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National
Qualifications
2017

X747/76/11

Mathematics
Paper 1
(Non-Calculator)

FRIDAY, 5 MAY

INSTRUCTIONS TO CANDIDATES

Candidates should enter their surname, forename(s), date of birth, Scottish candidate number and the name and Level of the subject at the top of their first answer sheet.

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly on your answer sheet.

Questions marked with an asterisk differ in some respects from those in the printed paper.

Marks are shown in square brackets at the end of each question or part question.

An OW in the margin indicates a new question.

A separate formulae sheet is provided.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin\,(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos\,(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

ATTEMPT ALL QUESTIONS

Total marks — 60

1. Functions f and g are defined on suitable domains by $f(x) = 5x$ and $g(x) = 2\cos x$.

(a) Evaluate $f(g(0))$. **[1 mark]**

(b) Find an expression for $g(f(x))$. **[2 marks]**

2. The point P(–2, 1) lies on the circle $x^2 + y^2 - 8x - 6y - 15 = 0$. Find the equation of the tangent to the circle at P. **[4 marks]**

3. Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$. **[2 marks]**

4. Find the value of k for which the equation $x^2 + 4x + (k-5) = 0$ has equal roots. **[3 marks]**

* **5.** Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

(a) Evaluate $\mathbf{u}.\mathbf{v}$. **[1 mark]**

(b) Refer to the diagram for Question 5 (b). Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}| = \sqrt{3}$. Calculate $\mathbf{u}.\mathbf{w}$. **[3 marks]**

6. A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$. Determine an expression for $h^{-1}(x)$. **[3 marks]**

7. A(–3, 5), B(7, 9) and C(2, 11) are the vertices of a triangle. Find the equation of the median through C. **[3 marks]**

8. Calculate the rate of change of $d(t)=\frac{1}{2t}$, $t \neq 0$, when $t = 5$. **[3 marks]**

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **[2 marks]**

(b) (i) Explain why this sequence approaches a limit as $n \rightarrow \infty$. **[1 mark]**

(ii) Calculate this limit. **[2 marks]**

***10.** Refer to the diagram for Question 10 (a). Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.

(a) Calculate the shaded area which is the area between the two curves between $x = 0$ and $x = 2$. **[5 marks]**

Refer to the diagram for Question 10 (b). The line passing through the points of intersection of the curves has equation $y = 1 - x$.

(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **[4 marks]**

11. A and B are the points (–7, 2) and (5, a). AB is parallel to the line with equation $3y - 2x = 4$. Determine the value of a. **[3 marks]**

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **[3 marks]**

13. Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}}dx,\ x<\frac{5}{4}$. **[4 marks]**

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$,where $k>0$ and $0<a<360$. **[4 marks]**

* (b) For the graph with equation $y=\sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$, describe the shape of the graph **[3 marks]**

***15.** Refer to the two diagrams for Question 15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y=f(x)$. The turning point is $(2,3)$.

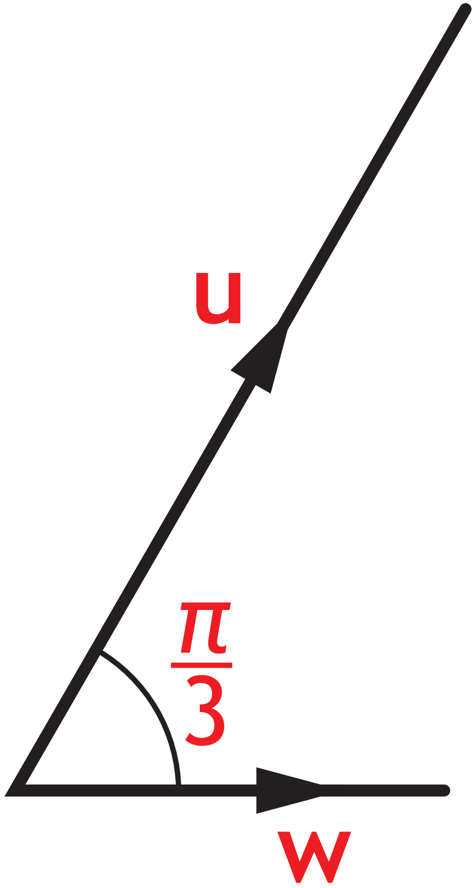
Diagram 2 shows part of the graph with equation $y=h(x)$. The turning point is $(7,6)$.

(a) Given that $h(x)=f(x+a)+b$. Write down the values of a and b. **[2 marks]**

(b) It is known that $\int_1^3 f(x)\,dx=4$. Determine the value of $\int_6^8 h(x)\,dx$. **[1 mark]**

(c) Given $f'(1)=6$, state the value of $h'(8)$. **[1 mark]**

[END OF QUESTION PAPER]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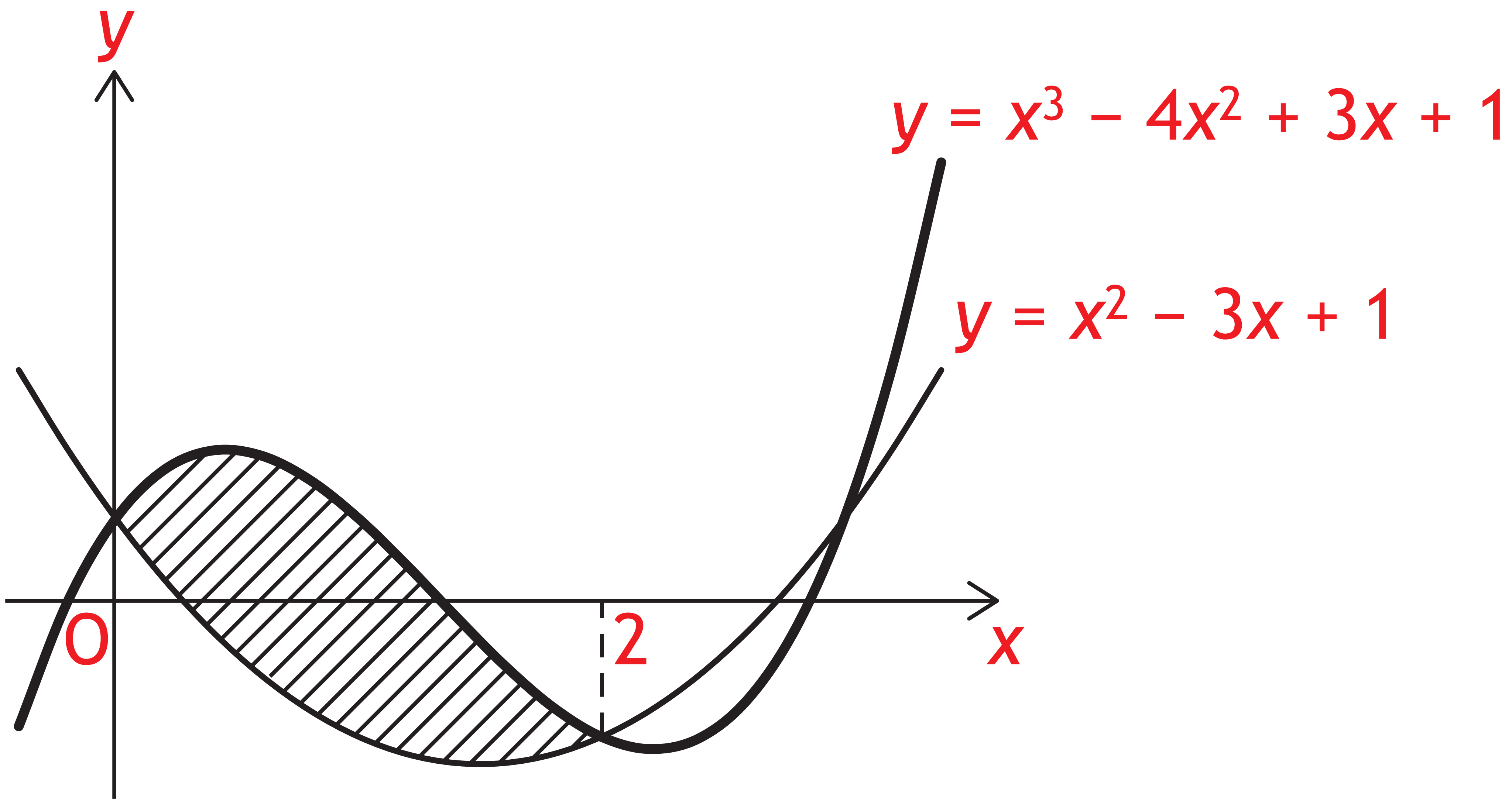
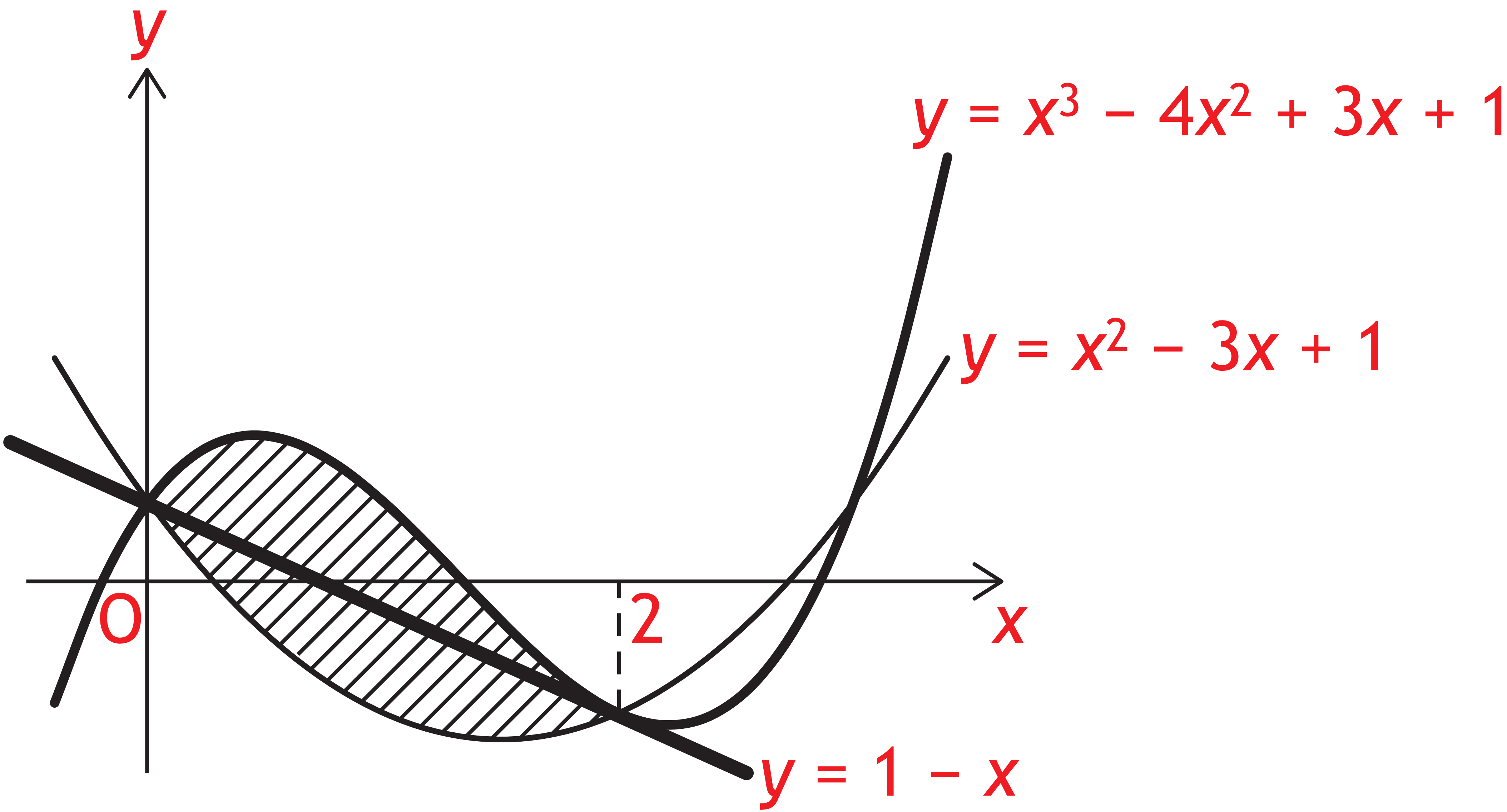
Q10a



Q10b



X747/76/11 Mathematics Higher 2017 Paper 1

Q15

Diagram 1

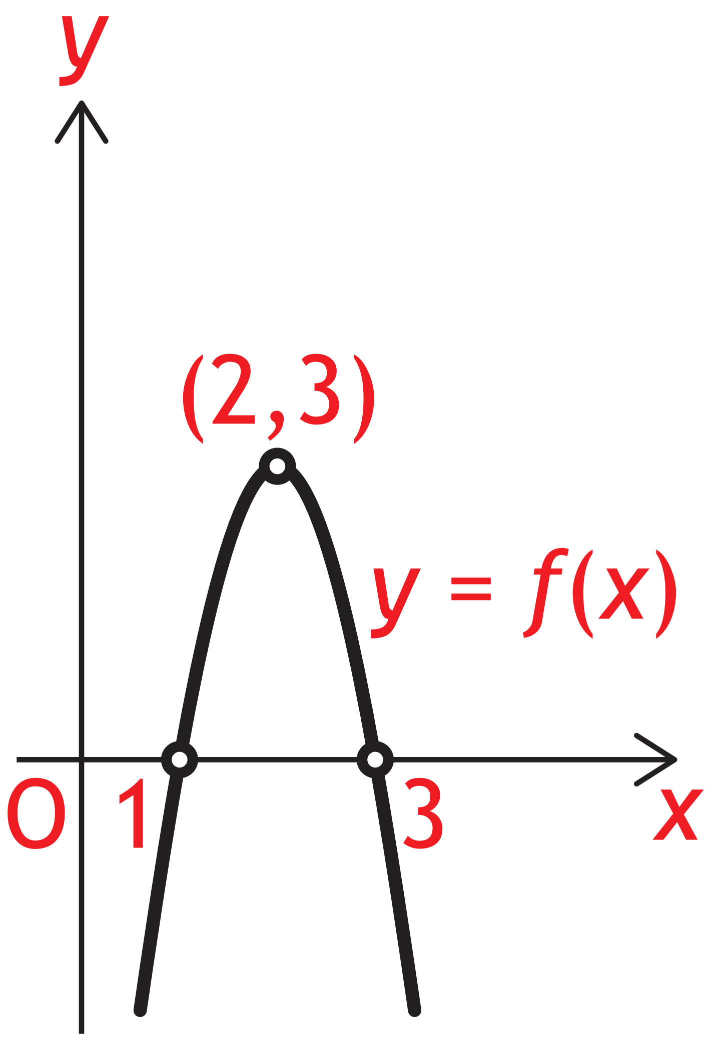
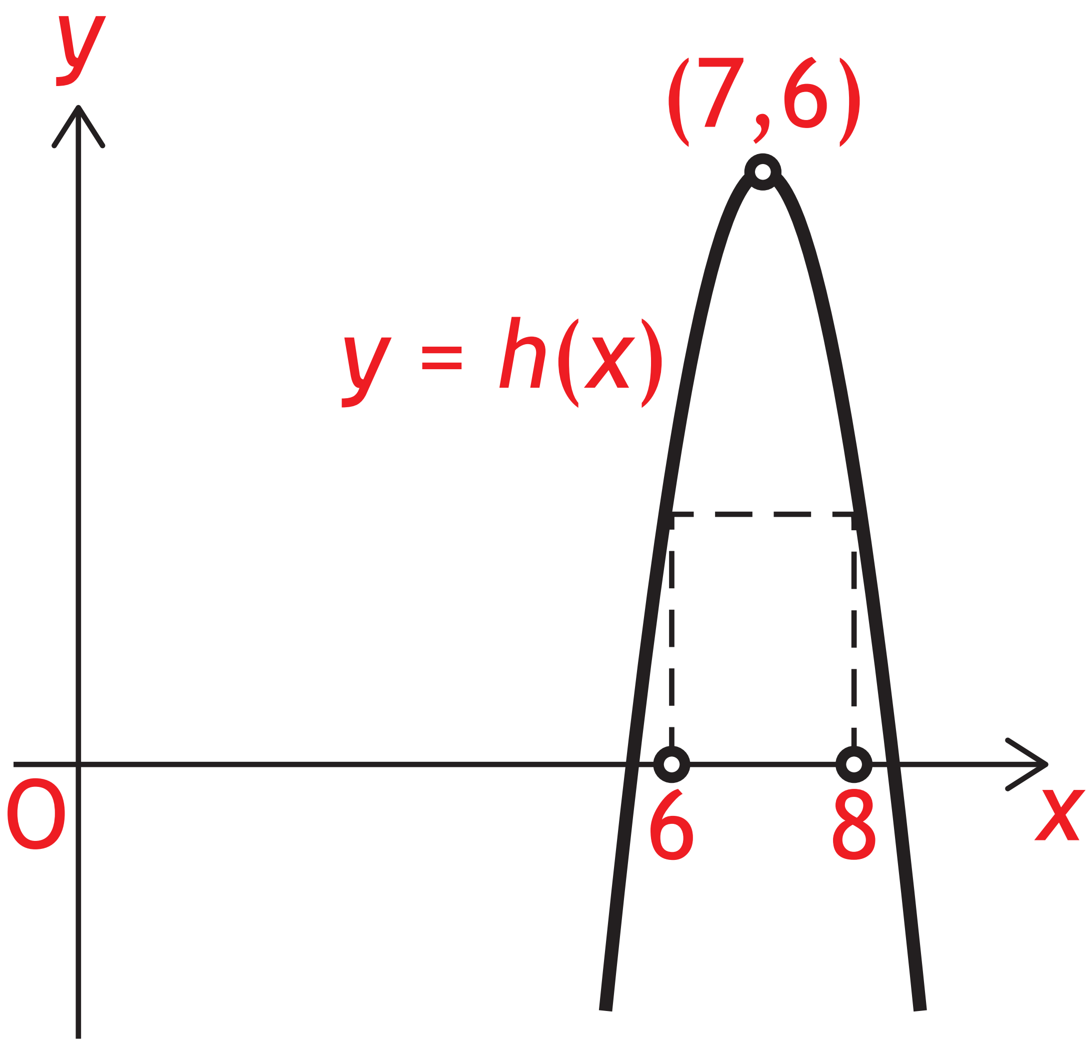


Diagram 2



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Mathematics
Paper 2

FRIDAY, 5 MAY

INSTRUCTIONS TO CANDIDATES

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Attempt ALL questions

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* 1. Refer to the diagram for Question 1. It shows triangle ABC. The coordinates of B are (3,0) and the coordinates of C are (9,–2). The broken line is the perpendicular bisector of BC.

(a) Find the equation of the perpendicular bisector of BC. **[4 marks]**

(b) The line AB makes an angle of 45° with the positive direction of the x-axis.

Find the equation of AB. **[2 marks]**

(c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. **[2 marks]**

2. (a) Show that $(x-1)$ is a factor of $f(x)=2x^3-5x^2+x+2$. **[2 marks]**

(b) Hence, or otherwise, solve $f(x)=0$. **[3 marks]**

* **3.** Refer to the diagram for Question 3. The line $y=3x$ intersects the circle with equation $(x-2)^2+(y-1)^2=25$.

Find the coordinates of the points of intersection. **[5 marks]**

4. (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. **[3 marks]**

(b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$. **[2 marks]**

(c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x. **[2 marks]**

* **5.** Refer to the diagram for Question 5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.

(a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. **[2 marks]**

The point S divides QR in the ratio 1:2.

(b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$. **[2 marks]**

(c) Hence, find the size of angle QPS. **[5 marks]**

6. Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$. **[5 marks]**

7. (a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. **[4 marks]**

(b) Hence, determine the greatest and least values of y in the interval $1 \le x \le 9$. **[3 marks]**

8. Sequences may be generated by recurrence relations of the form
$u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$. **[2 marks]**

(b) Determine the range of values of k for which $u_2 < u_0$. **[4 marks]**

*** 9.** Refer to the diagram for Question 9. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.

Find the values of k and n. **[5 marks]**

***10.** (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear. **[3 marks]**

Refer to the diagram for Question 10 (b). Three circles with centres A, B and C are drawn inside a circle with centre D as shown.

The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

$r_A = \sqrt{10}$

$r_B = 2r_A$

$r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **[4 marks]**

11. (a) Show that $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$. **[3 marks]**

(b) Hence, differentiate $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \dfrac{\pi}{2}$. **[3 marks]**

[END OF QUESTION PAPER]

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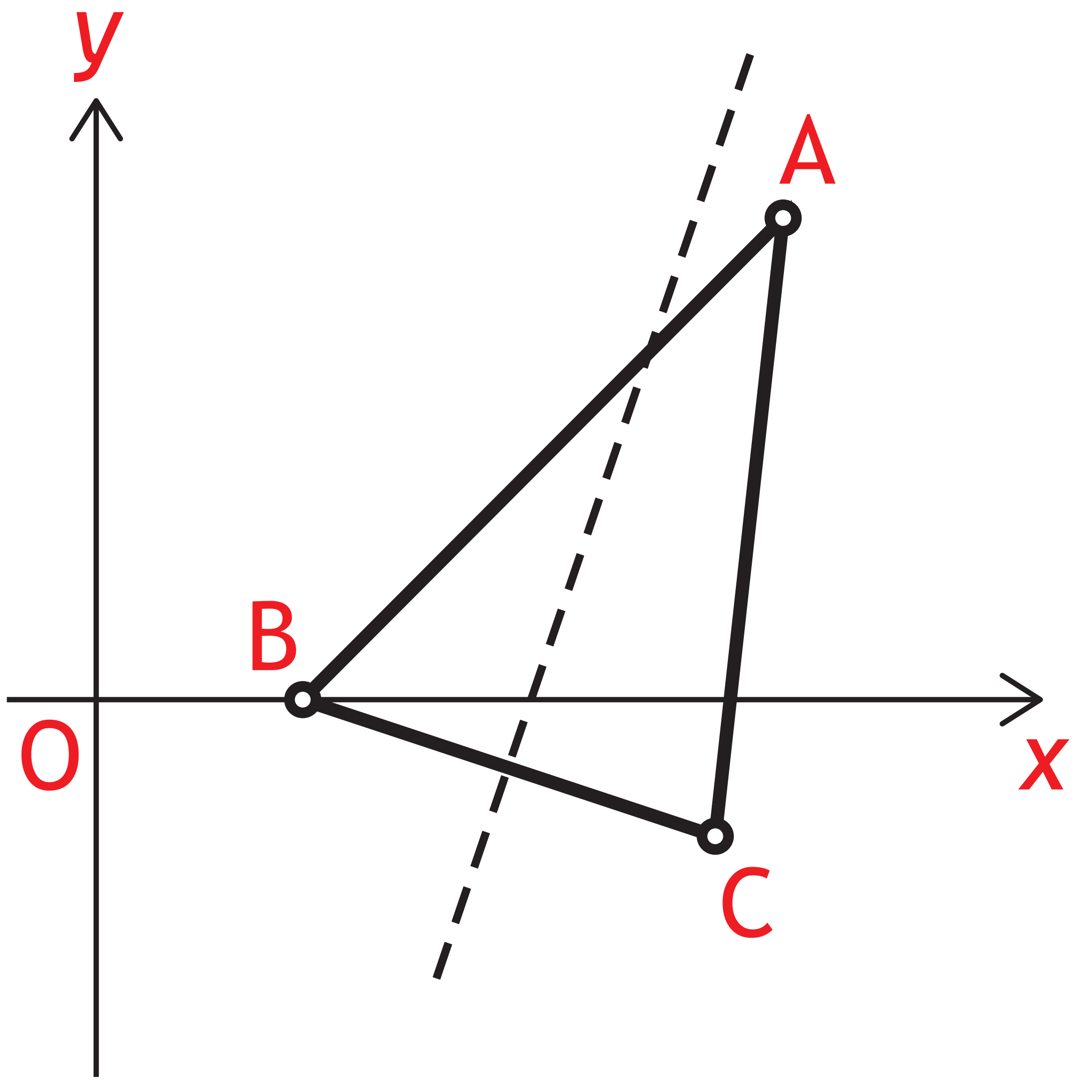
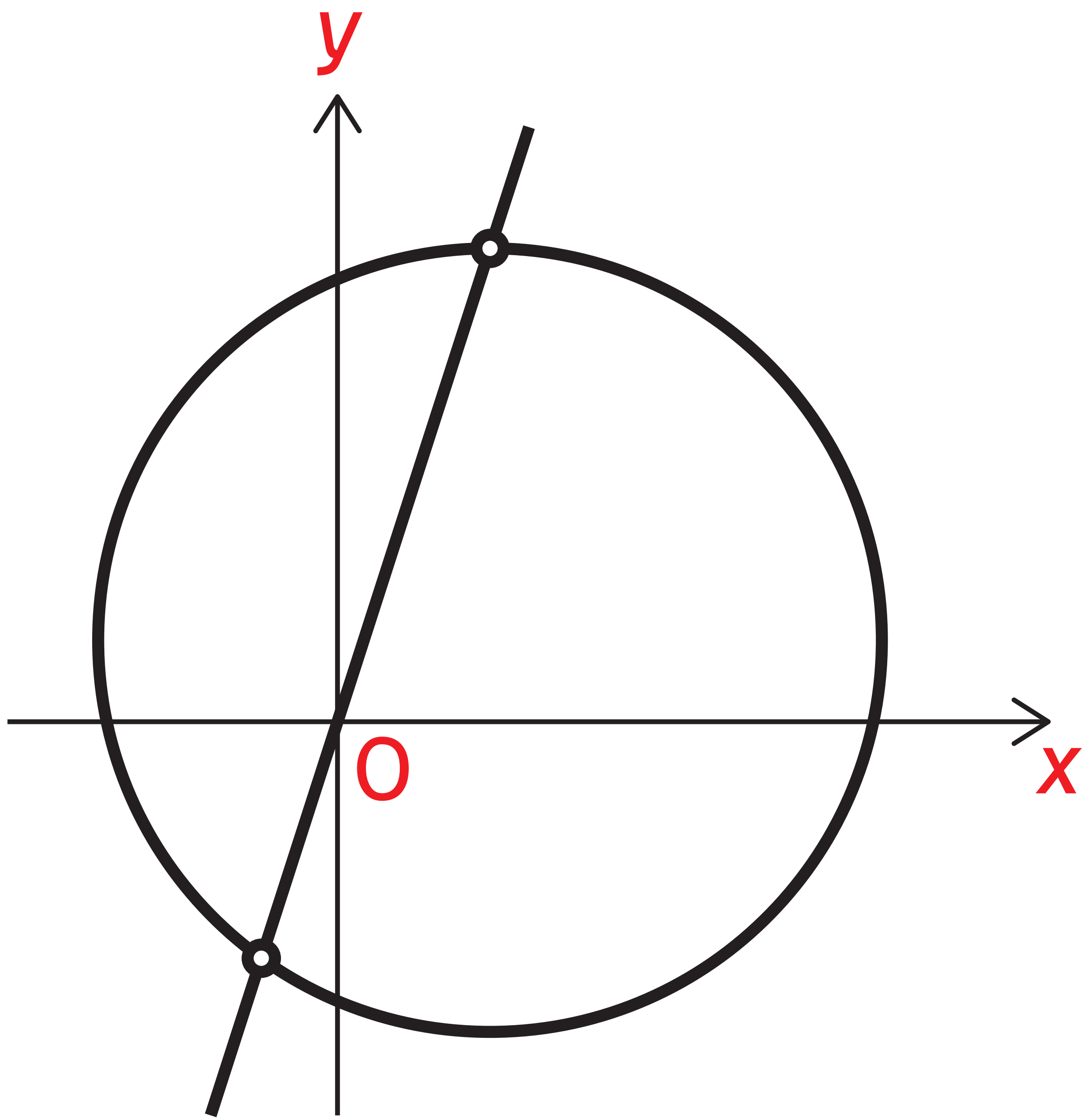
Q1



Q3



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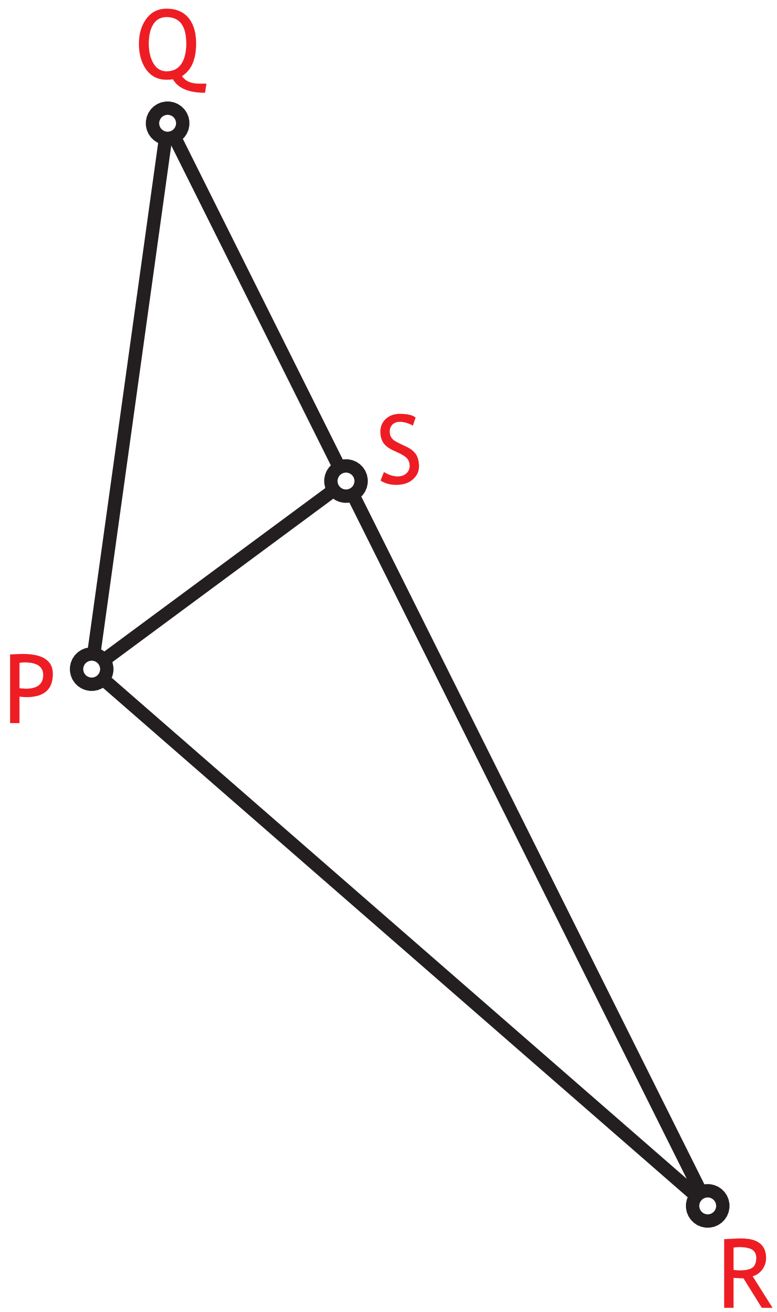
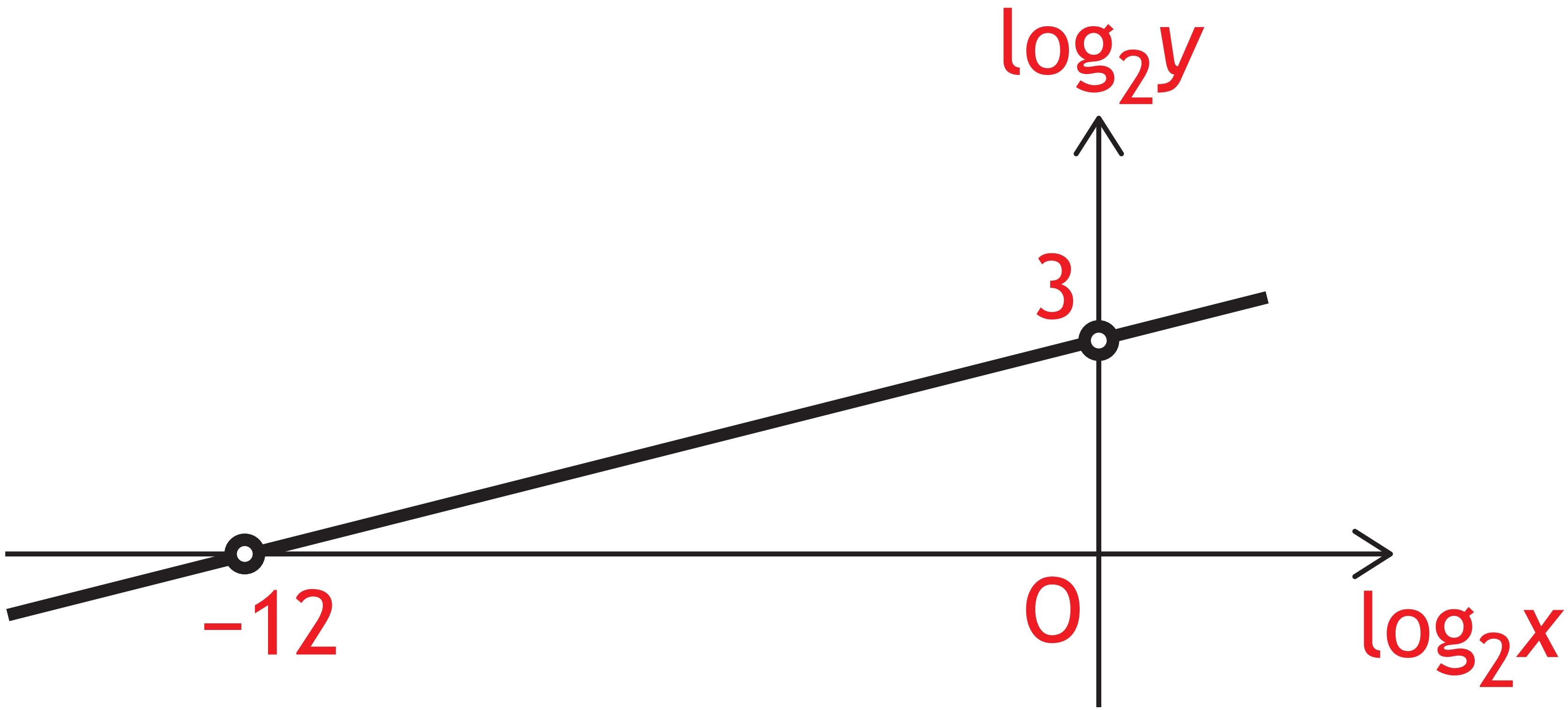
Q5



Q9



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Q10