2014 Applied Mathematics - Mechanics

Advanced Higher

Finalised Marking Instructions

Part One: General Marking Principles for Applied Mathematics – Mechanics – Advanced Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

(a) Marks for each candidate response must always be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question.

(b) Marking should always be positive ie, marks should be awarded for what is correct and not deducted for errors or omissions.

GENERAL MARKING ADVICE: Applied Mathematics – Mechanics – Advanced Higher

The marking schemes are written to assist in determining the "minimal acceptable answer" rather than listing every possible correct and incorrect answer. The following notes are offered to support Markers in making judgements on candidates' evidence, and apply to marking both end of unit assessments and course assessments.

These principles describe the approach taken when marking Advanced Higher Applied Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values/algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

Part Two: Marking Instructions for each Question

Section A

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	1		$I = Ft = mv - mu$ $3(4) = 2v - 0$ $v = 6\text{ms}^{-1}$ Conservation of Linear Momentum: $m u_1 + m_2u_2 = (m_1 + m_2)v$ $2(6) + m(0) = (2 + m)(3\cdot75)$ $m = \frac{12}{3\cdot75} - 2 = 1\cdot2\,\text{kg}$ **ALTERNATIVE SOLUTION** $F = ma$ $3 = 2a$ $a = \frac{3}{2}\,\text{m s}^{-2}$ u = 0 t = 4 a = $\frac{3}{2}$ v = u + at = 0 + $\frac{3}{2}(4) = 6\text{ m s}^{-1}$ Conservation of Linear Momentum: $m u_1 + m_2u_2 = (m_1 + m_2)v$ $2(6) + m(0) = (2 + m)(3\cdot75)$ $m = \frac{12}{3\cdot75} - 2 = 1\cdot2\,\text{kg}$	**4**	**M1**: Use of impulse to calculate velocity of impact **E1:** Value for velocity of impact **M1:** Conservation of Linear Momentum **E1:** Value of *m* **M1**: Use of Newton's 2nd Law to calculate acceleration **E1:** Stuva to calculate velocity of P before impact **M1:** Conservation of Linear Momentum **E1:** Value of m

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**2**		$T=\frac{2\pi}{\omega}\Rightarrow\frac{2\pi}{\omega}=\frac{14\pi}{5}\Rightarrow\omega=\frac{5}{7}$ $v^2=\omega^2(a^2-x^2)$ $2\cdot5^2=\frac{25}{49}(a^2-1\cdot2^2)$ $\frac{2\cdot5^2\times49}{25}+1\cdot2^2=a^2$ $a=3\cdot7$ metres $x=A\sin\omega t$ $1\cdot2=3\cdot7\sin\left(\frac{5t}{7}\right)$ $t=0\cdot46$ seconds	**4**	**E1:** Value of ω **M1:** Correct formula for velocity and amplitude and correct substitution **E1:** Value for amplitude **M1:** Use of formula to find displacement and answer

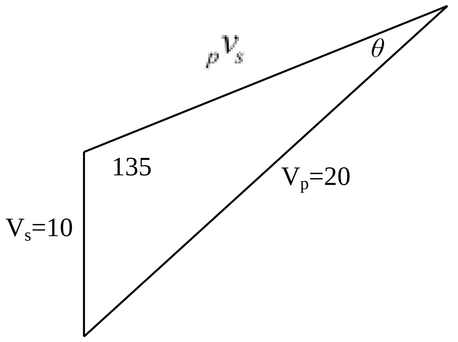
Question			Expected Answer(s)	Max Mark	Additional Guidance
A	3	(i) (ii)	$W = \int_0^{200} (F - 500)dx$ $= \int_0^{200} (3000 - 15x - 500)dx$ $= \left[2500x - \frac{15x^2}{2}\right]_0^{200}$ $= 200000 \text{ J} = 200 \text{ kJ}$ Work- Energy Principle: $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ $200000 = \frac{1}{2} \times 700 v^2$ $v = 23 \cdot 9 \text{m s}^{-1}$ ***Alternative solution for (ii)*** $F = ma \Rightarrow a = \frac{F}{m}$ $v\frac{dv}{dx} = \frac{1}{700}\int (2500 - 15x)dx$ $F = ma$ $v\frac{dv}{dx} = \frac{1}{700}(2500 - 15x)$ $\int vdv = \frac{1}{700}\int (2500 - 15x)dx$ $\frac{v^2}{2} = \frac{1}{700}\left[2500 - \frac{15x^2}{2}\right]_0^{200}$ $v = 23 \cdot 9 \text{ms}^{-1}$	2 2	**M1**: Method of finding work done $\int F dx$ (with limits or calculation of constant included later) **E1**: Correct answer **M1**: Use of Work – Energy Principle and substitution **E1**: Correct calculation of speed **M1**: Use of correct differential equation and substitution **E1**: Correct calculation of speed

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	4		↑ Equilibrium $2T\cos 30° + T\cos 50° = 2g$ $2T\cos 30° + T\cos 50° = 2g$ $T = \dfrac{2g}{2\cos 30° + \cos 50°} = 8 \cdot 25N$ $2T = 16 \cdot 5N$ → $F = \dfrac{mv^2}{r}$ $2T\sin 30° + T\ \sin 50° = \dfrac{2v^2}{r}$ $\tan 50° = \dfrac{r}{0 \cdot 3}$ $r = 0 \cdot 358$ $2T\sin 30° + T\sin 50° = \dfrac{2v^2}{0 \cdot 358}$ $2v^2 = 0 \cdot 358(16 \cdot 5 \times \dfrac{1}{2} + 8 \cdot 25 \times 0 \cdot 766)$ $v = 1 \cdot 61 \text{ m s}^{-1}$	**6**	**M1**: Consider equilibrium involving both tensions and weight **E1**: Correct substitution of components **E1**: Using conditions to find tension **M1**: Horizontal use of $F = \dfrac{mv^2}{r}$ (Consistent with M1 above) **E1**: Calculation of radius of circle **E1:** Algebraic manipulation to find v

Note:
If angular speed used can achieve 3/4.

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**5**		Perpendicular to slope: $R = Mg\cos\theta = \frac{4Mg}{5}$ Along slope: $F = ma$ $-\mu R - Mg\sin\theta = Ma$ $\frac{-4Mg}{4\times 5} - \frac{3Mg}{5} = Ma$ $a = \frac{-4g}{5}$ $(= -7\cdot 84)$ Motion under constant acceleration up slope to rest: $v^2 = u^2 + 2as$ $0 = u^2 - \frac{8gs}{5}$ $s = \frac{5u^2}{8g}$ Consider motion down slope: $F = ma$ $Mg\sin\theta - \frac{Mg\cos\theta}{4} = Ma$ $a = \frac{2g}{5}$ $(= 3\cdot 92)$ Constant acceleration down slope: $v^2 = u^2 + 2as$ $4u^2 = \frac{4gs}{5}$ $s = \frac{5u^2}{g}$ Distance AC = $\frac{5u^2}{g} - \frac{5u^2}{8g} = \frac{35u^2}{8g}$	**6**	**M1**: Equilibrium perpendicular to slope with equation **M1**: $F = ma$ along slope with equation **E1**: Calculation of acceleration **E1**: Calculation of displacement up slope to rest **E1**: Calculation of acceleration down slope **E1**: Calculation of displacement down slope and distance AC

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	6		**Method 1:**	6	
			$r_P = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad r_S = \begin{pmatrix} 15\cos 45° \\ 15\sin 45° \end{pmatrix}$ $v_P = \begin{pmatrix} 20\cos\theta \\ 20\sin\theta \end{pmatrix}$ $\quad v_s = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$		**M1**: Statements of displacements and velocity vectors at 3pm
			After time t $\quad r_P = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 20\cos\theta \\ 20\sin\theta \end{pmatrix} = \begin{pmatrix} 20t\cos\theta \\ 20t\sin\theta \end{pmatrix}$ $\mathrm{r}_s = \begin{pmatrix} 15\cos 45 \\ 15\sin 45 \end{pmatrix} + \mathrm{t}\begin{pmatrix} 0 \\ 10 \end{pmatrix} = \begin{pmatrix} 10\cdot 6 \\ 10\cdot 6 + 10t \end{pmatrix}$		**E1**: Statements of displacements after t hours
			At interception $r_P = r_s$ $20t\cos\theta = 10\cdot 6$ and $20t\sin\theta = 10\cdot 6 + 10t$ $t = \dfrac{10\cdot 6}{20\cos\theta}$ $\quad t = \dfrac{10\cdot 6}{20\sin\theta - 10}$		**M1**: Equate components
			$20\sin\theta - 10 = 20\cos\theta$ $\sin\theta - \cos\theta = \dfrac{1}{2}$ $\sqrt{2}\sin(\theta - 45)° = \dfrac{1}{2}$		**E1**: Algebraic manipulation
			$\theta = 65\cdot 7° \Rightarrow$ patrol vessel should steer on bearing $024\cdot 3°$		**E1**: Interpret answer to state bearing of interception
			$t = \dfrac{10\cdot 6}{20\cos 65\cdot 7} = 1\cdot 28$ hours = 1 hour 17 min Interception occurs at 4:17pm		**E1**: Calculation of time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	6		**(cont)** **Method 2:** $_{p}v_{s}$ must be in the direction *PS* for interception  $\frac{20}{\sin 135} = \frac{10}{\sin\theta}$ $\theta = \sin^{-1}\frac{10\sin 135}{20}$ = 20·7° Patrol vessel must sail (180 − 155·7) = 24·3º $v^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos 24{\cdot}3$ $v = 11{\cdot}6$ $t = \frac{15}{11{\cdot}6}$ = 1 hour 17 mins Interception occurs at 4:17pm		**M1**: For interception, relative velocity vector in direction PS **M1**: Correct diagram annotated **E1**: Use of trig **E1**: Interpret answer to state direction of interception **E1**: Find relative velocity **E1**: Calculation of time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	6		**(cont)** **Method 3:** $v_P = \begin{pmatrix} 20\cos\theta \\ 20\sin\theta \end{pmatrix} \quad v_s = \begin{pmatrix} 0 \\ 10 \end{pmatrix}$ ${}_p v_s = v_P - v_s = \begin{pmatrix} 20\cos\theta \\ 20\sin\theta - 10 \end{pmatrix}$ ${}_p v_s$ must be in the direction PS for interception $\begin{pmatrix} 20\cos\theta \\ 20\sin\theta - 10 \end{pmatrix} = k\begin{pmatrix} \cos 45° \\ \sin 45° \end{pmatrix}$ $k\cos 45 = 20\cos\theta$ $k\sin 45 = 20\sin\theta - 10$ $1 = \frac{20\sin\theta - 10}{20\cos\theta} \Rightarrow 20\cos\theta = 20\sin\theta - 10$ $\sin\theta - \cos\theta = \frac{1}{2}$ $\sqrt{2}\sin(\theta - 45)° = \frac{1}{2}$ $\theta = 65 \cdot 7° \Rightarrow$ patrol vessel should steer on bearing $024 \cdot 3°$ $t = \frac{10 \cdot 6}{20\cos 65 \cdot 7} = 1 \cdot 28$ hours = 1 hour 17 min Interception occurs at 4:17pm		**M1**: Statement of condition for interception **E1**: Expression for relative velocity vector **M1**: Equate components **E1**: Algebraic manipulation **E1**: Interpret answer to state bearing of interception **E1**: Calculation of time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**7**		$a_L = \frac{1}{9}g \quad a_B = g$ ↓ ${}_B a_L = g - \frac{1}{9}g = \frac{8g}{9}$ ${}_B v_L = \int {}_B a_L dt = \frac{8g}{9}t + c$ $t = 0, v = -3\cdot5 \quad \Rightarrow v = \frac{8g}{9}t - 3\cdot5$ ${}_B r_L = \int {}_B v_L dt = \frac{4g}{9}t^2 - 3\cdot5t + k$ $t = 0, r = -1 \quad \Rightarrow {}_B r_L = \frac{4g}{9}t^2 - 3\cdot5t - 1$ ${}_B r_L(t) = 0$ when ball hits floor $\frac{4g}{9}t^2 - 3\cdot5t - 1 = 0$ $4gt^2 - 31\cdot5t - 9 = 0$ $39\cdot2t^2 - 31\cdot5t - 9 = 0$ $t = \frac{31\cdot5 \pm \sqrt{(-31\cdot5)^2 - 4\times39\cdot2\times-9}}{78\cdot4}$ $t = 1\cdot03$ or $t = -0\cdot22$ $t = 1\cdot03$ seconds (reject negative answer)	**7**	**M1:** Find relative acceleration **M1**: Use of calculus to find relative velocity **E1:** Correct expression for relative velocity **E1:** Correct expression for relative displacement **M1:** Statement for conditions when ball hits floor of lift **E1:** Process of calculating time **E1**: Correct answer for time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	7		**(cont)** ***<u>Second solution using relative acceleration and stuva</u>*** $a_L = \frac{1}{9}g \quad a_B = g$ ${}_Ba_L = g - \frac{1}{9}g = \frac{8g}{9}$ $s = 1 \quad t = \quad u = -3\cdot5 \quad v = \quad a = \frac{8g}{9}$ $s = ut + \frac{1}{2}at^2 \qquad 1 = -3\cdot5t + \frac{4g}{9}t^2$ $4gt^2 - 31\cdot5t - 9 = 0 \qquad 39\cdot2t^2 - 31\cdot5t - 9 = 0$ $t = \frac{31\cdot5 \pm \sqrt{(-31\cdot5)^2 - 4\times39\cdot2\times-9}}{78\cdot4}$ $t = 1\cdot03$ or $t = -0\cdot22$ $t = 1\cdot03$ seconds (reject negative answer) ***Alternative solution*** Ball $a = -g$ $v = -gt + c$ $t = 0\ v = 3\cdot5 \Rightarrow c = 3\cdot5$ $r = \frac{-gt^2}{2} + 3\cdot5 + c_2$ $t = 0\ r = 0 \Rightarrow c_2 = 0$ $r = \frac{-gt^2}{2} + 3\cdot5$ Lift $a = \frac{-g}{9}$ $v = \frac{-gt}{9} + k$ $t = 0\ v = 0 \Rightarrow k = 0$ $r = \frac{-gt^2}{18} + k_2$ $t = 0\ r = -1 \Rightarrow k_2 = -1$ $r = \frac{-gt^2}{18} - 1$ $r_1 = r_2$ $\frac{-gt^2}{2} + 3\cdot5t = \frac{-gt^2}{18} - 1 \Rightarrow 8gt^2 - 63t - 18 = 0$ $4gt^2 - 31\cdot5t - 9 = 0 \qquad 39\cdot2t^2 - 31\cdot5t - 9 = 0$ $t = \frac{31\cdot5 \pm \sqrt{(-31\cdot5)^2 - 4\times39\cdot2\times-9}}{78\cdot4}$ $t = 1\cdot03$ or $t = -0\cdot22 \Rightarrow$ $t = 1\cdot03$ seconds (reject negative answer)		**M1** Show understanding of motion under constant relative acceleration **M1:** Find relative acceleration **M1 :**Consider motion ↓under constant relative acceleration **E1:** *stuva* and substitution **E1:** Correct quadratic equation **E1:** Process of calculating time **E1**: Correct answer for time **M1** Vertical motion of ball and lift separately **E1:** Correct expression for displacement of lift/ball after *t* secs **E1 :** Correct expression for displacement of otherafter *t* secs **M1:** Equating displacements **E1:** Correct quadratic equation **E1:** Process of calculating time **E1**: Correct answer for time

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	8	(a) (b)		3 7	
			At Q: total energy = $\frac{1}{2}mv^2 = 1\cdot5u^2$		**M1**: Consideration of conservation of energy.
			At top of circle total energy: $mgh + \frac{1}{2}mv^2$ $= 3g \times 1\cdot8 + \frac{3}{2}v^2 = 5\cdot4g + \frac{3}{2}v^2$		**E1**: Correct statements of energy at bottom and top of circle
			For complete circles $v > 0$: $1\cdot5u^2 > 5\cdot4g$ $u > \sqrt{\frac{18g}{5}}$ ms^{-1}		**M1**: For complete circles $v > 0$ and find u
			Height at any time: $0\cdot9(1-\cos\theta)$		**M1:** General expression for height at any time
			At rest (maximum height): Energy = $mgh = 3g \times 0\cdot9(1-\cos\theta)$		**M1**: Energy when rod is at rest
			If u = 4 : Energy at Q = $\frac{1}{2} \times 3 \times 4^2 = 24$		
			$24 = 3g \times 0\cdot9(1-\cos\theta)$ $\cos\theta = 0\cdot093$ $\theta = 84\cdot7°$		**E1**: Equate this with energy vertically below P
			Angle of oscillation = 169·4°		**E1**: Solve trig equation to find angle of oscillation
			Maximum tension when $\theta = 0$		**M1**: Understanding of maximum tension (stated or implied)
			$T - 3g = \frac{mv^2}{r}$ ↑		**M1**: Use of $F = \frac{mv^2}{r}$
			$T = 3g + \frac{3\times4^2}{0\cdot9} = 82\cdot7$N		**E1**: Calculation of Tension

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	9	(a)	**(i)** **(ii)** $v=\int a\,dt=\int 13\left(\frac{3}{8}-\frac{t}{16}\right)dt=13\left(\frac{3}{8}t-\frac{t^2}{32}\right)+c$ $t=0,0=0 \quad \Rightarrow c=0$ $v=13\left(\frac{3}{8}t-\frac{t^2}{32}\right)$ $t=\frac{5}{2}: v=13\left(\frac{3}{8}\times\frac{5}{2}--\frac{\left(\frac{5}{2}\right)^2}{32}\right)=9\cdot65\ \text{ms}^{-1}$ $\rightarrow: \ R=9\cdot65\cos25°\times t$ $\uparrow: s=ut+\frac{1}{2}at^2$ $0=9\cdot65\sin25°\times t-\frac{1}{2}gt^2$ $t(4\cdot08-4\cdot9t)=0$ $t=0$ or $t=0\cdot83$ $\rightarrow: \ R=9\cdot65\cos25°\times0\cdot83=7\cdot26\text{metres}$	3 3	**M1:** Integration to find expression for velocity **E1:** Substitution and correct expression **E1:** Substitute for *t* and correct answer for speed **M1:** Consider motion horizontally and vertically with substitution **E1:** Value of *t* **E1:** Value of *R*
A	9	(b)	**(i)** **(ii)** $\rightarrow 7\cdot51=10\cdot2\cos\theta\times t \qquad t=\frac{7\cdot51}{10\cdot2\cos\theta}$ $\uparrow: s=ut+\frac{1}{2}t^2$ $0=\frac{10\cdot2\sin\theta\times7\cdot51}{10\cdot2\cos\theta}-\frac{g}{2}\left(\frac{7\cdot51}{10\cdot2\cos\theta}\right)^2$ $7\cdot51\tan\theta-2\cdot656....\sec^2\theta=0$ $7\cdot51\tan\theta-2\cdot656....\tan^2\theta-2\cdot656....=0$ $\tan\theta=2\cdot41$ or $\tan\theta=0\cdot41$ $\theta=67\cdot2° \qquad \theta=22\cdot3°$ $\uparrow: v^2=u^2+2as$ $s=4\cdot51$ or $s=0\cdot76m$ Athlete cannot jump 4·51m vertically ⇒ Take-off angle ≈ 22·3°	3 2	**M1:** Consider motion → and expression for *t* **M1:** Consider motion vertically with this value of *t* and substitution **E1:** Solution of trig equation to give 2 angles of projection **E1:** Find two possible heights **E1:** Explanation of answer

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	9	(b)	**(cont)**		
			Method 2:		
			$\rightarrow 7\cdot51 = 10\cdot2\cos\theta \times t \quad t = \dfrac{7\cdot51}{10\cdot2\cos\theta}$		**M1:** Consider motion → and expression for *t*
			$\uparrow: s = ut + \dfrac{1}{2}t^2$		
			$0 = \dfrac{10\cdot2\sin\theta\times7\cdot51}{10\cdot2\cos\theta} - \dfrac{g}{2}\left(\dfrac{7\cdot51}{10\cdot2\cos\theta}\right)^2$		**M1:** Consider motion vertically with this value of *t* and substitution
			$\dfrac{7\cdot51\sin\theta}{\cos\theta} - \dfrac{2\cdot656....}{\cos^2\theta} = 0 \qquad [\times\cos^2\theta]$		
			$7\cdot51\sin\theta\cos\theta - 2\cdot656.... = 0$		
			$3\cdot755\sin2\theta - 2\cdot656.... = 0$		**E1:** Solution of trig equation to give 2 angles of projection
			$\sin2\theta = 0\cdot707....$		
			$2\theta = 45\cdot02.... \qquad 2\theta = 134\cdot976....$		
			$\theta = 22\cdot5° \qquad \theta = 67\cdot5°$		
			$\uparrow: v^2 = u^2 + 2as$		**E1:** Find two possible heights
			$s = 4\cdot51$ or $s = 0\cdot76m$		
			Athlete cannot jump 4·51m vertically ⇒ Take-off angle ≈ 22·5°		**E1:** Explanation of answer
			Method 3 (equation of a trajectory):		
			$y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$		**M1:** Consider equation of trajectory.
			$0 = 7\cdot51\tan\theta - \dfrac{g(7\cdot51^2)}{2(10\cdot2^2)\cos^2\theta}$		**E1:** When $y = 0$ $x = 7\cdot51$ and $u = 10\cdot2$ arrange in suitable form and prepare to solve
			$7\cdot51\tan\theta - 2\cdot656....\sec^2\theta = 0$		
			$7\cdot51\tan\theta - 2\cdot656....\tan^2\theta - 2\cdot656.... = 0$		
			$\tan\theta = 2\cdot41$ or $\tan\theta = 0\cdot41$		**E1:** Solution of trig equation to give 2 angles of projection
			$\theta = 67\cdot2° \qquad \theta = 22\cdot3°$		
			$\uparrow: v^2 = u^2 + 2as$		**E1:** Find two possible heights
			$s = 4\cdot51$ or $s = 0\cdot76m$		
			Athlete cannot jump 4·51m vertically ⇒ Take-off angle ≈ 22·3°		**E1:** Explanation of answer

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	**10**		$F=\frac{kmg}{v}$	**2** **8**	**M1**: $F=\frac{P}{v}$ Connect power and force and substitution.
			$\frac{kmg}{v}-mg=mv\frac{dv}{dx}$		**E1**: $F=ma$ and using $a=v\frac{dv}{dx}$
			$v^2\frac{dv}{dx}=g(k-v)$		
			$\int\frac{v^2}{k-v}dv=\int g dx$		**M1**: Integration to find displacement and separation of variables
			$\frac{-v^2}{2}-kv-k^2\ln\lvert k-v\rvert=gx+C$		
			$x=0 \quad v=0: -\frac{1}{2}(0^2)-k(0)-k^2\ln\lvert k-0\rvert=0+C$		**E1**: Process of Integration
			$\Rightarrow C=-k^2\ln\lvert k\rvert$		
			$gx=k^2\ln\left\lvert\frac{k}{k-v}\right\rvert-kv-\frac{1}{2}v^2$		**E1**: Substitution and simplification
			At height *h*: $v=u\Rightarrow gh=k^2\ln\left\lvert\frac{k}{k-u}\right\rvert-ku-\frac{1}{2}u^2$		**E1:** Final substitution and expression processed
			$mgh+\frac{1}{2}mu^2=mk^2\ln\left\lvert\frac{k}{k-u}\right\rvert-mku-\frac{mu^2}{2}+\frac{mu^2}{2}$		**M1**: Work done = Change of energy
			$=m\left[k^2\ln\left\lvert\frac{k}{k-u}\right\rvert-ku\right]$		**E1**: Algebraic manipulation
			$\frac{m\left[k^2\ln\left\lvert\frac{k}{k-u}\right\rvert-ku\right]}{mkg}=\frac{k}{g}\ln\left\lvert\frac{k}{k-u}\right\rvert-\frac{u}{g}$		**M1**: Time = $\frac{\text{Work}}{\text{Power}}$ **E1**: Final manipulation

Question			Expected Answer(s)	Max Mark	Additional Guidance
A	10		**(cont)** ***Alternative for last 2 marks:*** Work done = $\int_0^T Fv\,dt = \int_0^T \frac{kmg}{v} \times v\,dt = \int_0^T kmg\,dt = kmgT$ $kmgT = \frac{1}{2}mu^2 + mgh$ $= \frac{1}{2}mu^2 + m(k^2 \ln\left\|\frac{k}{k-u}\right\| - ku - \frac{1}{2}u^2)$ $T = \frac{k}{g}\ln\left\|\frac{k}{k-u}\right\| - \frac{u}{g}$		

[*END OF SECTION A*]

Section B (Mathematics for Applied Mathematics)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**1**		$y = 2x\sqrt{x-1}$ $\frac{dy}{dx} = 2x.\frac{d}{dx}\left(\sqrt{x-1}\right) + \sqrt{x-1} \times \frac{d}{dx}(2x)$ $= 2x.\frac{1}{2}(x-1)^{-\frac{1}{2}} + \sqrt{x-1} \times 2$ Gradient given by $\frac{dy}{dx}$ when x = 10, Gradient = $10.(9)^{-\frac{1}{2}} + \sqrt{9} \times 2$ $= \frac{28}{3}$	**4**	**1** product rule **1** first correct term **1** second correct term **1** evaluation (accept decimal equivalent to minimum of 3 sf)
B	**2**	**(a)**	$A + B = \begin{pmatrix} 4 & -7 & 6 \\ k-3 & 9 & -1 \\ 5 & 1 & 1 \end{pmatrix}$	**1**	**1** evaluation
B	**2**	**(b)**	$\det A = 1\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 3\begin{vmatrix} k & -1 \\ 5 & 0 \end{vmatrix} + 4\begin{vmatrix} k & 0 \\ 5 & 3 \end{vmatrix}$ = 1(0 + 3) −3(0 + 5) + 4(3k – 0) = 12k – 12	**2**	**1** form of determinant 1 evaluation
B	**2**	**(c)**	$BC = \begin{pmatrix} 3 & -10 & 2 \\ -3 & 9 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 & -6 \\ 1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}$ $= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	**1**	**1** evaluation
B	**2**	**(d)**	$BC = 3I$. $B = 3C^{-1}$ or $C = 3B^{-1}$	**2**	**1** identity matrix connection or mention of inverse **1** relationship correct

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**3**		$I = \int x \sin 3x \, dx$	**5**	
			$u = x \qquad dv = \sin 3x$ $du = 1 \qquad v = \int \sin 3x dx$		**1** evidence of integration by parts
			$= \frac{-1}{3} \cos 3x$		**1** correct choice of *u*, *dv*
			$I = x.\frac{-1}{3}\cos 3x - \int 1.\frac{-1}{3}\cos 3x \, dx$		**1** correct substitution
			$= \frac{-x}{3}\cos 3x + \frac{1}{3}\int \cos 3x \, dx$		
			$= \frac{-x}{3}\cos 3x + \frac{1}{9}\sin 3x$		**1** final integration correct
			$I_0^{2\pi} = \left[\frac{-x}{3}\cos 3x + \frac{1}{9}\sin 3x \right]_0^{2\pi}$		
			$= \left[\frac{-2\pi}{3}\cos 6\pi + \frac{1}{9}\sin 6\pi \right] - \left[0 + \frac{1}{9}\sin 0 \right]$		
			$= \frac{-2\pi}{3}$		1 evaluation

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**4**		$\sum_{r=1}^{80} 3r^2 = 3\sum_{r=1}^{80} r^2$ using $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ * $3\sum_{r=1}^{80} r^2 = 3\left(\frac{80(81)(2\cdot 80+1)}{6}\right)$ = 521,640	**2**	**1** correct substitution into * **1** evaluation (using incorrect formula – this mark available if of equivalent difficulty eg $\sum_{r=1}^{n} r^2 = \left(\frac{n(n+1)}{2}\right)^2$
B	**5**	**(a)**	$(e^x + 2)^4$ $= 1.(e^x)^4(2)^0 + 4(e^x)^3(2)^1 + 6(e^x)^2(2)^2$ $+ 4.(e^x)^1(2)^3 + 1.(e^x)^0(2)^4$ $= e^{4x} + 8e^{3x} + 24e^{2x} + 32e^x + 16$	**3**	Accept Binomial expansion ***or*** Pascal's Triangle **1** correct coefficients **1** correct powers of e^x and 2 **1** simplification
B	**5**	**(b)**	$\int (e^x + 2)^4 dx$ $= \int (e^{4x} + 8e^{3x} + 24e^{2x} + 32e^x + 16) dx$ $= \frac{e^{4x}}{4} = \frac{8e^{3x}}{3} + \frac{24e^{2x}}{2} + 32e^x + 16x + c$	**2**	**1** correct integration of composite function (at least one correct term involving composite exponential) **1** completion of integral (+ *c* not essential)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**6**	**(a)**	10 000 people.	**1**	
B	**6**	**(b)**	$\frac{10000}{N(20000-N)} = \frac{A}{N} + \frac{B}{20000-N}$ 10 000 = $A(20\,000 - N) + BN$ $A = \frac{1}{2}, \quad B = \frac{1}{2}$ Using $\frac{10000}{N(20000-N)} dN = dt$ gives $\frac{1}{2}\left(\frac{1}{N} + \frac{1}{20000-N}\right) dN = dt$ Integrating, $\int\left(\frac{1}{N} + \frac{1}{20000-N}\right) dN = \int 2dt$ $\ell nN - \ell n(20000-N) = 2t + c$ $\ell n\frac{N}{20000-N} = 2t + c$	**5**	**1** appropriate form of partial fractions **1** correct values of *A* and *B* **1** separate variables **1** starts integration eg $\int \frac{1}{N} dN$ correct **1** completes integration (moduli signs not required)

Question			Expected Answer(s)	Max Mark	Additional Guidance
B	**6**	**(c)**	Using $\ell n \dfrac{N}{20000-N} = 2t + c$ gives $\dfrac{N}{20000-N} = e^{2t+c}$ $\dfrac{N}{20000-N} = Ke^{2t}\left(\text{where } K = e^{c}\right)$ When $t = 0$, $N = 100$ $\dfrac{100}{19900} = K$ $K = \dfrac{1}{199}$ Hence $N = \left(20000 - N\right)\dfrac{e^{2t}}{199}$ $199N = (20\,000 - N)\,e^{2t}$ $N(199 + e^{2t}) = 20\,000e^{2t}$ $N = \dfrac{20000e^{2t}}{199 + e^{2t}}$	**4**	**1** accurately converts to exponential form (stating explicitly $K = e^{c}$ not required) **1** interprets initial condition **1** K valve **1** correctly gathers N terms

[*END OF SECTION B***]**

[*END OF QUESTION PAPER***]**