National
Qualifications
2023

2023 Statistics

Advanced Higher - Paper 2

Finalised Marking Instructions

General marking principles for Advanced Higher Statistics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$ ← This is a transcription error and so the mark is not awarded.

$$x=1$$ ← This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2+5x+7=9x+4$$

$$x-4x+3=0$$ ← This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$(x-3)(x-1)=0$$

$x=1$ or 3

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$ •6 $y = 5$ and $y = -7$

Vertical: •5 $x = 2$ and $y = 5$ •6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ *written as*

  $\left(x^3+2x^2+3x+2\right)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] appropriate strategy •[2] calculate probability	•[1] $P\left(Z > \frac{111-109}{7}\right)$ •[2] $P(Z > 0.29) = 0.3859$	**2**

Notes:
1. For •[2], answer of 0.3897 is not accepted. It comes from calculating $P(Z > 0.28)$
2. For •[2], if calculated exactly, $P\left(Z > \frac{2}{7}\right) = 0.3875$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] correct sampling distribution •[4] appropriate strategy •[5] calculate probability	•[3] $\bar{X} \sim N\left(109, \frac{49}{25}\right)$ •[4] $P(\bar{X} > 111) = P\left(Z > \frac{111-109}{\sqrt{\frac{49}{25}}}\right)$ •[5] 0.0764	**3**

Notes:
1. The notation of $\bar{X}$ is required to be seen in either •[3] or •[4]
2. For •[5], answer of 0.0778 is not accepted. It comes from calculating $P(Z > 1.42)$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[6] appropriate explanation	•[6] The sample mean's distribution is less spread out than the population's distribution of heights	**1**

Notes:

Commonly Observed Responses:
1. Candidates who refer to the Central Limit Theorem, gain 0 marks

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] correct assumption	•[1] the distribution of steps is symmetrical	6
			•[2] correct hypotheses	•[2] $H_0 : \eta = 300 \quad H_1 : \eta > 300$	
			•[3] correct test statistics	•[3] $n = 10 \quad W_- = 7.5 \quad W_+ = 47.5$	
			•[4] correct critical value	•[4] 5% 1-tail cv is 10	
			•[5] deal with H_0	•[5] 7.5<10 so we reject H_0 at the 5% significance level	
			•[6] appropriate conclusion	•[6] and conclude that there is evidence that the step counter from the mobile phone does over-count the median number of steps that Duncan takes.	

Notes:

1. For •[1], must include the context of 'steps'
2. For •[3], the only requirement is $W_- = 7.5$
3. For •[6], do not accept conclusions that are too definite. Phrasing must include 'evidence to conclude…', or 'evidence to suggest…', or similar

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•[1] appropriate distribution •[2] appropriate strategy •[3] calculate probability	•[1] $X \sim \text{B}(20, 0.02)$ •[2] $\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1)$ •[3] 0.0599	**3**

Notes:

1. For •[3], if 'answer only' is given, with no communication of the values of n and p that were used, then only gain 2 marks.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] state distribution •[5] appropriate justification •[6] appropriate approximation •[7] use of continuity correction •[8] calculate z •[9] calculate probability	•[4] $X \sim \text{B}(50, 0.504)$ •[5] $np = 25.2 > 5$, $nq = 24.8 > 5$ •[6] $X \approx \text{N}(25.2, 12.4992)$ •[7] $\text{P}(X \leq 30) = \text{P}\left(Z \leq \frac{30.5 - 25.2}{\sqrt{12.4992}}\right)$ •[8] $\text{P}(Z \leq 1.50)$ •[9] 0.9332	**6**

Notes:

1. Obtaining 0.9335 using the binomial distribution would gain only marks •[4] and •[9]
2. Mark •[4] can be awarded by implication from marks •[5] or •[6].
3. For •[5], both np and nq require to be evaluated and compared to 5.
4. For •[5], demonstration of $npq > 5$ gains 0 marks.

Commonly Observed Responses:

1. $\text{P}(X \leq 30) = \text{P}\left(Z \leq \frac{30 - 25.2}{\sqrt{12.4992}}\right) = \text{P}(Z \leq 1.36) = 0.9131$ loses mark •[7]
2. $\text{P}(X \leq 30) = \text{P}\left(Z \leq \frac{29.5 - 25.2}{\sqrt{12.4992}}\right) = \text{P}(Z \leq 1.22) = 0.8888$ loses mark •[7]

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] correct E_i •[2] calculate X^2 •[3] correct critical value •[4] deal with H_0	•[1] E_i are 80, 80 and 160 •[2] $X^2 = \frac{4}{80} + \frac{100}{80} + \frac{64}{160} = 1.7$ •[3] $\chi^2_{2,0.90} = 4.605$ •[4] as $1.7 < 4.605$, do not reject H_0	4

Notes:

1. For •[2], calculation method can only gain credit if $\sum E_i = 320$
2. Alternatively, •[3] p-value = 0.4274, and •[4] $0.4274 > 0.01$, do not reject H_0

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)	(i)	•[1] correct probability	•[1] $P(X = 4) = 1 - 9p$	**2**
			•[2] appropriate strategy	•[2] $\begin{cases} E(X) = 0p + 1p + 4p + 15p + 4(1-9p) \\ \quad = 4 - 16p \end{cases}$	
Notes: 1. Working backwards from $E(X) = 4 - 16p$ to $P(X = 4)$ not acceptable					
Commonly Observed Responses:					
		(ii)	•[3] calculate p	•[3] $4 - 16p = 3 \quad \Rightarrow \quad p = \frac{1}{16}$	**4**
			•[4] correct strategy for $V(X)$	•[4] $V(X) = E(X^2) - E^2(X)$	
			•[5] calculate $E(X^2)$	•[5] $E(X^2) = 16 - 90p = \frac{83}{8}$	
			•[6] calculate $V(X)$	•[6] $V(X) = \frac{11}{8}$	
Notes:					
Commonly Observed Responses: 1. Methods using $V(X) = \sum P(X = x)(x - E(X))^2$ can gain full marks, if completed correctly.					
	(b)		•[7] correct parameters	•[7] $E(Y) = V(Y) = 1$	**4**
			•[8] calculate $E(K)$	•[8] $E(K) = 2 - 3 + 3 = 2$	
			•[9] calculate $V(K)$	•[9] $V(K) = 4 + \frac{11}{8} + 0 = \frac{43}{8} = 5.375$	
			•[10] calculate $SD(K)$	•[10] $SD(K) = 2.318$	
Notes: 1. Mark •[7] can be awarded by implication from correct usage in both •[8] and •[9]					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
6.			•[1] state hypotheses	•[1] $H_0 : \mu = 50$ $H_1 : \mu > 50$	6
			•[2] correct sample mean and standard deviation	•[2] $\bar{x} = 51.2$ and $s = 4.696$	
			•[3] calculate test statistic	•[3] $z = \dfrac{51.2 - 50}{\frac{4.696}{\sqrt{75}}} = 2.21$	
			•[4] appropriate critical value	•[4] $z_{0.99} = 2.33$	
			•[5] deal with H_0	•[5] 2.21 < 2.33 so we do not reject H_0 at the 1% level of significance	
			•[6] appropriate conclusion	•[6] and conclude that there is insufficient evidence to support the midwife's theory	

Notes:

1. For •[4], p-value = P (Z > 2.21) = 0.0136
2. For •[4], also accept $t_{74,0.99} = 2.378$, from graphic calculator
3. For •[4], also accept p-value = P(t_{74} > 2.21) = 0.0150
4. For •[6], also accept 'and conclude that evidence suggests that the mean length of babies born to basketball players are significantly longer than the general population of full-term babies'
5. For •[6], do not accept conclusions that are too definite. Phrasing must include 'evidence to conclude…', or 'evidence to suggest…', or similar

Commonly Observed Responses:

1. Hypotheses that do not include the population mean and the number '50' gain 0 marks.

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•1 appropriate strategy •2 correct probability	•1 $P(W\mid T)=\dfrac{P(T\cap W)}{P(T)}$ •2 $=\dfrac{0.035}{0.2}=0.175$	**2**

Notes:

1. Mark •1 can implied by mark •2

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•3 appropriate strategy •4 calculate probability •5 calculate probability •6 appropriate strategy •7 calculate probability	•3 $P(B)=\dfrac{P(C\cap B)}{P(C)}$ •4 $=\dfrac{0.12}{0.3}=0.4$ •5 P (A) = 1 – 0.4 = 0.6 •6 $P(J\cap A)=P(J)P(A)$ •7 = 0.5 × 0.6 = 0.3	**5**

Notes:

1. Drawing a tree diagram is also an acceptable strategy.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] appropriate hypotheses •[2] correct test statistic •[3] correct value of t •[4] appropriate critical value •[5] deal with H_o •[6] appropriate conclusion	•[1] $H_0: \rho = 0 \quad H_1: \rho \neq 0$ •[2] $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ •[3] $= \frac{0.652\sqrt{23}}{\sqrt{1-0.652^2}} = 4.124$ •[4] 0.1% cv is $t_{23,\ 0.9995}$ = 3.768 •[5] $4.124 > 3.768$ so we reject H_0 (at the 0.1% significance level) •[6] conclude that there is (very strong) evidence of a **linear** association between body mass and plasma volume in healthy women	6

Notes:

1. For •[1], also accept hypothesis phrased in terms of correlation, in the context
2. For •[1], do not accept 'H_0: … linearly associated'
3. Alternatively, •[4] p-value = 0.000413
4. For •[6], do not accept conclusions that are too definite. Phrasing must include 'evidence to conclude…', or 'evidence to suggest…', or similar

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•1 state hypotheses	•1 $H_0 : \mu_d = 0$ $H_1 : \mu_d > 0$	6
			•2 appropriate strategy	•2 $t_{n-1} = \dfrac{\overline{x}_d}{s_d / \sqrt{n}}$	
			•3 calculate test statistic	•3 $t = \dfrac{0.45 - 0}{0.927 / \sqrt{12}} = 1.68$	
			•4 correct critical value	•4 $t_{11,0.95} = 1.796$	
			•5 deal with H_0	•5 $1.68 < 1.796$ so we do not reject H_0 at the 5% level of significance	
			•6 appropriate conclusion	•6 There is insufficient evidence that the mean difference in distance run is greater than zero	

Notes:

1. For •1, hypotheses must be phrased in terms of the mean difference and the number 0.
2. For •2, do not accept $t_{n-1} = \dfrac{\overline{x}_d}{\sigma / \sqrt{n}}$
3. Alternatively, for •4 p-value = $P(t_{11} > 1.68) = 0.0606$
4. For •5, candidates must clearly state their chosen level of significance in their solution
5. For •6, also accept 'insufficient evidence that the runners ran further with a fitness tracker'
6. For •6, do not accept references to running for longer, or running faster
7. For •6, do not accept conclusions that are too definite. Phrasing must include 'evidence to conclude…', or 'evidence to suggest…', or similar

Question			Generic scheme	Illustrative scheme	Max mark
9.	(continued)				

Commonly Observed Responses:

Candidate A – performed Wilcoxon Signed Rank Test (paired data)
Mark •[1] requires H_0: median difference = 0 and H_1: median difference > 0
Mark •[2] not available
Mark •[3] requires $W\_$ = 20.5 or W_+ = 57.5
Mark •[4] requires critical value of 17 (for 5% with n = 12)
Marks •[5], •[6] available only if consistent with previous workings, and must mention 'medians' rather than 'means'

Candidate B – performed one-sample z-test on mean differences (paired data)
Mark •[1] requires $H_0 : \mu_d = 0$ and $H_1 : \mu_d > 0$
Marks •[2] and •[3] not available
Mark •[4] requires $z_{0.95} = 1.64$ or p-value = 0.0463
Marks •[5], •[6] available only if consistent with previous workings

Candidate C – performed two-sample z-test for a difference in population means (non-paired data)
Mark •[1] requires $H_0 : \mu_{with} = \mu_{without}$ $H_1 : \mu_{with} > \mu_{without}$
Marks •[2] and •[3] not available
Mark •[4] requires $z_{0.95}$ = 1.64 or p-value = 0.3420
Marks •[5], •[6] available only if consistent with previous workings

Candidate D – performed two-sample t-test for a difference in population means (non-paired data)
Mark •[1] requires $H_0 : \mu_{with} = \mu_{without}$ $H_1 : \mu_{with} > \mu_{without}$
Marks •[2] and •[3] not available
Mark •[4] requires $t_{22,0.95}$ = 1.717 or p-value = 0.3503
Marks •[5], •[6] available only if consistent with previous workings

	(b)	(i)	•[7] appropriate comment	•[7] the differences do not appear to be symmetrical about a central value, therefore an assumption of normality would not be appropriate	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)	(ii)	•[8] appropriate test •[9] appropriate comment	•[8] a Wilcoxon Signed Rank test could be considered. •[9] the differences would be assumed to be symmetrical, therefore this test would not be suitable	**2**

Notes:

1. For •[8], consideration should be given to the candidate's response to part (a). If they had decided that the data was non-paired, then accept 'Mann-Whitney Test'

2. For •[9], consideration should be given to the candidate's response to part (b)(i). If they had decided that the assumption of normality was valid, then accept 'the differences would be assumed to be symmetrical, therefore this test would be suitable'

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] state hypotheses •[2] appropriate strategy •[3] correct z value •[4] appropriate critical value •[5] deal with H_0 •[6] appropriate comment	•[1] $H_0 : p = 0.624 \quad H_1 : p \neq 0.624$ •[2] $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}}$ •[3] $= \dfrac{\dfrac{23312}{37878} - 0.624}{\sqrt{\dfrac{0.624 \times (1 - 0.624)}{37878}}} = -3.43553$ •[4] $z_{0.0025} = -2.81$ •[5] $-3.44 < -2.81$, so we reject H_0 (at the 0.5% level of significance) •[6] and conclude that there is evidence of the proportion of homeless veterans in sheltered accommodation in 2018 being different to that from 2010-2017	**6**

Notes:

1. For •[2] and •[3] the alternative use of the normal approximation to the binomial distribution with continuity correction and a p-value of 0.000603 (or z = -3.43023) is acceptable

2. For •[3] if a candidate uses $\hat{p}$ = 0.615 (the 3 decimal place approximation to the exact fraction) then they will obtain a z-value = -3.62, but this does not change the conclusion

3. Alternatively, for •[4] p-value = 2 × P(Z < −3.44) = 2 × 0.000296 = 0.000591

4. Using tables for •[4] p-value = 2 × P(Z < −3.44) = 2 × 0.0003 = 0.0006

5. For •[6], do not accept conclusions that are too definite. Phrasing must include ‘evidence to conclude…’, or ‘evidence to suggest…’, or similar

Commonly Observed Responses:

Candidate A – performed a z-test for a difference in population proportions

Mark •[1] requires $H_0 : p_1 = p_2$ and $H_1 : p_1 \neq p_2$
Marks •[2] and •[3] not available
Mark •[4] requires $z_{0.0025} = -2.81$
Marks •[5], •[6] available only if consistent with previous workings

Question			Generic scheme	Illustrative scheme	Max mark
11.			•[1] interpret first probability statement •[2] interpret second probability statement •[3] appropriate strategy •[4] calculate μ and σ	•[1] $\frac{24-\mu}{\sigma}=1.64$ •[2] $\frac{17-\mu}{\sigma}=-1.28$ •[3] $24-1.64\sigma=17+1.28\sigma$ •[4] μ = 20.07 and σ = 2.397	**4**

Notes:

1. Answers of $\mu \approx 20.0655$ and $\sigma \approx 2.3920$ may be obtained when working with z-values taken from a graphic calculator.
2. For •[3], also accept similar equations that have eliminated one variable

Commonly Observed Responses:

1. If $\frac{17-\mu}{\sigma}=1.28$ is used, then μ = −7.889 and σ = 19.444 gaining maximum of 3 marks.

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] correct strategy •[2] correct substitution •[3] calculate interval •[4] appropriate explanation	•[1] $\hat{p} \pm z_{0.995}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ •[2] $0.55 \pm 2.58\sqrt{\frac{0.55 \times 0.45}{100}}$ •[3] $(0.422, 0.678)$ •[4] the calculation involves a normal approximation to the binomial distribution	**4**

Notes:

1. For •[1], $\hat{p} \pm t_{n,0.995}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ gains 0 marks.
2. For •[4], responses that refer to the sample size gain 0 marks.
3. For •[4], do not accept 'binomial approximation to the normal distribution'

Commonly Observed Responses:

1. $55 \pm 2.58\sqrt{\frac{0.55 \times 0.45}{100}} = (54.872, 55.128)$ loses mark •[2] only.

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[5] appropriate strategy •[6] correct substitution •[7] calculate n	•[5] lower limit > 0.50 •[6] $0.55 - 2.58\sqrt{\frac{0.55 \times 0.45}{n}} > 0.50$ •[7] $n = 659$	**3**

Notes:

1. For •[5], also accept 'lower limit = 0.50' and 'lower limit ≥ 0.50'
2. For •[5], do not accept 'lower limit < 0.50' or 'lower limit ≤ 0.50'
3. For •[5], do not accept expressions that include the upper limit. ie $\hat{p} \pm z_{0.995}\sqrt{\frac{\hat{p}\hat{q}}{n}} > 0.50$

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]