◆ partial fractions</li> <li>◆ calculus skills through techniques of differentiation</li> <li>◆ calculus skills through techniques of integration</li> <li>◆ solving differential equations</li> </ul> <p>Candidates can use a calculator.</p>
<b>Percentage of marks across the paper</b>	<p>Approximately 25–45% of the overall marks relate to forces, energy and momentum.</p> <p>Approximately 25–45% of the overall marks relate to straight-line, periodic and parabolic motion.</p> <p>Approximately 25–45% of the overall marks relate to mathematical techniques for mechanics.</p>
<b>Type of question</b>	The question paper contains short-answer and extended-response questions, set in contexts.
<b>Type of question paper</b>	Semi-structured question papers: separate question paper and answer booklet. The answer booklet is structured with spaces for answers.
<b>Proportion of level ‘C’ questions</b>	Many questions use a stepped approach to ensure that there are opportunities for candidates to demonstrate their abilities beyond level ‘C’. Approximately 65% of the marks are available for level ‘C’ responses.
<b>Balance of skills</b>	Operational and reasoning skills are assessed in the question paper. Some questions assess only operational skills (approximately 65% of the marks), but other questions assess operational and reasoning skills (approximately 35% of the marks).

# Administrative information

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## History of changes

Version	Description of change	Date
2.0	Course support notes; skills, knowledge and understanding with suggested learning and teaching contexts; and question paper brief added as appendices.	May 2019

Note: please check SQA's website to ensure you are using the most up-to-date version of this document.

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