National
Qualifications
SPECIMEN ONLY

S802/77/11

Mathematics of Mechanics

Date — Not applicable

Duration — 3 hours

Total marks — 100

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate. Any rounded answer should be accurate to an appropriate number of significant figures unless otherwise stated.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

S80277 11

FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a\sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Coordinates of the centre of mass of a uniform lamina, area A square units, bounded by the equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by

$$\bar{x} = \frac{1}{A}\int_a^b xy\,dx \qquad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}y^2\,dx$$

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax) + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$

MARKS

Total marks — 100

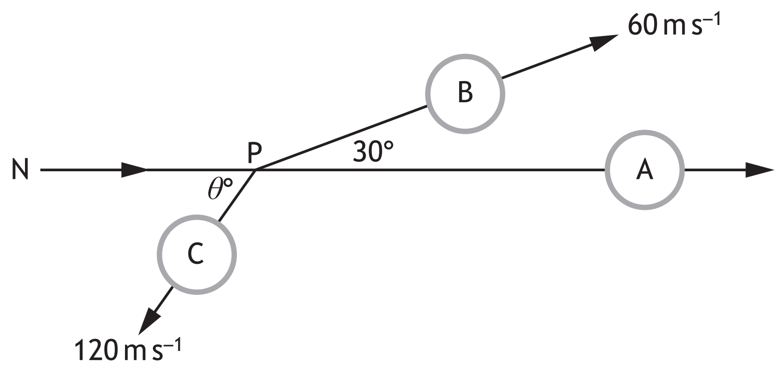
Attempt ALL questions

Note that $g\,\text{m s}^{-2}$ denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be 9·8 m s^{-2}.

1. A particle has velocity $\mathbf{v} = (3\sin 2t)\mathbf{i} + (\cos 2t - 3)\mathbf{j}\,\text{m s}^{-1}$ where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in horizontal and vertical directions respectively.
Find the magnitude of the acceleration of the particle when $t = \frac{\pi}{6}$. **4**

2. A shell of mass 20 kg is travelling in a horizontal plane along the line NP at 100 m s^{-1}.

At P, it breaks into 3 pieces A, B and C of masses 12 kg, 6 kg, and 2 kg respectively.



Find the size of the acute angle $\theta°$, and the speed of A. **4**

3. If $f(x) = \dfrac{\ln x}{2x^2}$, $x \neq 0$, find $f'(x)$.

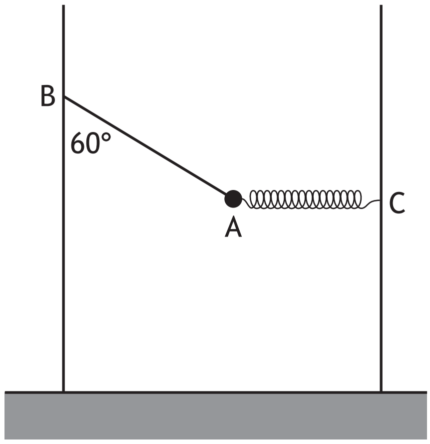
Fully simplify your answer. **3**

[Turn over

MARKS

4. A particle of mass 1 kg is held in equilibrium by a light, inextensible wire AB and a light spring AC.

AC is horizontal and AB is inclined at 60° to the vertical.



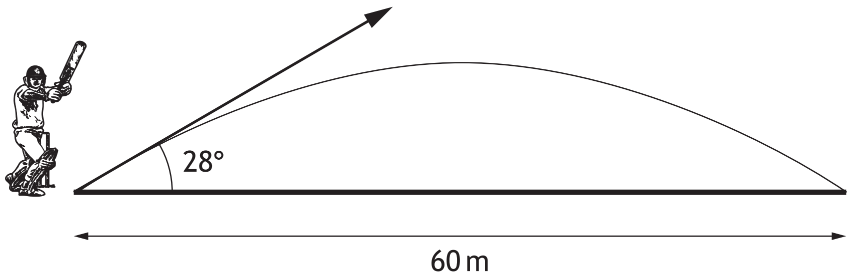
(a) Find the tension in the wire AB, and calculate the tension in the spring AC. **3**

(b) The spring has modulus of elasticity 40 Newtons and natural length 10 centimetres.

Calculate the distance AC. **2**

5. Express $\frac{3x^2+4x+17}{(x-3)(x^2+5)}$ as a sum of partial fractions. **4**

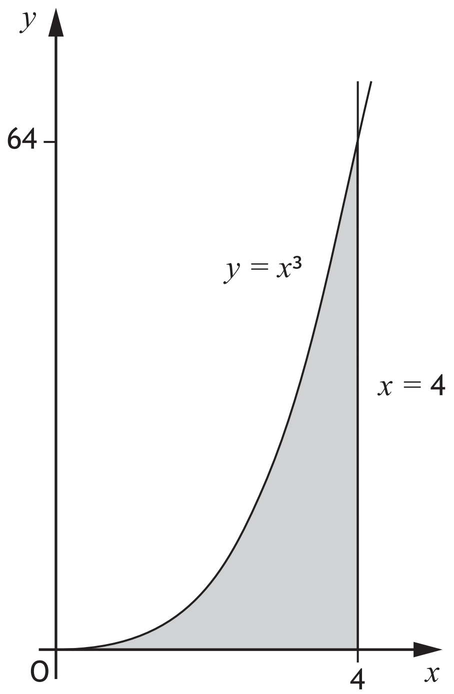
6. A cricket batsman hits a shot from ground level. The ball lands on the boundary which is 60 metres away.



If the angle of flight to the horizontal ground is 28° at the instant the ball leaves the bat, calculate the initial speed of the shot. **5**

MARKS

7. A uniform lamina is bounded by the curve $y = x^3$, the line $x = 4$ and the x-axis.



(a) Find the coordinates of the centre of mass of the lamina. **4**

(b) The lamina is suspended in equilibrium from its right-angled corner.

Find the angle between the longer of its two straight sides and the vertical. **2**

8. A function is defined as $f(x) = e^{\sec^2 x}$ where $0 \le x < \frac{\pi}{2}$.

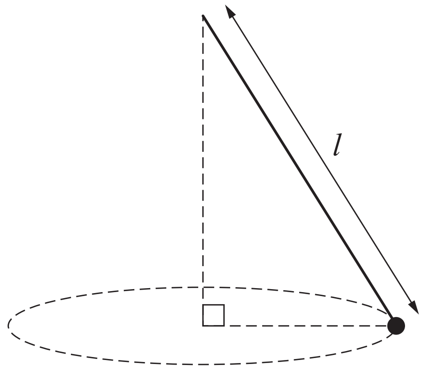
Find the value of $f'\left(\frac{\pi}{4}\right)$. **3**

[Turn over

MARKS

9. A mass m kg is attached to one end of a light inextensible string of length l metres.

The other end of the string is fixed and the mass is spun in a horizontal circle so that the path of the string forms a conical pendulum.



The angular speed of the mass is ω radians per second.

Given that the length of the string is double the radius of the horizontal circle, show that

$$\omega^2 = \frac{2g}{\sqrt{3}\,l}$$ 5

10. The motion of a particle is defined by the equations

$$x = t(t+4) \text{ and } y = t(1-t)^3$$

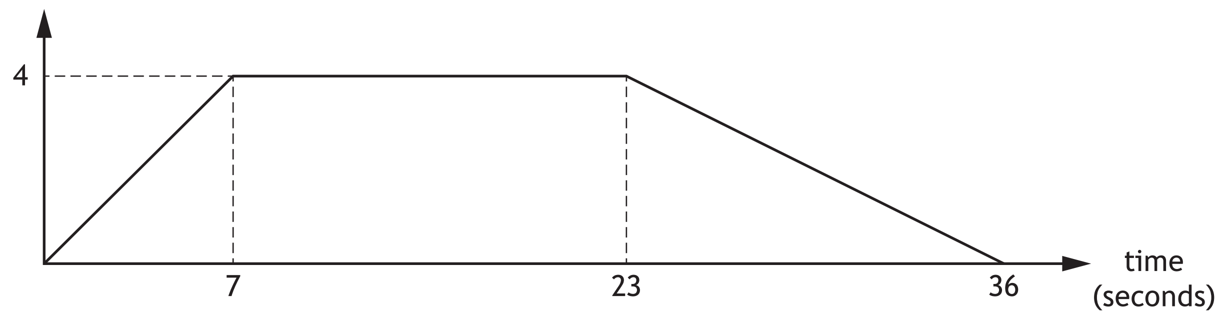
where t is the time elapsed since the start of motion.

Find the speed of the particle when $t = 3$. 4

MARKS

11. A lift and its occupants are limited to a mass of 1800 kg. It is to be drawn up and down a lift-shaft by an engine, using an inextensible cable.

A speed/time graph for the lift's ascent is shown.



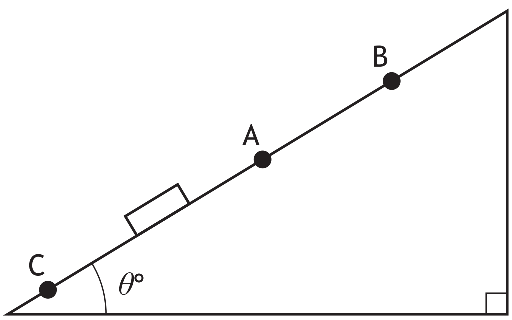
(a) Find the vertical distance travelled by the lift. **2**

(b) Find the power of the engine when the lift is fully loaded and travelling with a constant speed. **2**

(c) Find the maximum power generated during the ascent. **2**

12. A body of mass M kg moves up a rough plane inclined at $\theta°$ to the horizontal, where $\tan\theta° = \frac{3}{4}$. Its speed is u m s^{-1} as it passes through A.

After momentarily coming to rest at B, it passes through C with speed $2u$ m s^{-1} when moving down the slope.



The coefficient of friction between the mass and the slope is $\frac{1}{4}$.

Show that $AC = \dfrac{35u^2}{8g}$. **6**

[Turn over

MARKS

13. Find the exact value of the integral $\int_0^{\sqrt{5}} \frac{2x^3}{\sqrt{x^2+4}}\,dx$, using the substitution $u = x^2 + 4$. **6**

14. A fishing boat, A, leaves a harbour with a constant speed of 10 km h^{-1} moving on a bearing of 060°.

At the same time another fishing boat, B, is 12 km due east of A, moving with a constant speed of $10\sqrt{3}$ km h^{-1} on a bearing of 330°.

(a) Show that the relative position of boat A to boat B, t hours after A has left the harbour, can be written as ${}_{\text{A}}\mathbf{r}_{\text{B}} = (10\sqrt{3}t - 12)\mathbf{i} - 10t\,\mathbf{j}$. **2**

(b) Determine how long the two boats would be within 7 km of each other.

Give your answer to the nearest minute. **5**

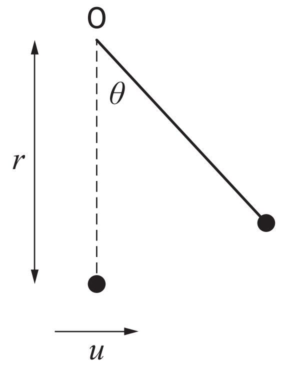
15. The displacement x metres of a point on a hinged door after t seconds is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 0.$$

Solve this differential equation, given that initially the displacement is 1 metre and the velocity is 2 m s^{-1}. **5**

MARKS

16. A light inextensible string of length r metres has one end attached to a fixed point O and the other end is attached to a particle of mass m kilograms.



From its equilibrium position, the particle is given a horizontal velocity u m s^{-1}.

(a) (i) Show that the tension, T, in the string can be expressed as

$$T = \frac{mu^2}{r} + mg(3\cos\theta - 2)$$

where θ is the angle between the string and the downward vertical through O. **4**

(ii) Determine a condition for u in terms of r and g, so that the particle executes a complete circle. **2**

(b) In some circumstances, the string will go slack as it moves in a vertical circle.

Given that the value of u is $2\sqrt{rg}$, find an expression in terms of r for the height of the particle above its starting position when this occurs. **3**

17. A body of mass of 750 g is attached to a light elastic string of natural length 50 cm and modulus of elasticity 150 N. The mass hangs vertically with one end of the string attached to the ceiling.

(a) Find the extension in the string when the body hangs in equilibrium. **2**

The body is released from a position 2 cm below the equilibrium position.

(b) (i) Show that the body moves with simple harmonic motion modelled by $\ddot{x} = -400x$ where x metres is the displacement from the equilibrium position. **3**

(ii) Find the speed of the body when it is 0·5 cm above the point of release. **2**

(c) On another occasion, the body is pulled down 3 cm below the equilibrium position. Explain why in this case the subsequent motion is not simple harmonic. **1**

[Turn over

MARKS

18. A particle P of mass 3 kg falls from rest under gravity.

Throughout its motion it experiences a resistance of $0\cdot25v^2$ newtons per unit mass, where v is its speed in metres per second after t seconds.

(a) Calculate the distance travelled by the mass in reaching a speed of 5 m s^{-1}. **5**

A second particle Q of mass 5 kg starts from rest at the origin and moves in a horizontal straight line with acceleration $2t\,\mathbf{i}$ m s^{-1}, where t is the time in seconds from the start of the motion and $\mathbf{i}$ is the unit vector in the direction of motion.

(b) Given that the work done by the total force acting on Q during the first a seconds is equal to that done on particle P in reaching its speed of 5 m s^{-1}, calculate the value of a. **5**

[END OF SPECIMEN QUESTION PAPER]

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S802/77/11

Mathematics of Mechanics

Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

General marking principles for Advanced Higher Mathematics of Mechanics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

This is a transcription error and so the mark is not awarded.

$$x = 1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x=2$	$x=-4$
•6	$y=5$	$y=-7$

Horizontal: •5 $x=2$ and $x=-4$
•6 $y=5$ and $y=-7$

Vertical: •5 $x=2$ and $y=5$
•6 $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as
  $(x^3+2x^2+3x+2)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$
  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ differentiate to give expression for acceleration with one term correct •² complete differentiation •³ substitute to give acceleration in vector form •⁴ calculate magnitude of acceleration	•¹ $6\cos 2t\mathbf{i}+\ldots$ **or** $-2\sin 2t\mathbf{j}+\ldots$ •² $(6\cos 2t)\mathbf{i}-(2\sin 2t)\mathbf{j}$ •³ $6\cos\frac{\pi}{3}\mathbf{i}-2\sin\frac{\pi}{3}\mathbf{j}$ •⁴ $2\sqrt{3}$ $[3\cdot46\ldots]$	**4**
2.			•¹ state momentum before breaking with direction indicated •² state momentum after breaking •³ apply the conservation of linear momentum to find value of $\theta°$ •⁴ calculate the speed of A	•¹ eg $\begin{pmatrix}2000\\0\end{pmatrix}$ •² $12\begin{pmatrix}v\\0\end{pmatrix}+6\begin{pmatrix}60\cos 30°\\60\sin 30°\end{pmatrix}+2\begin{pmatrix}-120\cos\theta°\\-120\sin\theta°\end{pmatrix}$ •³ $48\cdot6$ •⁴ 154	**4**

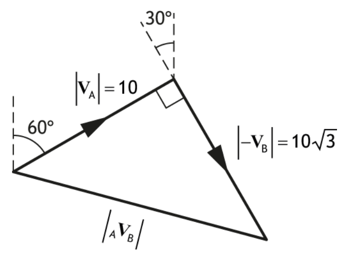
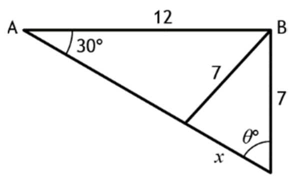
Question			Generic scheme	Illustrative scheme	Max mark
3.			**Method 1**	**Method 1**	3
			•1 use of quotient rule with one term in numerator and denominator correct	•1 eg $\dfrac{2x^2\times\frac{1}{x}-\ldots}{\left(2x^2\right)^2}$	
			•2 complete differentiation	•2 $\dfrac{2x^2\times\frac{1}{x}-\ln x\times(4x)}{\left(2x^2\right)^2}$	
			•3 simplify fully	•3 $\dfrac{1-2\ln x}{2x^3}$	
			Method 2	**Method 2**	3
			•1 express as product and start differentiation correctly	•1 eg $\frac{1}{2}(\ln x)\left(-2x^{-3}\right)+\ldots$	
			•2 complete differentiation	•2 $\frac{1}{2}(\ln x)\left(-2x^{-3}\right)+\frac{1}{2}\left(x^{-2}\right)\left(\frac{1}{x}\right)$	
			•3 simplify fully	•3 $\dfrac{-\ln x}{x^3}+\dfrac{1}{2x^3}$ or $\dfrac{1-2\ln x}{2x^3}$	
4.	(a)		•1 resolve forces vertically	•1 $T_{AB}\sin 30°-mg=0$	3
			•2 calculate tension in AB	•2 $2g$	
			•3 resolve the forces to find the tension in the spring AC	•3 $\sqrt{3}g$ $(16\cdot97\ldots)$	
	(b)		•4 use Hooke's law with substitution	•4 eg $T_{AC}=\dfrac{40x}{0\cdot10}$	2
			•5 calculate the extension and hence the length of AC	•5 $x=0\cdot0424\ldots\Rightarrow \text{AC}=0\cdot142\,(\text{m})$	
5.			•1 write as a sum of fractions	•1 $\dfrac{A}{x-3}+\dfrac{Bx+C}{x^2+5}$	4
			•2 rewrite with no denominator and calculate one value	•2 $A\left(x^2+5\right)+(Bx+C)(x-3)=3x^2+4x+17$ eg $A=4$	
			•3 calculate second value	•3 $B=-1$ or $C=1$	
			•4 calculate final value and rewrite original function as sum of partial fractions	•4 $\dfrac{4}{x-3}+\dfrac{1-x}{x^2+5}$	

Question			Generic scheme	Illustrative scheme	Max mark
6.			•1 use range to find expression for time of flight	•1 $60 = u\cos 28° \times t$	5
			•2 use equations of motion with constant acceleration vertically	•2 $0 = ut\sin 28° - \frac{g}{2}t^2$	
			•3 rearrange to give expression for t and substitution	•3 $t = \frac{60}{u\cos 28°}$ $0 = u\sin 28° \times \frac{60}{u\cos 28°} - \frac{g}{2}\left(\frac{60}{u\cos 28°}\right)^2$	
			•4 process algebra	•4 $\frac{g}{2}\left(\frac{60}{u\cos 28°}\right)^2 = 60\tan 28°$	
			•5 find initial speed	•5 26·6	
7.	(a)		•1 find the area under the curve	•1 64	4
			•2 state the integral for $\bar{x}$ with substitution and limits	•2 $\frac{1}{64}\int_0^4 x^4 dx$	
			•3 state the integral for $\bar{y}$ with substitution and limits	•3 $\frac{1}{64}\int_0^4 \frac{x^6}{2} dx$	
			•4 state centre of mass	•4 $(3{\cdot}2, 18{\cdot}3)$ or $\left(\frac{16}{5}, \frac{128}{7}\right)$	
	(b)		•5 interpret equilibrium to find horizontal displacement of COM	•5 0·8	2
			•6 calculate angle	•6 2·5	
8.			•1 start to use chain rule to find derivative	•1 $e^{\sec^2 x} \times \frac{d}{dx}\sec^2 x$	3
			•2 complete the differentiation	•2 $2\sec^2 x \tan x \; e^{\sec^2 x}$	
			•3 evaluate at $x = \frac{\pi}{4}$	•3 $4e^2$ (29·55…)	

Question			Generic scheme	Illustrative scheme	Max mark
9.			•1 denote quantities appropriately (via diagram or otherwise) and resolve vertically	•1 $T\cos\theta = mg$	5
			•2 use Newton's 2nd law horizontally with circular motion	•2 $T\sin\theta = mr\omega^2$	
			•3 eliminate T and m	•3 $\tan\theta = \frac{r\omega^2}{g}$	
			•4 use $l = 2r$ to find a value for $\tan\theta$ or evaluate θ	•4 $\tan\theta = \frac{1}{\sqrt{3}}$ or $\theta = \frac{\pi}{6}$	
			•5 complete proof	•5 $\frac{1}{\sqrt{3}} = \frac{l\omega^2}{2g} \rightarrow \omega^2 = \frac{2g}{\sqrt{3}l}$	
10.			•1 find $\frac{dx}{dt}$	•1 $2t+4$	4
			•2 find $\frac{dy}{dt}$	•2 $(1-t)^3 - 3t(1-t)^2$	
			•3 evaluate derivatives when $t = 3$	•3 $\frac{dx}{dt}(t=3) = 10$ **and** $\frac{dy}{dt}(t=3) = -44$	
			•4 substitute into appropriate formula and calculate speed	•4 $\sqrt{10^2 + (-44)^2} = 45 \cdot 12\ldots$	
11.	(a)		•1 know to find area under graph	•1 eg $\frac{1}{2}(7\times 4) + (16\times 4) + \frac{1}{2}(13\times 4)$	2
			•2 calculate distance	•2 104	
	(b)		•3 state tension in cable	•3 $1800g$	2
			•4 use relationship between power, force and velocity to calculate power	•4 70560	
	(c)		•5 use Newton's 2nd law to find tension under acceleration	•5 18668·57…	2
			•6 calculate maximum power	•6 74674	

Question			Generic scheme	Illustrative scheme	Max mark
12.			•1 consider equilibrium perpendicular to slope with equation •2 use Newton's 2nd law along the slope with equation •3 calculate acceleration up the slope •4 calculate the displacement up slope to rest •5 calculate acceleration down the slope •6 calculate displacement down slope and complete proof	•1 $R = Mg\cos\theta$ •2 $-\mu R - Mg\sin\theta = Ma$ •3 $-\frac{4g}{5}$ •4 $\frac{5u^2}{8g}$ •5 $\frac{2g}{5}$ •6 $s = \frac{5u^2}{g} \rightarrow \text{AC} = \frac{5u^2}{g} + -\frac{5u^2}{8g} = \frac{35u^2}{8g}$	**6**
Alternative Method					
			•1 consider equilibrium perpendicular to slope with equation •2 find KE at A, and PE at B •3 equate work done to change in energy between A and B •4 calculate the displacement up slope to rest •5 equate work done to change in energy between B and C •6 calculate displacement down slope and complete proof	•1 $R = Mg\cos\theta$ •2 $E_K(A) = \frac{1}{2}Mu^2,\ E_P(B) = \frac{3Mgs}{5}$ •3 $\frac{1}{4}\left(\frac{4Mg}{5}\right)s = \frac{1}{2}Mu^2 - \frac{3Mgs}{5}$ •4 $\frac{5u^2}{8g}$ •5 $\frac{1}{4}\left(\frac{4Mg}{5}\right)s = \frac{3Mgs}{5} - 2Mu^2$ •6 $s = \frac{5u^2}{g} \rightarrow \text{AC} = \frac{5u^2}{g} + -\frac{5u^2}{8g} = \frac{35u^2}{8g}$	**6**

Question			Generic scheme	Illustrative scheme	Max mark
13.			•¹ differentiate u with respect to x	•¹ $\frac{du}{dx}=2x$	**6**
			•² evaluate new limits	•² $x=0 \Rightarrow u=4,\ x=\sqrt{5} \Rightarrow u=9$	
			•³ state new integral	•³ $\int_4^9 \frac{u-4}{u^{\frac{1}{2}}}\,du$	
			•⁴ express in integrable form	•⁴ $\int_4^9 \left(u^{\frac{1}{2}}-4u^{-\frac{1}{2}}\right)du$	
			•⁵ integrate	•⁵ $\left[\frac{2}{3}u^{\frac{3}{2}}-8u^{\frac{1}{2}}\right]_4^9$	
			•⁶ state exact value	•⁶ $\frac{14}{3}$	

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		•1 obtain equations for the velocity of boat A and boat B •2 state initial positions and obtain expressions for the positions of boat A and boat B at time t and complete	•1 $\mathbf{v}_A = 10\sin 60\mathbf{i} + 10\cos 60\mathbf{j}$ $\mathbf{v}_B = -10\sqrt{3}\sin 30\mathbf{i} + 10\sqrt{3}\cos 30\mathbf{j}$ •2 $\mathbf{r}_A = 5\sqrt{3}t\mathbf{i} + 5t\mathbf{j}$ $\mathbf{r}_B = \left(12 - 5\sqrt{3}t\right)\mathbf{i} + 15t\mathbf{j}$ ${}_A\mathbf{r}_B = \mathbf{r}_A - \mathbf{r}_B = \left(10\sqrt{3}t - 12\right)\mathbf{i} - 10t\mathbf{j}$	**2**
	(b)		•3 obtain expression for the magnitude relative distance between boats A and B •4 equate distance expression to 7km and obtain quadratic equation in standard form •5 find values for t •6 interpret inequality •7 state time interval rounded to the nearest minute	•3 $\left\|{}_A\mathbf{r}_B\right\| = \sqrt{\left(10\sqrt{3}t - 12\right)^2 + \left(10t\right)^2}$ •4 $400t^2 - 240\sqrt{3}t + 95 = 0$ •5 $t = 0{\cdot}339,\ 0{\cdot}700$ •6 $400t^2 - 240\sqrt{3}t + 95 \le 0$ stated or implied by •7 •7 22 minutes	**5**
Alternative Method					
			•1 find expression for ${}_A\mathbf{v}_B$ •2 find expression for ${}_A\mathbf{r}_B$	•1 ${}_A\mathbf{v}_B = 10\sqrt{3}\,\mathbf{i} - 12\,\mathbf{j}$ •2 ${}_A\mathbf{r}_B = -12\mathbf{i} + t\left(10\sqrt{3}\,\mathbf{i} - 12\,\mathbf{j}\right)$ $= \left(10\sqrt{3}\,t - 12\right)\mathbf{i} - 12t\,\mathbf{j}$	**2**
			•3 find the magnitude of the relative velocity ${}_A\mathbf{v}_B$ •4 draw diagram to indicate relative position vectors or otherwise •5 find appropriate angles for when displacement is 7 km	•3  $\left\|{}_A\mathbf{v}_B\right\| = 20$ •4  •5 $\dfrac{12}{\sin\theta°} = \dfrac{7}{\sin 30°} \rightarrow \theta = 59{\cdot}0°$ or $\left(121°\right)$	**5**

Question			Generic scheme	Illustrative scheme	Max mark
			•6 find distance between above points •7 find time to travel this distance at relative speed	•6 $\frac{x}{\sin 62°} = \frac{7}{\sin 59°} \rightarrow x = 7{\cdot}21$ •7 22 minutes	
15.			•1 set up auxiliary equation •2 solve and state general solution with repeated root •3 use initial condition to find value of A •4 differentiate to use initial condition •5 substitute to obtain B and particular solution	•1 $m^2 + 8m + 16 = 0$ •2 $m = -4$ $x = Ae^{-4t} + Bte^{-4t}$ •3 $A = 1$ •4 $-4Ae^{-4t} + Be^{-4t} - 4Bte^{-4t}$ •5 $B = 6$, $x = e^{-4t} + 6te^{-4t}$	**5**
16.	(a)	(i)	•1 consider energy at two positions •2 use conservation of energy •3 find expression for velocity •4 $F = ma$ radially and show expression for tension	•1 at starting position $E_k = \frac{1}{2}mu^2$ elsewhere $E_k + E_p = \frac{1}{2}mv^2 + mg(r - r\cos\theta)$ •2 $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(r - r\cos\theta)$ •3 $v^2 = u^2 - 2rg(1 - \cos\theta)$ •4 $T - mg\cos\theta = \frac{mv^2}{r}$ and complete	**4**
		(ii)	•5 interpret condition for full circles •6 find expression for u	•5 $T \geq 0$ when $\theta = \pi$ and substitute •6 $u \geq \sqrt{5rg}$	**2**
	(b)		•7 interpret condition for string going slack and substitute •8 find ratio •9 find height	•7 $T = 0$ and substitute •8 $\cos\theta = -\frac{2}{3}$ •9 $h = \frac{5}{3}r$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
17.	(a)		•¹ consider body in equilibrium and Hooke's law •² evaluate equilibrium extension	•¹ $T = mg : \frac{150e}{0\cdot5} = 0\cdot75g$ •² $e = 0\cdot0245 = 2\cdot45$ cm	**2**
	(b)	(i)	•³ apply $F = ma$ vertically •⁴ apply Hooke's law in extension with substitution •⁵ complete to prove SHM	•³ $mg - T = m\ddot{x}$ •⁴ $mg - \frac{150(0\cdot0245 + x)}{0\cdot5} = m\ddot{x}$ $g - 400(0\cdot0245 + x) = \ddot{x}$ •⁵ $\ddot{x} = -400x$ SHM as eg $\ddot{x} = -kx$	**3**
		(ii)	•⁶ use equation for speed with substitution •⁷ find the value of speed	•⁶ $v^2 = 400(0\cdot02^2 - 0\cdot015^2)$ •⁷ $v = 0\cdot265\ldots$	**2**
	(c)		•⁸ statement about extension that allows tension in the string	•⁸ $3\text{ cm} > 2\cdot45\text{ cm}$ so string is not in tension throughout	**1**

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)		•[1] use Newton's 2nd law with appropriate formula for acceleration •[2] separate variables with substitution •[3] complete integration •[4] substitute to find value of c or use limits •[5] substitute for v to give displacement	•[1] $3g-0\cdot75v^2=3v\frac{dv}{dx}$ •[2] eg $\int dx=\int\frac{v}{g-0\cdot25v^2}dv$ •[3] $x=-2\ln\left\|g-0\cdot25v^2\right\|+c$ •[4] $x=-2\ln\left\|g-0\cdot25v^2\right\|+2\ln g$ •[5] $x=2\cdot030\ldots$	**5**
	(b)		•[6] integrate expression for acceleration to find expression for v •[7] find work done by variable force •[8] calculate change of energy •[9] use work-energy principle •[10] evaluate value of a	$v=\int 2t\mathbf{i}\,dt=t^2\mathbf{i}+c$ •[6] $t=0$ $v=0$ $c=0$ $v=t^2\mathbf{i}$ •[7] $\int_0^a \mathbf{F.v}\,dt=\int_0^a 10t^3\,dt=\frac{5a^4}{2}$ •[8] $3g(2\cdot03)-\frac{1}{2}(3)(5^2)$ •[9] $\frac{5a^4}{2}=22\cdot2$ •[10] $1\cdot726\ldots$	**5**

[END OF SPECIMEN MARKING INSTRUCTIONS]