Statistics

Advanced Higher

Statistical Formulae and Tables

For use in National Qualification Courses.

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Contents

STATISTICAL FORMULAE

Probability Distributions

	Discrete			Continuous	
Distribution	Uniform	Binomial	Poisson	Uniform	Normal
Parameters	$\mathrm{U}(k)$	$\mathrm{B}(n, p)$	$\mathrm{Po}(\lambda)$	$\mathrm{U}(a,b)$	$\mathrm{N}(\mu, \sigma^2)$
pf/pdf	$\frac{1}{k}$	${}^nC_x\, p^x (1-p)^{n-x}$	$\frac{e^{-\lambda}\lambda^x}{x!}$	$\frac{1}{b-a}$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
Mean	$\frac{k+1}{2}$	np	λ	$\frac{a+b}{2}$	μ
Variance	$\frac{k^2-1}{12}$	$np(1-p)$	λ	$\frac{(b-a)^2}{12}$	σ^2

Western Electric Company Rules

To determine when a process may be out of statistical control we may use the Western Electric Company Rules:

- Any single data point falls outside a 3σ limit
- Two out of three consecutive points fall beyond the same 2σ limit
- Four out of five consecutive points fall beyond the same 1σ limit
- Eight consecutive points fall on the same side of the centre line

Sums of Squares and Products

$$S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}$$

$$S_{yy} = \sum(y_i - \bar{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}$$

$$S_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

Sample Standard Deviation

$$s = \sqrt{\frac{S_{xx}}{n-1}}$$

Correlation

Product moment correlation coefficient $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

Linear Regression

The linear model is $Y_i = \alpha + \beta x_i + \varepsilon_i$ where ε_i are independent, $\mathrm{E}(\varepsilon_i) = 0$ and $\mathrm{V}(\varepsilon_i) = \sigma^2$.

Least squares estimates, a and b, for α and β respectively are given by

$b = \dfrac{S_{xy}}{S_{xx}}$ and $a = \overline{y} - b\overline{x}$.

The sum of squared residuals is given by $SSR = S_{yy} - \dfrac{S_{xy}^2}{S_{xx}}$

and an estimate for σ^2 is $s^2 = \dfrac{SSR}{n-2}$.

If additionally $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$ then

a $100(1-\alpha)\%$ prediction interval for $Y_i \mid x_i$ is given by $\hat{Y}_i \pm t_{n-2,1-\alpha/2}\, s\sqrt{1 + \dfrac{1}{n} + \dfrac{(x_i - \overline{x})^2}{S_{xx}}}$ and

a $100(1-\alpha)\%$ confidence interval for $\mathrm{E}(Y_i \mid x_i)$ is given by $\hat{Y}_i \pm t_{n-2,1-\alpha/2}\, s\sqrt{\dfrac{1}{n} + \dfrac{(x_i - \overline{x})^2}{S_{xx}}}$.

Hypothesis Test Statistics

z-test for a difference in population means:

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim \mathrm{N}(0,1)$$

t-test for a difference in population means:

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{s\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \sim t_{n_1+n_2-2} \text{ where } s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

z-test for a difference in population proportions:

$$\frac{p_1 - p_2}{\sqrt{pq\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} \sim \mathrm{N}(0,1) \text{ where } p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

Chi-squared test for goodness-of-fit and contingency tables

$$\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_v^2$$

where at least 80% of the E_i should be at least 5 and none should be less than 1.

To test the null hypothesis that the population product moment correlation coefficient $\rho = 0$ use the test statistic $t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

and to test the null hypothesis that the slope parameter $\beta = 0$ use the test statistic $t = \dfrac{b\sqrt{S_{xx}}}{s}$

TABLE 1: BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $F(x) = \mathrm{P}(X \leq x)$ where X has the binomial distribution $\mathrm{B}(n, p)$.
Omitted entries to the left and right of tabulated values are 1·0000 and 0·0000 respectively, to four decimal places.

p		0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
$n = 4$	$x = 0$	0·8145	0·6561	0·5220	0·4096	0·3164	0·2401	0·1785	0·1296	0·0915	0·0625
	1	0·9860	0·9477	0·8905	0·8192	0·7383	0·6517	0·5630	0·4752	0·3910	0·3125
	2	0·9995	0·9963	0·9880	0·9728	0·9492	0·9163	0·8735	0·8208	0·7585	0·6875
	3		0·9999	0·9995	0·9984	0·9961	0·9919	0·9850	0·9744	0·9590	0·9375
$n = 6$	$x = 0$	0·7351	0·5314	0·3771	0·2621	0·1780	0·1176	0·0754	0·0467	0·0277	0·0156
	1	0·9672	0·8857	0·7765	0·6554	0·5339	0·4202	0·3191	0·2333	0·1636	0·1094
	2	0·9978	0·9842	0·9527	0·9011	0·8306	0·7443	0·6471	0·5443	0·4415	0·3438
	3	0·9999	0·9987	0·9941	0·9830	0·9624	0·9295	0·8826	0·8208	0·7447	0·6563
	4		0·9999	0·9996	0·9984	0·9954	0·9891	0·9777	0·9590	0·9308	0·8906
	5				0·9999	0·9998	0·9993	0·9982	0·9959	0·9917	0·9844
$n = 8$	$x = 0$	0·6634	0·4305	0·2725	0·1678	0·1001	0·0576	0·0319	0·0168	0·0084	0·0039
	1	0·9428	0·8131	0·6572	0·5033	0·3671	0·2553	0·1691	0·1064	0·0632	0·0352
	2	0·9942	0·9619	0·8948	0·7969	0·6785	0·5518	0·4278	0·3154	0·2201	0·1445
	3	0·9996	0·9950	0·9786	0·9437	0·8862	0·8059	0·7064	0·5941	0·4770	0·3633
	4		0·9996	0·9971	0·9896	0·9727	0·9420	0·8939	0·8263	0·7396	0·6367
	5			0·9998	0·9988	0·9958	0·9887	0·9747	0·9502	0·9115	0·8555
	6				0·9999	0·9996	0·9987	0·9964	0·9915	0·9819	0·9648
	7						0·9999	0·9998	0·9993	0·9983	0·9961
$n = 10$	$x = 0$	0·5987	0·3487	0·1969	0·1074	0·0563	0·0282	0·0135	0·0060	0·0025	0·0010
	1	0·9139	0·7361	0·5443	0·3758	0·2440	0·1493	0·0860	0·0464	0·0233	0·0107
	2	0·9885	0·9298	0·8202	0·6778	0·5256	0·3828	0·2616	0·1673	0·0996	0·0547
	3	0·9990	0·9872	0·9500	0·8791	0·7759	0·6496	0·5138	0·3823	0·2660	0·1719
	4	0·9999	0·9984	0·9901	0·9672	0·9219	0·8497	0·7515	0·6331	0·5044	0·3770
	5		0·9999	0·9986	0·9936	0·9803	0·9527	0·9051	0·8338	0·7384	0·6230
	6			0·9999	0·9991	0·9965	0·9894	0·9740	0·9452	0·8980	0·8281
	7				0·9999	0·9996	0·9984	0·9952	0·9877	0·9726	0·9453
	8						0·9999	0·9995	0·9983	0·9955	0·9893
	9								0·9999	0·9997	0·9990

TABLE 1: BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION (continued)

p	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
$n = 12$ $x = 0$	0·5404	0·2824	0·1422	0·0687	0·0317	0·0138	0·0057	0·0022	0·0008	0·0002
1	0·8816	0·6590	0·4435	0·2749	0·1584	0·0850	0·0424	0·0196	0·0083	0·0032
2	0·9804	0·8891	0·7358	0·5583	0·3907	0·2528	0·1513	0·0834	0·0421	0·0193
3	0·9978	0·9744	0·9078	0·7946	0·6488	0·4925	0·3467	0·2253	0·1345	0·0730
4	0·9998	0·9957	0·9761	0·9274	0·8424	0·7237	0·5833	0·4382	0·3044	0·1938
5		0·9995	0·9954	0·9806	0·9456	0·8822	0·7873	0·6652	0·5269	0·3872
6		0·9999	0·9993	0·9961	0·9857	0·9614	0·9154	0·8418	0·7393	0·6128
7			0·9999	0·9994	0·9972	0·9905	0·9745	0·9427	0·8883	0·8062
8				0·9999	0·9996	0·9983	0·9944	0·9847	0·9644	0·9270
9						0·9998	0·9992	0·9972	0·9921	0·9807
10							0·9999	0·9997	0·9989	0·9968
11									0·9999	0·9998
$n = 14$ $x = 0$	0·4877	0·2288	0·1028	0·0440	0·0178	0·0068	0·0024	0·0008	0·0002	0·0001
1	0·8470	0·5846	0·3567	0·1979	0·1010	0·0475	0·0205	0·0081	0·0029	0·0009
2	0·9699	0·8416	0·6479	0·4481	0·2811	0·1608	0·0839	0·0398	0·0170	0·0065
3	0·9958	0·9559	0·8535	0·6982	0·5213	0·3552	0·2205	0·1243	0·0632	0·0287
4	0·9996	0·9908	0·9533	0·8702	0·7415	0·5842	0·4227	0·2793	0·1672	0·0898
5		0·9985	0·9885	0·9561	0·8883	0·7805	0·6405	0·4859	0·3373	0·2120
6		0·9998	0·9978	0·9884	0·9617	0·9067	0·8164	0·6925	0·5461	0·3953
7			0·9997	0·9976	0·9897	0·9685	0·9247	0·8499	0·7414	0·6047
8				0·9996	0·9978	0·9917	0·9757	0·9417	0·8811	0·7880
9					0·9997	0·9983	0·9940	0·9825	0·9574	0·9102
10						0·9998	0·9989	0·9961	0·9886	0·9713
11							0·9999	0·9994	0·9978	0·9935
12								0·9999	0·9997	0·9991
13										0·9999
$n = 16$ $x = 0$	0·4401	0·1853	0·0743	0·0281	0·0100	0·0033	0·0010	0·0003	0·0001	
1	0·8108	0·5147	0·2839	0·1407	0·0635	0·0261	0·0098	0·0033	0·0010	0·0003
2	0·9571	0·7892	0·5614	0·3518	0·1971	0·0994	0·0451	0·0183	0·0066	0·0021
3	0·9930	0·9316	0·7899	0·5981	0·4050	0·2459	0·1339	0·0651	0·0281	0·0106
4	0·9991	0·9830	0·9209	0·7982	0·6302	0·4499	0·2892	0·1666	0·0853	0·0384
5	0·9999	0·9967	0·9765	0·9183	0·8103	0·6598	0·4900	0·3288	0·1976	0·1051
6		0·9995	0·9944	0·9733	0·9204	0·8247	0·6881	0·5272	0·3660	0·2272
7		0·9999	0·9989	0·9930	0·9729	0·9256	0·8406	0·7161	0·5629	0·4018
8			0·9998	0·9985	0·9925	0·9743	0·9329	0·8577	0·7441	0·5982
9				0·9998	0·9984	0·9929	0·9771	0·9417	0·8759	0·7728
10					0·9997	0·9984	0·9938	0·9809	0·9514	0·8949
11						0·9997	0·9987	0·9951	0·9851	0·9616
12							0·9998	0·9991	0·9965	0·9894
13								0·9999	0·9994	0·9979
14									0·9999	0·9997

TABLE 1: BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION (continued)

p	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
$n = 18$ $x = 0$	0·3972	0·1501	0·0536	0·0180	0·0056	0·0016	0·0004	0·0001		
1	0·7735	0·4503	0·2241	0·0991	0·0395	0·0142	0·0046	0·0013	0·0003	0·0001
2	0·9419	0·7338	0·4797	0·2713	0·1353	0·0600	0·0236	0·0082	0·0025	0·0007
3	0·9891	0·9018	0·7202	0·5010	0·3057	0·1646	0·0783	0·0328	0·0120	0·0038
4	0·9985	0·9718	0·8794	0·7164	0·5187	0·3327	0·1886	0·0942	0·0411	0·0154
5	0·9998	0·9936	0·9581	0·8671	0·7175	0·5344	0·3550	0·2088	0·1077	0·0481
6		0·9988	0·9882	0·9487	0·8610	0·7217	0·5491	0·3743	0·2258	0·1189
7		0·9998	0·9973	0·9837	0·9431	0·8593	0·7283	0·5634	0·3915	0·2403
8			0·9995	0·9957	0·9807	0·9404	0·8609	0·7368	0·5778	0·4073
9			0·9999	0·9991	0·9946	0·9790	0·9403	0·8653	0·7473	0·5927
10				0·9998	0·9988	0·9939	0·9788	0·9424	0·8720	0·7597
11					0·9998	0·9986	0·9938	0·9797	0·9463	0·8811
12						0·9997	0·9986	0·9942	0·9817	0·9519
13							0·9997	0·9987	0·9951	0·9846
14								0·9998	0·9990	0·9962
15									0·9999	0·9993
16										0·9999
$n = 20$ $x = 0$	0·3585	0·1216	0·0388	0·0115	0·0032	0·0008	0·0002			
1	0·7358	0·3917	0·1756	0·0692	0·0243	0·0076	0·0021	0·0005	0·0001	
2	0·9245	0·6769	0·4049	0·2061	0·0913	0·0355	0·0121	0·0036	0·0009	0·0002
3	0·9841	0·8670	0·6477	0·4114	0·2252	0·1071	0·0444	0·0160	0·0049	0·0013
4	0·9974	0·9568	0·8298	0·6296	0·4148	0·2375	0·1182	0·0510	0·0189	0·0059
5	0·9997	0·9887	0·9327	0·8042	0·6172	0·4164	0·2454	0·1256	0·0553	0·0207
6		0·9976	0·9781	0·9133	0·7858	0·6080	0·4166	0·2500	0·1299	0·0577
7		0·9996	0·9941	0·9679	0·8982	0·7723	0·6010	0·4159	0·2520	0·1316
8		0·9999	0·9987	0·9900	0·9591	0·8867	0·7624	0·5956	0·4143	0·2517
9			0·9998	0·9974	0·9861	0·9520	0·8782	0·7553	0·5914	0·4119
10				0·9994	0·9961	0·9829	0·9468	0·8725	0·7507	0·5881
11				0·9999	0·9991	0·9949	0·9804	0·9435	0·8692	0·7483
12					0·9998	0·9987	0·9940	0·9790	0·9420	0·8684
13						0·9997	0·9985	0·9935	0·9786	0·9423
14							0·9997	0·9984	0·9936	0·9793
15								0·9997	0·9985	0·9941
16									0·9997	0·9987
17										0·9998

TABLE 2: POISSON CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $F(x) = \mathrm{P}(X \leq x)$ where X has the Poisson distribution $\mathrm{Po}(\lambda)$.

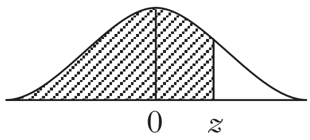
Omitted entries to the left and right of tabulated values are 1·0000 and 0·0000 respectively, to four decimal places.

λ	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	4·5	5·0
$x = 0$	0·6065	0·3679	0·2231	0·1353	0·0821	0·0498	0·0302	0·0183	0·0111	0·0067
1	0·9098	0·7358	0·5578	0·4060	0·2873	0·1991	0·1359	0·0916	0·0611	0·0404
2	0·9856	0·9197	0·8088	0·6767	0·5438	0·4232	0·3208	0·2381	0·1736	0·1247
3	0·9982	0·9810	0·9344	0·8571	0·7576	0·6472	0·5366	0·4335	0·3423	0·2650
4	0·9998	0·9963	0·9814	0·9473	0·8912	0·8153	0·7254	0·6288	0·5321	0·4405
5		0·9994	0·9955	0·9834	0·9580	0·9161	0·8576	0·7851	0·7029	0·6160
6		0·9999	0·9991	0·9955	0·9858	0·9665	0·9347	0·8893	0·8311	0·7622
7			0·9998	0·9989	0·9958	0·9881	0·9733	0·9489	0·9134	0·8666
8				0·9998	0·9989	0·9962	0·9901	0·9786	0·9597	0·9319
9					0·9997	0·9989	0·9967	0·9919	0·9829	0·9682
10					0·9999	0·9997	0·9990	0·9972	0·9933	0·9863
11						0·9999	0·9997	0·9991	0·9976	0·9945
12							0·9999	0·9997	0·9992	0·9980
13								0·9999	0·9997	0·9993
14									0·9999	0·9998
15										0·9999

λ	5·5	6·0	6·5	7·0	7·5	8·0	8·5	9·0	9·5	10·0
$x = 0$	0·0041	0·0025	0·0015	0·0009	0·0006	0·0003	0·0002	0·0001	0·0001	
1	0·0266	0·0174	0·0113	0·0073	0·0047	0·0030	0·0019	0·0012	0·0008	0·0005
2	0·0884	0·0620	0·0430	0·0296	0·0203	0·0138	0·0093	0·0062	0·0042	0·0028
3	0·2017	0·1512	0·1118	0·0818	0·0591	0·0424	0·0301	0·0212	0·0149	0·0103
4	0·3575	0·2851	0·2237	0·1730	0·1321	0·0996	0·0744	0·0550	0·0403	0·0293
5	0·5289	0·4457	0·3690	0·3007	0·2414	0·1912	0·1496	0·1157	0·0885	0·0671
6	0·6860	0·6063	0·5265	0·4497	0·3782	0·3134	0·2562	0·2068	0·1649	0·1301
7	0·8095	0·7440	0·6728	0·5987	0·5246	0·4530	0·3856	0·3239	0·2687	0·2202
8	0·8944	0·8472	0·7916	0·7291	0·6620	0·5925	0·5231	0·4557	0·3918	0·3328
9	0·9462	0·9161	0·8774	0·8305	0·7764	0·7166	0·6530	0·5874	0·5218	0·4579
10	0·9747	0·9574	0·9332	0·9015	0·8622	0·8159	0·7634	0·7060	0·6453	0·5830
11	0·9890	0·9799	0·9661	0·9467	0·9208	0·8881	0·8487	0·8030	0·7520	0·6968
12	0·9955	0·9912	0·9840	0·9730	0·9573	0·9362	0·9091	0·8758	0·8364	0·7916
13	0·9983	0·9964	0·9929	0·9872	0·9784	0·9658	0·9486	0·9261	0·8981	0·8645
14	0·9994	0·9986	0·9970	0·9943	0·9897	0·9827	0·9726	0·9585	0·9400	0·9165
15	0·9998	0·9995	0·9988	0·9976	0·9954	0·9918	0·9862	0·9780	0·9665	0·9513
16	0·9999	0·9998	0·9996	0·9990	0·9980	0·9963	0·9934	0·9889	0·9823	0·9730
17		0·9999	0·9998	0·9996	0·9992	0·9984	0·9970	0·9947	0·9911	0·9857
18			0·9999	0·9999	0·9997	0·9993	0·9987	0·9976	0·9957	0·9928
19					0·9999	0·9997	0·9995	0·9989	0·9980	0·9965
20						0·9999	0·9998	0·9996	0·9991	0·9984
21							0·9999	0·9998	0·9996	0·9993
22								0·9999	0·9999	0·9997
23									0·9999	0·9999

TABLE 3: STANDARD NORMAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(Z \leq z)$ where Z has the standard normal distribution $N(0, 1)$.



z	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
0·0	0·5000	0·5040	0·5080	0·5120	0·5160	0·5199	0·5239	0·5279	0·5319	0·5359
0·1	0·5398	0·5438	0·5478	0·5517	0·5557	0·5596	0·5636	0·5675	0·5714	0·5753
0·2	0·5793	0·5832	0·5871	0·5910	0·5948	0·5987	0·6026	0·6064	0·6103	0·6141
0·3	0·6179	0·6217	0·6255	0·6293	0·6331	0·6368	0·6406	0·6443	0·6480	0·6517
0·4	0·6554	0·6591	0·6628	0·6664	0·6700	0·6736	0·6772	0·6808	0·6844	0·6879
0·5	0·6915	0·6950	0·6985	0·7019	0·7054	0·7088	0·7123	0·7157	0·7190	0·7224
0·6	0·7257	0·7291	0·7324	0·7357	0·7389	0·7422	0·7454	0·7486	0·7517	0·7549
0·7	0·7580	0·7611	0·7642	0·7673	0·7704	0·7734	0·7764	0·7794	0·7823	0·7852
0·8	0·7881	0·7910	0·7939	0·7967	0·7995	0·8023	0·8051	0·8078	0·8106	0·8133
0·9	0·8159	0·8186	0·8212	0·8238	0·8264	0·8289	0·8315	0·8340	0·8365	0·8389
1·0	0·8413	0·8438	0·8461	0·8485	0·8508	0·8531	0·8554	0·8577	0·8599	0·8621
1·1	0·8643	0·8665	0·8686	0·8708	0·8729	0·8749	0·8770	0·8790	0·8810	0·8830
1·2	0·8849	0·8869	0·8888	0·8907	0·8925	0·8944	0·8962	0·8980	0·8997	0·9015
1·3	0·9032	0·9049	0·9066	0·9082	0·9099	0·9115	0·9131	0·9147	0·9162	0·9177
1·4	0·9192	0·9207	0·9222	0·9236	0·9251	0·9265	0·9279	0·9292	0·9306	0·9319
1·5	0·9332	0·9345	0·9357	0·9370	0·9382	0·9394	0·9406	0·9418	0·9429	0·9441
1·6	0·9452	0·9463	0·9474	0·9484	0·9495	0·9505	0·9515	0·9525	0·9535	0·9545
1·7	0·9554	0·9564	0·9573	0·9582	0·9591	0·9599	0·9608	0·9616	0·9625	0·9633
1·8	0·9641	0·9649	0·9656	0·9664	0·9671	0·9678	0·9686	0·9693	0·9699	0·9706
1·9	0·9713	0·9719	0·9726	0·9732	0·9738	0·9744	0·9750	0·9756	0·9761	0·9767
2·0	0·9772	0·9778	0·9783	0·9788	0·9793	0·9798	0·9803	0·9808	0·9812	0·9817
2·1	0·9821	0·9826	0·9830	0·9834	0·9838	0·9842	0·9846	0·9850	0·9854	0·9857
2·2	0·9861	0·9864	0·9868	0·9871	0·9875	0·9878	0·9881	0·9884	0·9887	0·9890
2·3	0·9893	0·9896	0·9898	0·9901	0·9904	0·9906	0·9909	0·9911	0·9913	0·9916
2·4	0·9918	0·9920	0·9922	0·9925	0·9927	0·9929	0·9931	0·9932	0·9934	0·9936
2·5	0·9938	0·9940	0·9941	0·9943	0·9945	0·9946	0·9948	0·9949	0·9951	0·9952
2·6	0·9953	0·9955	0·9956	0·9957	0·9959	0·9960	0·9961	0·9962	0·9963	0·9964
2·7	0·9965	0·9966	0·9967	0·9968	0·9969	0·9970	0·9971	0·9972	0·9973	0·9974
2·8	0·9974	0·9975	0·9976	0·9977	0·9977	0·9978	0·9979	0·9979	0·9980	0·9981
2·9	0·9981	0·9982	0·9982	0·9983	0·9984	0·9984	0·9985	0·9985	0·9986	0·9986
3·0	0·9987	0·9987	0·9987	0·9988	0·9988	0·9989	0·9989	0·9989	0·9990	0·9990
3·1	0·9990	0·9991	0·9991	0·9991	0·9992	0·9992	0·9992	0·9992	0·9993	0·9993
3·2	0·9993	0·9993	0·9994	0·9994	0·9994	0·9994	0·9994	0·9995	0·9995	0·9995
3·3	0·9995	0·9995	0·9995	0·9996	0·9996	0·9996	0·9996	0·9996	0·9996	0·9997
3·4	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9998
3·5	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998
3·6	0·9998	0·9998	0·9999	0·9999	0·9999	0·9999	0·9999	0·9999	0·9999	0·9999

TABLE 4: PERCENTAGE POINTS OF THE STANDARD NORMAL DISTRIBUTION

The entries in the table are such that for the standard normal distribution $\mathrm{P}(Z > z_p) = p$.

p	z_p
0·500	0·00
0·250	0·67
0·100	1·28
0·050	1·64
0·025	1·96
0·010	2·33
0·005	2·58
0·001	3·09
0·0005	3·29

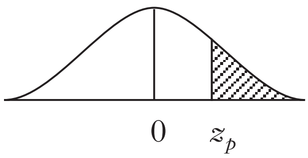
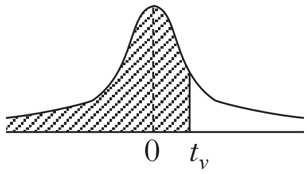


TABLE 5: STUDENT'S t DISTRIBUTION

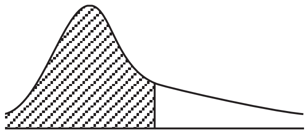
The table entry t_v is such that $P(T \leq t_v) = q$ where T has the Student t distribution with v degrees of freedom.



q	0·900	0·950	0·975	0·990	0·995	0·999	0·9995
$v = 1$	3·078	6·314	12·706	31·821	63·656	318·289	636·578
2	1·886	2·920	4·303	6·965	9·925	22·328	31·600
3	1·638	2·353	3·182	4·541	5·841	10·214	12·924
4	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	1·476	2·015	2·571	3·365	4·032	5·894	6·869
6	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	1·314	1·703	2·052	2·473	2·771	3·421	3·689
28	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	1·311	1·699	2·045	2·462	2·756	3·396	3·660
30	1·310	1·697	2·042	2·457	2·750	3·385	3·646
31	1·309	1·696	2·040	2·453	2·744	3·375	3·633
32	1·309	1·694	2·037	2·449	2·738	3·365	3·622
33	1·308	1·692	2·035	2·445	2·733	3·356	3·611
34	1·307	1·691	2·032	2·441	2·728	3·348	3·601
35	1·306	1·690	2·030	2·438	2·724	3·340	3·591
36	1·306	1·688	2·028	2·434	2·719	3·333	3·582
37	1·305	1·687	2·026	2·431	2·715	3·326	3·574
38	1·304	1·686	2·024	2·429	2·712	3·319	3·566
39	1·304	1·685	2·023	2·426	2·708	3·313	3·558
40	1·303	1·684	2·021	2·423	2·704	3·307	3·551
∞	1·282	1·645	1·960	2·327	2·576	3·091	3·291

TABLE 6: THE CHI-SQUARED DISTRIBUTION

The table entry χ_v^2 is such that $\mathrm{P}\left(X_v^2 \le \chi_v^2\right) = q$ where X_v^2 has the chi-squared distribution with v degrees of freedom.



q	0·900	0·950	0·975	0·990	0·995	0·999	0·9995
$v = 1$	2·706	3·841	5·024	6·635	7·879	10·827	12·115
2	4·605	5·991	7·378	9·210	10·597	13·815	15·201
3	6·251	7·815	9·348	11·345	12·838	16·266	17·731
4	7·779	9·488	11·143	13·277	14·860	18·466	19·998
5	9·236	11·070	12·832	15·086	16·750	20·515	22·106
6	10·645	12·592	14·449	16·812	18·548	22·457	24·102
7	12·017	14·067	16·013	18·475	20·278	24·321	26·018
8	13·362	15·507	17·535	20·090	21·955	26·124	27·867
9	14·684	16·919	19·023	21·666	23·589	27·877	29·667
10	15·987	18·307	20·483	23·209	25·188	29·588	31·419
11	17·275	19·675	21·920	24·725	26·757	31·264	33·138
12	18·549	21·026	23·337	26·217	28·300	32·909	34·821
13	19·812	22·362	24·736	27·688	29·819	34·527	36·477
14	21·064	23·685	26·119	29·141	31·319	36·124	38·109
15	22·307	24·996	27·488	30·578	32·801	37·698	39·717
16	23·542	26·296	28·845	32·000	34·267	39·252	41·308
17	24·769	27·587	30·191	33·409	35·718	40·791	42·881
18	25·989	28·869	31·526	34·805	37·156	42·312	44·434
19	27·204	30·144	32·852	36·191	38·582	43·819	45·974
20	28·412	31·410	34·170	37·566	39·997	45·314	47·498
21	29·615	32·671	35·479	38·932	41·401	46·796	49·010
22	30·813	33·924	36·781	40·289	42·796	48·268	50·510
23	32·007	35·172	38·076	41·638	44·181	49·728	51·999
24	33·196	36·415	39·364	42·980	45·558	51·179	53·478
25	34·382	37·652	40·646	44·314	46·928	52·619	54·948

TABLE 7: THE WILCOXON SIGNED RANK TEST

The table gives the critical value of W, for a selection of significance levels, where n is the sample size for the test.

1-tail 2-tail n	0·05 0·10	0·025 0·05	0·01 0·02	0·005 0·01
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger samples, a normal approximation for W may be employed where

$$\mathrm{E}(W)=\frac{1}{4}n(n+1) \text{ and } \mathrm{V}(W)=\frac{1}{24}n(n+1)(2n+1).$$

TABLE 8: THE MANN-WHITNEY TEST

Samples of sizes $m \leq n$ have sum of ranks W_m and W_n and W is the smaller of W_m and $m(m+n+1) - W_m$
Critical values of W are given for a selection of significance levels.

1-tail	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01
2-tail	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$			$m = 7$			$m = 8$		
3	6	-	-															
4	6	-	-	11	10													
5	7	6	-	12	11	10	19	17	16									
6	8	7	-	13	12	11	20	18	17	28	26	24						
7	8	7	6	14	13	11	21	20	18	29	27	25	39	36	34			
8	9	8	6	15	14	12	23	21	19	31	29	27	41	38	35	51	49	45
9	10	8	7	16	14	13	24	22	20	33	31	28	43	40	37	54	51	47
10	10	9	7	17	15	13	26	23	21	35	32	29	45	42	39	56	53	49
11	11	9	7	18	16	14	27	24	22	37	34	30	47	44	40	59	55	51
12	11	10	8	19	17	15	28	26	23	38	35	32	49	46	42	62	58	53
13	12	10	8	20	18	15	30	27	24	40	37	33	52	48	44	64	60	56
14	13	11	8	21	19	16	31	28	25	42	38	34	54	50	45	67	62	58
15	13	11	9	22	20	17	33	29	26	44	40	36	56	52	47	69	65	60

1-tail	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01
2-tail	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02
n	$m = 9$			$m = 10$			$m = 11$			$m = 12$			$m = 13$			$m = 14$		
9	66	62	59															
10	69	65	61	82	78	74												
11	72	68	63	86	81	77	100	96	91									
12	75	71	66	89	84	79	104	99	94	120	115	109						
13	78	73	68	92	88	82	108	103	97	125	119	113	142	136	130			
14	81	76	71	96	91	85	112	106	100	129	123	116	147	141	134	166	160	152
15	84	79	73	99	94	88	116	110	103	133	127	120	152	145	138	171	164	156
16	87	82	76	103	97	91	120	113	107	138	131	124	156	150	142	176	169	161
17	90	84	78	106	100	93	123	117	110	142	135	127	161	154	146	182	174	165
18	93	87	80	110	103	96	127	121	113	146	139	131	166	158	150	187	179	170
19	96	90	83	113	107	99	131	124	116	150	143	134	171	163	154	192	183	174
20	99	93	85	117	110	102	135	128	119	155	147	138	175	167	158	197	188	178

1-tail	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01	0·05	0·025	0·01
2-tail	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02	0·1	0·05	0·02
n	$m = 15$			$m = 16$			$m = 17$			$m = 18$			$m = 19$			$m = 20$		
15	192	184	176															
16	197	190	181	219	211	202												
17	203	195	186	225	217	207	249	240	230									
18	208	200	190	231	222	212	255	246	235	280	270	259						
19	214	205	195	237	228	218	262	252	241	287	277	265	313	303	291			
20	220	210	200	243	234	223	268	258	246	294	283	271	320	309	297	348	337	324

For larger samples, a normal approximation for W may be employed where

$$\mathrm{E}(W) = \frac{1}{2}m(m+n+1) \text{ and } \mathrm{V}(W) = \frac{1}{12}mn(m+n+1).$$

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