2013 Applied Mathematics – Statistics

Advanced Higher

Finalised Marking Instructions

Part One: General Marking Principles for Applied Mathematics – Statistics – Advanced Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

(a) Marks for each candidate response must <u>always</u> be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question. If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader/Principal Assessor.

(b) Marking should always be positive ie, marks should be awarded for what is correct and not deducted for errors or omissions.

GENERAL MARKING ADVICE: Applied Mathematics – Statistics – Advanced Higher

The marking schemes are written to assist in determining the "minimal acceptable answer" rather than listing every possible correct and incorrect answer. The following notes are offered to support Markers in making judgements on candidates' evidence, and apply to marking both end of unit assessments and course assessments.

These principles describe the approach taken when marking Advanced Higher Applied Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values / algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

In the detailed Marking Instructions which follow, marks are shown alongside the line for which they are awarded. There are two codes used M and E. The code M indicates a method mark, so in question B3, **M1** means a method mark for understanding integration by parts. The code E refers to 'error', so in question B6(b), up to 2 marks can be awarded but 1 mark is lost for each error.

Part Two: Marking Instructions for each Question

Section A (Statistics 1 and 2)

Question			Solution	Max Mark	Additional Guidance
A	**1**	**a**	**Give an example of both a random and a non-random method of sampling from a population and state an advantage and a disadvantage of each.** Simple random sampling is a random method of sampling a population, an advantage being that it ensures a representative sample. A disadvantage is that a list of all population members is required. Quote sampling is a non-random method, an advantage being that no list of population members is required. A disadvantage is that the sample may not be representative of the population. **In chain-referral, or "snowball" sampling, used in social research, the researcher first identifies a member of the population of interest to include in the sample and to interview. The first member is then asked to refer the interviewer to a second member of the population for inclusion in the sample and so on.**	**4**	
A	**1**	**b**	**State a disadvantage of this type of sampling.** A disadvantage is that the sample may not be representative eg a member of a population might not refer the interviewer to a population member with views that differ radically from his/her own.	**1**	

Question			Solution	Max Mark	Additional Guidance
A	2		**The random variable *X* has the binomial distribution with parameters 25 and 0·2.**		
A	2	a	**Write down an expression for P(*X*<4) and evaluate it correct to four decimal places.** $P(X < 4) = P(X \leq 3)$ $= \binom{25}{0} 0 \cdot 8^{25} + \binom{25}{1} 0 \cdot 8^{24}\, 0 \cdot 2 + \binom{25}{2} 0 \cdot 8^{23}\, 0 \cdot 2 + \binom{25}{3} 0 \cdot 8^{22}\, 0 \cdot 2^{3}$ = 0·2340	2	
		b	**Use a normal approximation to estimate the required probability, both with and without continuity correction. Comment in relation to the widely quoted rule of thumb for the reliability of this approximation.** $\mu = np = 25 \times 0 \cdot 2 = 5$ $\sigma = \sqrt{npq} = \sqrt{25 \times 0 \cdot 2 \times 0 \cdot 8} = 2$ With continuity correction:- $P(X \leq 3) \approx P(Z \leq \frac{3 \cdot 5 - 5}{2}) = P(Z \leq -0 \cdot 75) = 0 \cdot 2266$ Without continuity correction:- $P(X \leq 3) \approx P(Z \leq \frac{3 - 5}{2}) = P(Z \leq -1) = 0 \cdot 1587$ The rule of thumb we use is that *np* and *nq* both exceed 5. This is very nearly true in this case (*np*=5) and the approximation with continuity correction is still accurate to 2 decimal places.	5	

Question			Solution	Max Mark	Additional Guidance
A	**3**		**The standard score for a random variable *X* is defined as** $Z = \frac{X-\mu}{\sigma}$ **where** μ **= E (*X*) and** σ **= V (*X*)**		
A	**3**	**a**	**Use the laws of expectation and variance to determine the mean and standard deviation of *Z*.** E (*Z*) = E $\left[\frac{1}{\sigma}(X-\mu)\right]=\frac{1}{\sigma}\left[(E(X)-E(\mu)\right]=\frac{1}{\sigma}(\mu-\mu)=0$ V (*Z*) = V $\left[\frac{1}{\sigma}(X-\mu)\right]=\frac{1}{\sigma^2}\mathrm{V}(X-\mu)=\frac{1}{\sigma^2}\mathrm{V}(X)=\frac{\sigma^2}{\sigma^2}=1$ **In national examinations in Mathematics and Music the mean marks were 50 and 65 respectively and the standard deviations 10 and 15 respectively. A student scored 60 in Mathematics and 80 in Music.**	**3**	
A	**3**	**b**	**Calculate this student's standard scores and comment.** $Z_{Maths} = \frac{60-50}{10}=1$ $Z_{music} = \frac{80-65}{15}=1$ In terms of standard scores the student's performance was equally good in the two subjects.	**2**	

Question			Solution	Max Mark	Additional Guidance
A	4		**Weight gain in lambs of a breed of sheep reared on particular diet regime is known to have mean 160g / day and standard deviation 24g / day. A random sample of lambs of this breed were fed the diet supplemented by fish meal and the weight gains were:** **218, 201, 143, 184, 172, 193, 163, 216, 127, 163, 156, and 173 g/day**		
A	4	a	**Stating any assumptions required, use a z-test to determine whether or not the data provide any evidence that the fish meal supplement lead to a change in mean weight gain for the breed.** Assume that, for the supplemented diet, weight gain has a normal distribution with standard deviation 24. $H_0 : \mu = 160$ and $H_1 : \mu \neq 160$ $z = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$ $= \dfrac{175 \cdot 75 - 160}{24 / \sqrt{12}} = 2 \cdot 27$ Since 2·27 > 1·96, H_0 is rejected at the 5% level of significance and the data provide evidence of a change in the mean weight gain.	5	
A	4	b	**Alternatively, using a student's t-distribution, a 95% confidence interval for the mean weight gain of lambs of this breed, fed with supplemented diet, is (158·0, 193·5)** **Comment on this result and also on the use of a t-interval** Since the confidence interval includes 160 the null hypothesis cannot be rejected. Use of the t-distribution requires the assumption of normality but no assumption on variability.	2	

<table>
<tr><th colspan="3">Question</th><th>Solution</th><th>Max Mark</th><th>Additional Guidance</th></tr>
<tr><td>A</td><td>5</td><td></td><td>At the Vienna General Hospital in 1843 there were 5799 births in the two maternity clinics. Maternal mortality due to puerperal fever was of major concern to one of the physicians, Dr Ignaz Semelweiss. Classification of the births yielded the following contingency table

<table>
<tr><td></td><td></td><td colspan="2">Clinic in which birth took place</td></tr>
<tr><td></td><td></td><td>No. 1</td><td>No. 2</td></tr>
<tr><td rowspan="2">Maternal Outcome</td><td>Death from puerperal fever</td><td>274</td><td>164</td></tr>
<tr><td>No death from puerperal fever</td><td>2786</td><td>2575</td></tr>
</table>
</td><td></td><td></td></tr>
<tr><td>A</td><td>5</td><td>a</td><td>Carry out a formal test of association between maternal outcome and clinic in which the birth took place.

Observed and expected frequencies (bracketed) are as follows:-

<table>
<tr><td></td><td>No.1</td><td>No.2</td></tr>
<tr><td>Death from puerperal fever</td><td>274
(231·12)</td><td>164
(206·88)</td></tr>
<tr><td>No death from puerperal fever</td><td>2786
(2828·88)</td><td>2575
(2532·12)</td></tr>
</table>

H_0: there is no association between clinic and maternal outcome

H_1: there is an association

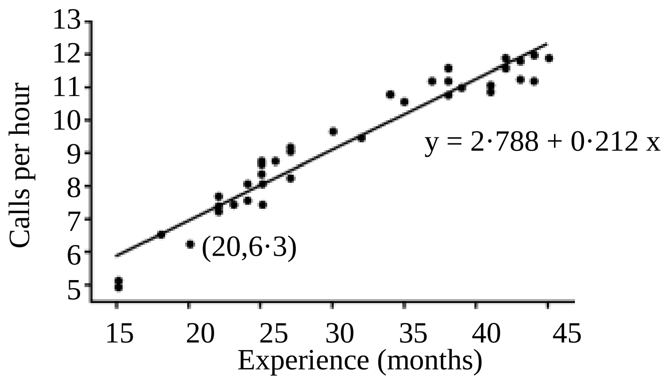
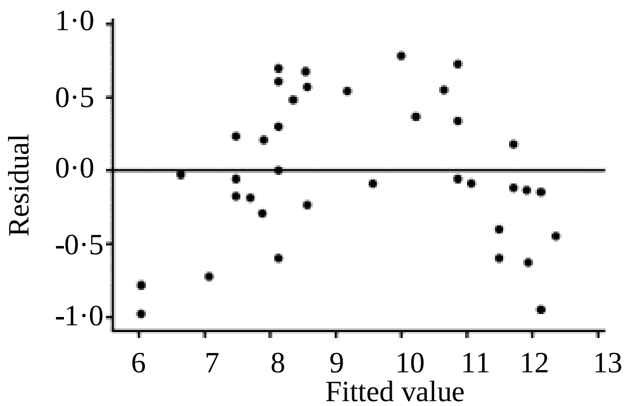
$$X^2 = \sum \frac{(O-E)^2}{E}$$

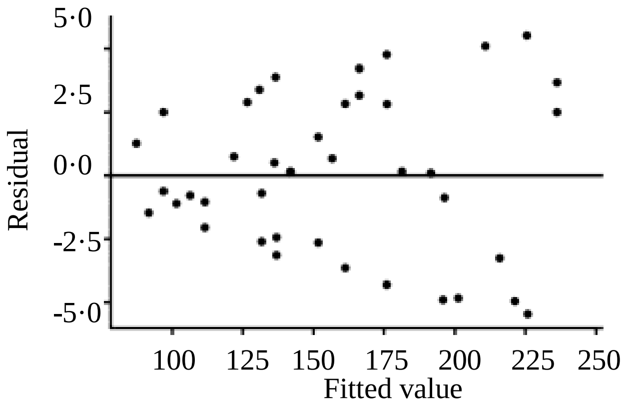
$$x^2 = \frac{(274-231\cdot 12)^2}{231\cdot 12} + \frac{(164-206\cdot 88)^2}{206\cdot 88}$$

$$+\frac{(2786-2828\cdot 88)^2}{2828\cdot 88} + \frac{(2575-2532\cdot 12)^2}{2532\cdot 12}$$

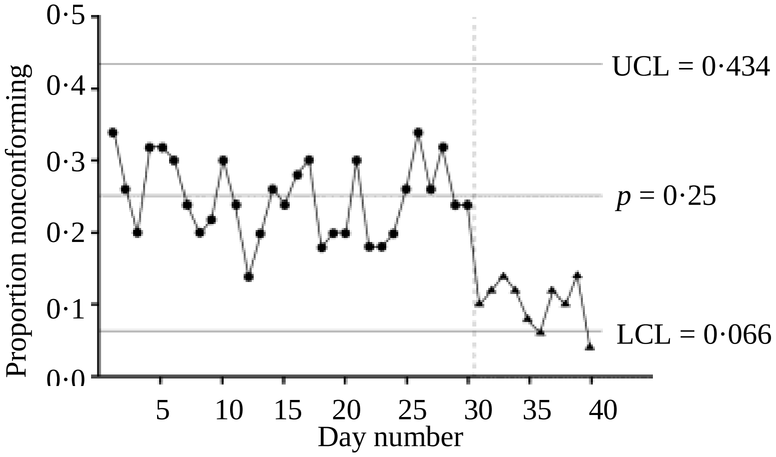
= 18·22

The critical value of chi-squared with 1 d.f. at the 0·1% level is 10·827. Since 18·22 exceeds this value the null hypothesis would be rejected and there is strong evidence of an association between clinic and maternal outcome.</td><td>4</td><td></td></tr>
</table>

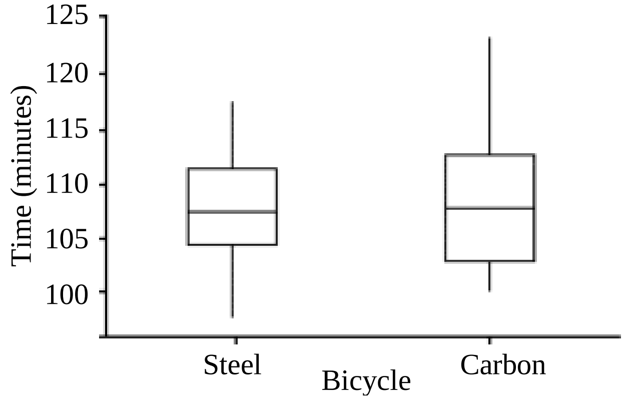
Question			Solution	Max Mark	Additional Guidance
A	5		**(cont)**		
A	5	b	**Summarise your findings, including numerical information, in a concise form that could be understood by someone with no knowledge of statistics.** The proportion of births with maternal death from puerperal fever as an outcome were 274/3060 = 9% and 164/2739 = 6% respectively for Clinics 1 and 2. The analysis indicates that the proportion of births with maternal death from puerperal fever was significantly higher at Clinic 1.	2	
A	6		**At a large call centre the resource management team carried out an investigation of the relationship between experience of staff, as measured by months in post, and productivity, as measured by completed calls per hour over a month. The data for a random sample of 40 staff are displayed in the scatter plot below, annotated with the equation of the least squares regression line and the coordinates of one of the data points.** 		
A	6	a	**Calculate the residual for the data point given** **The plot of residuals against fitted values is shown below.**  Residual = Data value – Fitted value = $y_i - (a + bx_i)$ $= 6{\cdot}3 - (2{\cdot}788 + 0{\cdot}212 \times 20) = -0{\cdot}728$	2	

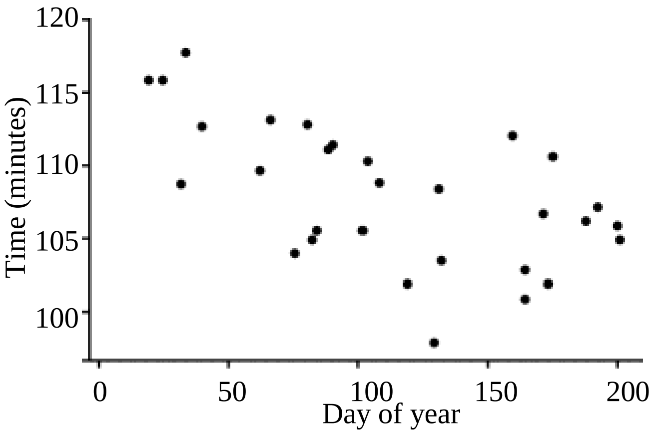
Question			Solution	Max Mark	Additional Guidance
A	6		**(cont)**		
A	6	b	**State why this plot indicates a problem with the fitted model. How might the model be improved?** The plot has an arched ∩ shape rather than the ideal random scatter centred on zero. A non-linear model might be better eg a quadratic. **In another investigation the team obtained the following residual plot.** 	2	
A	6	c	**State the assumption of the standard linear model that appears to be violated in this case and what action might be taken to deal with it.** The plot has a wedge < shape suggesting that the assumption that the $V(\varepsilon_i) = \sigma^2$, a constant, is violated. Transformation of the data might yield a better model.	2	

Question			Solution	Max Mark	Additional Guidance
A	7		**A computer system is subject to random attacks from both Attila and Berserker malware programmes. The mean number of attacks per day from each of these is λ and the two types of attack occur independently of each other.**		
A	7	a	**State the distribution of the number of attacks per day from each type of malware and show that over a period of *n* days the expected number of days on which attacks occur is $n(1 - e^{-2\lambda})$** The distribution in both cases is Po (λ). P(day is attack free) = P(no A attack) P(no B attack) $= e^{-\lambda} \times e^{-\lambda} = e^{-2\lambda}$ $\therefore$ P(attack occurs on a day) = $1 - e^{-2\lambda}$ The expected number of days out of *n* in which attacks occur $= n\,(1 - e^{-\lambda})$	3	
A	7	b	**Show that $\frac{1-e^{-\lambda}}{1-e^{-2\lambda}} = \frac{1}{1+e^{-\lambda}}$ and hence show that on a day when the system was subjected to attack the probability that it was due to just one of the two types of malware is $\frac{2}{e^{\lambda}+1}$** $\frac{1-e^{-\lambda}}{1-e^{-2\lambda}} = \frac{1-e^{-\lambda}}{(1-e^{-\lambda})(1+e^{-\lambda})} = \frac{1}{1+e^{-\lambda}}$ $\text{P (Attack by one type} \mid \text{Attack has occurred)}$ $\frac{\text{P (Attack is by one type} \cap \text{Attack has occurred)}}{\text{P(Attack has occured)}}$ $\frac{\text{P}(\bar{A}B) + \text{P}(A\bar{B})}{\text{P (Attack)}}$ $= \frac{e^{-\lambda}(1-e^{-\lambda}) + (1-e^{-\lambda})e^{-\lambda}}{1-e^{-2\lambda}}$ $= \frac{2e^{-\lambda}(1-e^{-\lambda})}{(1-e^{-2\lambda})}$ $= \frac{2e^{-\lambda}}{1+e^{-\lambda}}$ $= \frac{2}{e^{\lambda}+1}$	5	

Question			Solution	Max Mark	Additional Guidance
A	8	a	**Using the variance of the random variable $X \sim B(n, p)$, obtain the standard deviation of the proportion of the success X/n, justifying your method.** **During a 30-day period of manufacturing In-Plane Switching displays for tablet computers, the proportion of nonconforming displays at final inspection was estimated to be 0·25. The proportions nonconforming in random samples of 50 taken daily over that period of 30 days are displayed in the control chart shown (circular symbols)**  $V(X) = npq$ $V(X/n) = \frac{1}{n^2}V(X) = \frac{npq}{n^2} = \frac{pq}{n}$ Standard error of proportion = $\sqrt{\frac{pq}{n}}$	3	
A	8	b	**Confirm the values of the 3-sigma control chart limits.** **At the end of the 30-day period, modifications were made to the manufacturing process. Data for the subsequent 10-day period are also displayed in the control chart (triangular symbols).** Chart limits are given by $p \pm 3\sqrt{\frac{pq}{n}}$ $0.250 \pm 3\sqrt{\frac{0 \cdot 25 \times 0 \cdot 75}{50}} = 0.250 \pm 0.184$ or (0·066, 0·434)	2	

Question			Solution	Max Mark	Additional Guidance
A	**8**	**c**	**Explain how the chart provides evidence of improvement.** **Following the modification, the proportion of nonconforming displays was estimated to be 0·10. The process manager wished to continue to monitor the process with a view to further reducing the proportion of nonconforming displays.** The proportion of non-conforming displays appears to be much lower after the modifications. The occurrence of two points below the lower limit provides evidence of a reduction in the proportion nonconforming ie of improvement.	**3**	
A	**8**	**d**	**Show that, with sample size 50, a negative value would now be obtained for the lower chart limit. Determine the minimum sample size that would yield a non-negative lower limit and state why such a lower limit is desirable.** $\text{LCL} = 0{\cdot}100 - 3\sqrt{\frac{0\cdot1\times0\cdot9}{50}} = 0{\cdot}100 - 0{\cdot}127 < 0$ $\text{LCL} > 0 \Rightarrow 0{\cdot}100 - 3\sqrt{\frac{0\cdot1\times0\cdot9}{n}} > 0$ $\Rightarrow 0{\cdot}100 > 3\sqrt{\frac{0\cdot1\times0\cdot9}{n}} \Rightarrow 0{\cdot}100^2 > \frac{9\times0\cdot1\times0\cdot9}{n}$ $\Rightarrow n > 9 \times 0{\cdot}1 \times \frac{0.9}{0.01}$ = 81 so minimum size is 82 Such a limit enables evidence of further improvement to be indicated through the occurrence of a point below the LCL.	**4**	

Question			Solution	Max Mark	Additional Guidance
A	9		**On the days between mid-January 2010 and mid-July 2010 when he cycled to work, Dr Jeremy Groves randomly allocated either his steel frame bicycle or his carbon frame bicycle for his journey. The time the bicycle was moving for the 27-mile round trip was recorded using a bicycle computer. Analysis of his data was published in the Royal Statistical Society magazine *Significance*. The data is used with permission of Dr Groves who believed that the lighter carbon frame would lead to shorter times. He made 30 journeys on the steel bicycle and 26 on the carbon one and boxplot of the times are shown below**		
A	9	a	Comment on the boxplots  The display indicates that the median journey times are very similar suggesting that his belief is incorrect. **The data were ranked with the shortest time being allocated rank 1 and the rank sum for the steel bicycle was 856·5**		
A	9	b	**Perform a formal test to evaluate the data for any evidence of a difference in median times** $H_o: \eta_{Steel} = \eta_{Carbon}$ $H_1: \eta_{Steel} \neq \eta_{Carbon}$ $W = 856{\cdot}5$ $E(W) = \frac{1}{2} 30\,(30 + 26 + 1) = 855$ $V(W) = \frac{1}{12} 30 \times 26\,(30 + 26 + 1) = 60{\cdot}87^2$ $z = \frac{W - E(W)}{\sqrt{V(W)}} = \frac{856{\cdot}5 - 0{\cdot}5 - 855}{60{\cdot}87} = 0{\cdot}02$ The critical value is 1·96 > 0·02 so we accept H_o at the 5% level ie there is no evidence of a difference in median times	5	

Question			Solution	Max Mark	Additional Guidance
A	**9**		**(cont)** **A non–parametric test of the null hypothesis that the variances of time for the two bicycles are equal, with a two-sided alternative, yielded a p-value of 0·198**		
A	**9**	**c**	**Explain the relevance of this to the test performance in (b).** The Mann-Whitney test assumes that the two distributions have the same shape and hence the same variance. Since the p-value exceeds 0·05 the null hypothesis of equal variance cannot be rejected so the required assumption is tenable. **A scatter plot of time versus day of the year for the steel bicycle is shown below, the sample product moment correlation coefficient being -0·589.** 	**2**	
A	**9**	**d**	**Show that there is strong evidence of a non-zero population correlation coefficient and comment in relation to the test in (*b*)** $H_o : \rho = 0 \qquad H_1 : \rho \neq 0$ $t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \dfrac{-0 \cdot 589\sqrt{28}}{\sqrt{1-(-0 \cdot 589)^2}}$ = -3·86 with 28 degrees of freedom The critical region for level of significance 0·1% is t < -3·674. As -3·86 lies in the critical region we would reject Ho, furnishing strong evidence of a non-zero correlation. A Mann-Whitney test is used to compare two distributions but the correlation of time with day of the year suggests that it is not appropriate to refer to a distribution of times for each type of bicycle. (Also accept a comment on the apparent non-linear fit)	**4**	

Section B

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**1**		**Given that $y = \sin(e^{5x})$, find $\frac{dy}{dx}$** $\frac{dy}{dx} = \cos e^{5x} \times \frac{d}{dx}(e^{5x})$ $= \cos e^{5x} \times 5e^{5x}$ $= 5e^{5x} \cos e^{5x}$	**2**	**1** First application of chain rule. **1** Second application of chain rule.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**2**		**Matrices are given as** **A =** $\begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}$ **B =** $\begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$ **C =** $\begin{pmatrix} y & 3 \\ -1 & 2 \end{pmatrix}$		
B	**2**	**a**	**Write A^2 – 3B as a single matrix** $A^2 - 3B = \begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix} - 3\begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 16 & 6x \\ 0 & 4 \end{pmatrix} - 3\begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 16 & 6x \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 15 & 3 \\ 0 & 3 \end{pmatrix}$ $= \begin{pmatrix} 1 & 6x-3 \\ 0 & 1 \end{pmatrix}$	**2**	**1** A^2 correct. **1** For correct evaluation of 3B and simplify.
B	**2**	**b**	**(i) Given that C is non-singular, find C^{-1}, the inverse of C.** det C = 2y + 3 $C^{-1} = \frac{1}{2y+3}\begin{pmatrix} 2 & -3 \\ 1 & y \end{pmatrix}$	**2**	**1** Determinant correct. **1** Inverse correct.
B	**2**	**b**	**(ii) For what value of *y* would matrix C be singular?** 2y + 3 = 0 for C singular $y = -\frac{3}{2}$	**1**	**1** y value correct.
Notes:					

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**3**		**Use integration by parts to obtain** $\int \frac{\ln x}{x^3} dx$ **where** $x > 0$	**4**	
			$u = \ln x,\ dv = \frac{1}{x^3} dx$		
			$du = \frac{1}{x} dx,\ v = \int \frac{1}{x^3} dx$		**M1** Understand integration by parts.
			$v = -\frac{1}{2x^2}$		
			$I = \ln x . -\frac{1}{2x^2} - \int -\frac{1}{2x^2} . \frac{1}{x} dx$		**1** Integrates *dv* and substitutes correctly.
			$= -\frac{\ln x}{2x^2} + \frac{1}{2}\int \frac{dx}{x^3}$		**1** Correctly combines *v* and *du*.
			$= -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c$		**1** Correctly integrates second term.

Notes:
3.1 Treat omission of "+*c*" as bad form: do not penalise.
3.2 Negative indices for *x* equally acceptable.

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	4	a	**State the results for** $\sum_{r=1}^{n} r$ **and** $\sum_{r=1}^{n} r^3$ **in terms of *n*.** **Hence show that** $\sum_{r=1}^{n}(r^3 - 3r) = \frac{n(n+1)(n-2)(n+3)}{4}$	4	
			$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$		**1** Both formulae correct.
			$\sum_{r=1}^{n}(r^3 - 3r) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r$		**1** Correct separation.
			$= \frac{n^2(n+1)^2}{4} - \frac{3n(n+1)}{2}$		**1** Substitution.
			$= \frac{n(n+1)}{4}[n(n+1) - 6]$		
			$= \frac{n}{4}(n+1)(n^2 + n - 6)$ **Note:** This or equivalent intermediate algebra required for this mark.		**1** Algebra correct.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**4**		**(cont)**		
B	**4**	**b**	**Use the above result to evaluate** $\sum_{r=5}^{15}(r^3-3r)$	**2**	
			$\sum_{r=5}^{15}(r^3-3r) = \sum_{r=1}^{15}(r^3-3r) - \sum_{r=1}^{4}(r^3-3r)$		**1** Correct limits.
			$= \frac{15\times16\times18\times13}{4} - \frac{4\times5\times2\times7}{4}$		
			= 14 040 – 70		
			= 13 970		**1** Correct evaluation.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**5**		**Find the general solution of the differential equation**	**6**	
			$\mathbf{\frac{1}{x}\frac{dy}{dx}+2y=6, x\neq 0}$		
			$\frac{dy}{dx}+2xy=6x$		**1** Multiplies through by *x*.
			I.F = $e^{\int 2x} = e^{x^2}$		**1** Correct integrating factor.
			$e^{x^2}\frac{dy}{dx} + e^{x^2}2xy = 6x \,.\, e^{x^2}$		
			$\frac{d}{dx}(e^{x^2}.y) = 6x \,.\, e^{x^2}$		**1** Recognises LHS as exact differential of *g* × I.F.
			$\int \frac{d}{dx}(e^{x^2}.y)\,dx = \int 6x \,.\, e^{x^2}\,dx$		**1** Knows to integrate.
			$e^{x^2} \,.\, y = 3\,e^{x^2} + c$		**1** Integrates correctly.[2]
			$y = 3 + \frac{c}{e^{x^2}}$		**1** Divides through by e^{x^2}.

Notes:
5.1 Final answer of $y = 3 + ce^{-x^2}$ also correct.
5.2 "+*c*" required for mark here.

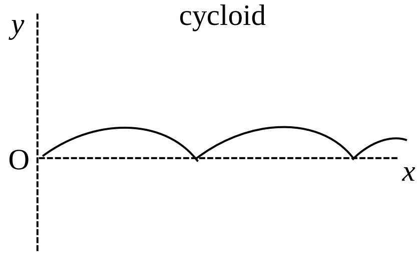
Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**5**		**Alternative:**		
			$\mathbf{\frac{1}{x}\frac{dy}{dx} = 6 - 2y}$		
			$\frac{dy}{6-2y} = x\,dx$		**1** Separates variables.
			$-\frac{1}{2}\ln\lvert 6-2y\rvert = \frac{1}{2}x^2 + k$		**1** Integrates LHS. **1** Integrates RHS (constant on either side).
			$\ln\lvert 6-2y\rvert = -x^2 - 2k$		**1** Prepares for exponential.
			$6 - 2y = Ae^{-x^2}$		**1** Converts form to exponential.[3,4]
			$-2y = Ae^{-x^2} - 6$		
			$y = \frac{1}{2}Ae^{-x^2} + 3$		**1** Rearranges to make y subject.[3,5]
			$y = 3 + \frac{C}{e^{x^2}}$		

Notes:

5.3 Any constant acceptable. Therefore, term containing constant can be positive or negative.

5.4 $6 - 2y = e^{-x^2-c}$ a valid alternative for this mark.

5.5 Either of last two lines valid for award of final mark.

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**6**		**The cycloid curve below is defined by the parametric equations** $\boldsymbol{x = t - \sin t, y = 1 - \cos t.}$ 		
B	**6**	**a**	**Find** $\frac{dy}{dx}$ **in terms of** ***t***	**2**	
			$\frac{dy}{dt} = \sin t, \ \frac{dx}{dt} = 1 - \cos t$		**1** Appropriate differentiation.
			$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sin t}{1 - \cos t}$		**1** Correct use.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**6**		**(cont)**		
B	**6**	**b**	**Show that the value of $\frac{d^2y}{dx^2}$ is always negative, in the case where $0 < t < 2\pi$**	**5**	
			$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$		**M1** Correct application of method.
			$= \frac{(1-\cos t)\cos t - \sin t(\sin t)}{(1-\cos t)^2} \div (1-\cos t)$		
			$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^3}$		**2E1** Differentiates /substitutes correctly.
			$= \frac{-[(\cos^2 t + \sin^2 t) - \cos t]}{(1-\cos t)^3}$		**1** Uses $\sin^2 t + \cos^2 t = 1$ and simplifies.
			$= \frac{-(1-\cos t)}{((1-\cos t))^3}$		
			$= -\frac{1}{(1-\cos t)^2} < 0$		**1** Clear explanation.
			Hence		
			$\frac{d^2y}{dx^2} < 0$, for $0 < t < 2\pi$		

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**6**	**c**	**A particle follows the path of the cycloid where *t* is the time elapsed since the particle's motion commenced.** **Calculate the speed of the particle when *t* = $\frac{\pi}{3}$.** Speed = $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 1$	**2**	**1** Correct formula. **1** Applies correct values to obtain a speed of 1.

Notes:

[END OF MARKING INSTRUCTIONS]