

X100/12/03

NATIONAL TUESDAY, 6 MAY
QUALIFICATIONS 2.50 PM – 4.00 PM
2014

MATHEMATICS
HIGHER
Paper 2

Read carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

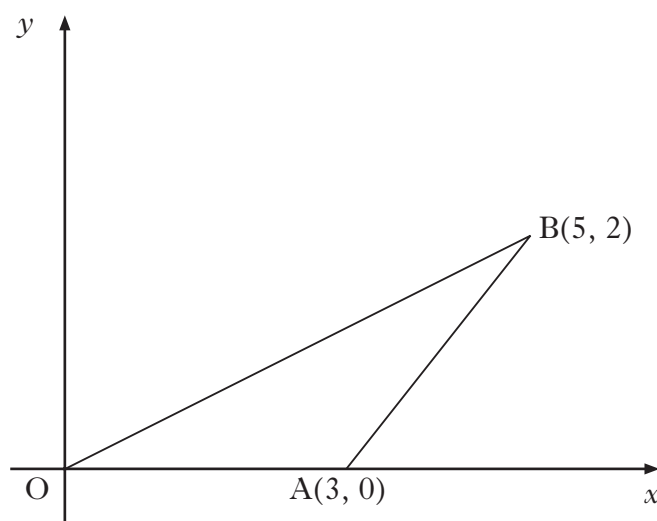
Table of standard derivatives:

| $f(x)$ | $f'(x)$ |
|-----------|--------------|
| $\sin ax$ | $a \cos ax$ |
| $\cos ax$ | $-a \sin ax$ |

Table of standard integrals:

| $f(x)$ | $\int f(x) dx$ |
|-----------|----------------------------|
| $\sin ax$ | $-\frac{1}{a} \cos ax + c$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + c$ |

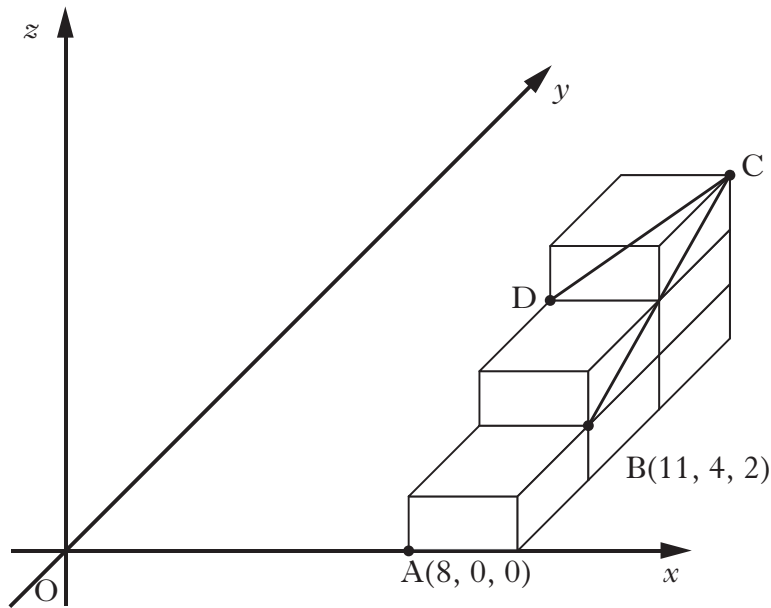
1. A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



- (a) Obtain the equation of the perpendicular bisector of AB. 4
- (b) The median from A has equation $y + 2x = 6$.
Find T, the point of intersection of this median and the perpendicular bisector of AB. 2
- (c) Calculate the angle that AT makes with the positive direction of the x -axis. 2
2. A curve has equation $y = x^4 - 2x^3 + 5$.
Find the equation of the tangent to this curve at the point where $x = 2$. 4
3. Functions f and g are defined on suitable domains by
- $$f(x) = x(x - 1) + q \text{ and } g(x) = x + 3.$$
- (a) Find an expression for $f(g(x))$. 2
- (b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots. 4

[Turn over

4. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points $(8, 0, 0)$ and $(11, 4, 2)$ respectively.

- (a) State the coordinates of C and D. 2
- (b) Determine the components of \vec{CB} and \vec{CD} . 2
- (c) Find the size of the angle BCD. 5
5. Given that $\int_4^t (3x + 4)^{-\frac{1}{2}} dx = 2$, find the value of t . 5
6. Solve the equation
- $$\sin x - 2 \cos 2x = 1 \quad \text{for } 0 \leq x < 2\pi. \quad \text{5}$$

7. Land enclosed between a path and a railway line is being developed for housing. This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents 300 m^2 of land.

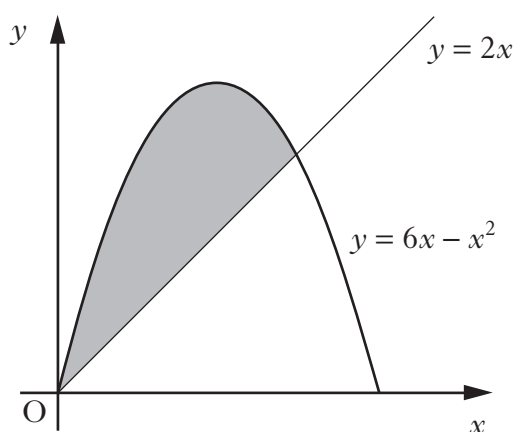


Diagram 1

- (a) Calculate the area of land being developed. 5
- (b) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

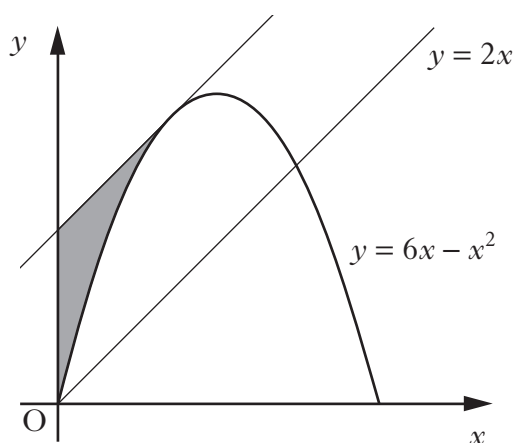


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park. 5

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8. Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p .

5

9. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

- (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion.

3

- (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$.

2

- (c) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.

3

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