WEDNESDAY, 18 MAY
1:00 PM - 3:30 PM

## Total marks - 75

You may refer to the Advanced Higher Engineering Science Data Booklet.
SECTION 1 - 35 marks
Attempt ALL questions.

## SECTION 2 - 40 marks

Attempt ALL questions.
Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.
For questions 3 (b), 4 (a) and 9 (a), write your answers clearly in the worksheets provided in the answer booklet.
Show all working and units where appropriate.
The number of significant figures expressed in a final answer should be equivalent to the least significant data value given in the question. Answers that have two more figures or one less figure than this will be accepted.

Use blue or black ink. Sketches, diagrams and graphs may be drawn in pencil.
Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

## SECTION 1 - 35 marks

## Attempt ALL questions

1. A beam has the symmetrical cross-section shown below.


Note that the second moment of area of a triangle about its base is given by the equation below.

$$
I_{x x}=\frac{b h^{3}}{12}
$$



Calculate the second moment of area of the cross-section about axis $\mathrm{X}-\mathrm{X}$.
2. The electricity supply to the UK is based on a collection of fossil fuel, nuclear, hydro and renewable energy power stations connected to a distribution grid.
(a) Explain why it is necessary to include hydro-electric pump storage systems as part of the electrical supply for the UK.

An innovative solution to the storage of electricity is to use a large mass that can be raised or lowered in a disused coal mine shaft. The mass is raised by an electric motor when surplus electricity is available; when electricity is required the mass can fall, driving a generator as it does so.


The motor has an efficiency of $96 \%$, the generator $94 \%$ and there is a $3 \%$ loss in the mechanical system when raising and a $3 \%$ loss when lowering.
(b) Calculate the amount of electricity available from this storage system for every 1 unit of electricity produced prior to storage.
3. Floating wind power is an emerging technology with a UK industry generation ambition of 5 GW in the near future. The expected generation in 2021 was 80 MW .


The graph below shows the predicted change in capital costs for the various components of the system as they progress from prototype to commercial production.

Note: CAPEX/MW (£m) — CAPital EXpenditure per Mega Watt

(a) Describe what capital costs are and explain why there is a predicted downward trend in the capital costs.
3. (continued)

During the installation of a turbine prototype, a project manager divided the project into eight stages, $\mathrm{A}-\mathrm{H}$, identified the precedents for each stage and the planned duration of each stage. The information is detailed in the precedence table below.

| Stage | Precedents | Duration |
| :---: | :---: | :---: |
| A | ---- | 4 |
| B | ---- | 1 |
| C | B | 4 |
| D | A, B | 7 |
| E | C | 3 |
| F | E | 6 |
| G | A, C | 7 |
| H | D, F, G | 3 |

(b) Using the precedence table and the worksheet for question 3 (b)
(i) complete the Gantt chart for stages G and H of the project, showing the
timing of each stage including floats
(ii) identify the critical path for this project.
(c) Describe how the project manager would use the information in the Gantt chart to manage the project effectively.
4. A simple audio amplifier circuit is being tested by a student. The circuit diagram is shown below.


The MOSFET must be biased in the saturation region.
(a) Draw the load line for the circuit on the worksheet for question 4 (a) and identify the most suitable value for $\mathrm{V}_{\mathrm{GS}}$ to allow maximum output voltage swing.

Design rule: The combined resistance of $R_{1}$ in parallel with $R_{2}$ is $100 \mathrm{k} \Omega$.
(b) Calculate suitable values for $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$.
5. A pulse-width modulation (PWM) signal from a microcontroller is amplified by a MOSFET circuit. The amplified PWM signal is shown below.


The MOSFET dissipates power when conducting and when switching on and off.
The graphs below show a magnified view of the $\mathrm{V}_{\mathrm{DS}}$ and $\mathrm{I}_{\mathrm{DS}}$ values when a MOSFET switches on and off.

$$
\mathrm{T}_{\text {on }}=24 \mathrm{~ns}
$$


time (ns)
$\mathrm{T}_{\text {off }}=28 \mathrm{~ns}$

time (ns)

When each MOSFET switches on and off it experiences a switching energy loss given by the formula below.

$$
E_{\text {Switching }}=\frac{1}{2} V_{D S(o f f)}\left(T_{o n} I_{D S(o n)}+T_{o f f} I_{D S(o n)}\right)
$$

The switching power is calculated using the formula below.

$$
P_{\text {Switching }}=E_{\text {Switching }} \times \text { frequency }
$$

The frequency is the number of times that the MOSFET switches on and off in each second.

Determine the MOSFET switching power loss for this MOSFET.
6. A touchscreen system used to control a robotic arm is shown below. When the user touches the screen, current is drawn from electrodes at each corner and is 'earthed' through their finger.


The current flow from each corner is processed by an integrator circuit, similar to the one shown below, and a voltage signal is produced.


Assume that the op-amps are ideal. At $\mathrm{t}=0 \mathrm{~s}$ both $\mathrm{V}_{2}$ and $\mathrm{V}_{\text {out }}$ are 0 V .
For one position on the screen, $\mathrm{V}_{\text {in }}=5.0 \mathrm{~V}$.
The graph of $\mathrm{V}_{2}$ is shown below.

6. (continued)
(a) Calculate the value of capacitor, C .

The capacitor, $C$, calculated in part (a) is replaced with a $33 \mu \mathrm{~F}$ capacitor.
(b) Describe how this change in capacitance affects the time taken for op-amp 1 to saturate.
(c) Determine, in its simplest form, the expression for $\mathrm{V}_{\text {out }}$ in terms of t .
7. A weightlifter lifts a total mass of 190 kg on a bar that measures 1.50 m between the loads. The weightlifter's hands are both 0.50 m from the loads. The bar is made from a low-alloy steel tube: it has an external diameter of 29.0 mm and a second moment of area of $21 \times 10^{3} \mathrm{~mm}^{4}$.


The deflection of a simply supported beam with two equally spaced vertical loads is given by the formula below.

$$
\delta_{\max }=\frac{P a}{24 E I}\left(3 L^{2}-4 a^{2}\right)
$$

(a) Calculate the maximum deflection of the bar due to the applied loads.

The diagram below is a sketch of the shape of the bending moment diagram for the bar as it is being held above the weightlifter, as described above.
The magnitude of the bending moment reaches a maximum where the weightlifter grips the bar.

(b) Determine the maximum stress in the bar when it is stationary above the weightlifter.
7. (continued)

The ultimate tensile and compressive stresses for low-alloy steels lie in the range $680-2400 \mathrm{Nmm}^{-2}$.
(c) Compare the suitability of low-alloy steel for this application at both ends of the range in terms of the safety of the weightlifter. You should refer to your answer in part (b).

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## Attempt ALL questions


8. A high-speed passenger train is powered from a single overhead $25 \mathrm{kV}, 50 \mathrm{~Hz} \mathrm{AC}$ cable. A block diagram of the locomotive electrical system is given below.

(a) State the names given to sub-systems $A$ and $B$ and explain the need for both in order to drive the locomotive's motors.

As the high-speed passenger train begins to accelerate, a step-down transformer on the locomotive draws 5.2 MVA of power at full load from the overhead 25 kV AC supply cable ( 1 MVA is equivalent to 1 MW ). The transformer output voltage is 1.5 kV .

The power loss in the transformer core is 2.5 kW . Currents in the primary and secondary windings also cause power loss. The primary winding of the transformer has a resistance of $482 \mathrm{~m} \Omega$ and the secondary winding has a resistance of $1.74 \mathrm{~m} \Omega$.
(b) Calculate the current in the secondary winding and hence the resistive power loss in the secondary winding.
8. (continued)

A student investigating AC motor speed control uses a form of pulse-width modulation (PWM) to control a single-phase AC motor as shown below.

## PWM <br> AC signal



The period of the PWM signal matches the period of the AC output signal. Varying the PWM signal period allows the speed of the motor to be varied.
The program opposite produces a PWM signal of 59.5 Hz which is used to control the speed of an AC motor. Twelve values are stored in a data table. The values are used to adjust the duty cycle and produce the PWM signal. Elsewhere, the microcontroller has been configured to produce a PWM signal with a frequency of 15.6 kHz on the output pin.
8. (continued)

| Arduino | PBasic |
| :---: | :---: |
| ```int i; int duty; int table[] = {128, 192, 238, 255, 238, 192, 128, 64, 18, 0, 18, 64}; void setup() { pinMode(6, OUTPUT); } void loop() { for (i=1; i <= 12; i = i + 1) { analogWrite(6, table[i]); delayMicroseconds(1400); } }``` | ```symbol i = b0 symbol duty = b1 table (128, 192, 238, 255, 238, 192, 128, 64, 18, 0, 18, 64) main: for i=1 to 12 readtable i, duty pwmduty b.6, duty pauseus }14 next i goto main``` |
| Command analogWrite(pin, value) <br> Parameters <br> pin: the Arduino pin to write to <br> value: the duty cycle: between 0 <br> (always off) and 255 (always on) <br> delayMicroseconds (value) <br> value: the number of microseconds to pause | Command <br> pwmduty, pin, duty <br> Parameters <br> pin: a variable/constant which is the pin to write to <br> duty: the duty cycle: between 0 (always off) and 255 (always on) <br> pauseus value <br> value: multiples of 10 microseconds to pause |

## Example

$$
50 \% \text { duty cycle }- \text { duty value }=128
$$


$25 \%$ duty cycle - duty value $=64$

(c) Explain how the period, T , of the signal could be varied within the program.

PWM duty values could be calculated within the program during each loop rather than reading them from a data table, as in the program above.
(d) Explain which of the two methods would produce the most accurate output waveform.
8. (continued)

The two axles on a bogie of a high-speed locomotive are each driven by an AC electric motor acting through a gear train, as shown below.


The gear train is shown below. For each gear, the pitch circle diameter where the gear teeth mesh is proportional to the number of teeth on the gear. The pitch circle diameter of the gear on the bogie axle is 480 mm . All gears have a pressure angle of $20^{\circ}$. For a time at which the train is moving at constant speed, the motor produces 80 kW at 1800 revs $\mathrm{min}^{-1}$.

(e) Calculate the magnitude of the force acting on the motor shaft gear at the point at which it meshes and indicate, on a sketch, the direction of this force.
8. (continued)

The forces on the gear and on the axle are shown below for a time where the passenger train is running at a constant speed. The gear force is 48.6 kN acting at $20^{\circ}$ and the wheel forces are 122 kN vertically and 14.4 kN horizontally (for each wheel).

(f) Determine the magnitude and direction of the reaction at bearing $B$ for the given axle loading conditions.
9. A free-body diagram for one leg of the frame that supports a turbine that is under water is shown below. It is simply supported with a uniformly distributed load representing the ballast to hold the structure on the seabed. The turbine load is represented as two vertical point loads.

(a) Draw the shear force diagram for the loaded beam on the worksheet for question 9 (a).

The maximum bending moment acting on the beam occurs at a point between the 25.0 kN and 50.0 kN point loads. A distance, $x$, is measured to a point along the length of the beam from its left-hand end.
(b) (i) Construct the equation to define the bending moment as a function of length along the beam, $x$, for the section of the beam between the two point loads $(4 \leq x \leq 6)$.
(ii) Determine, by differentiating the bending moment equation, the distance from the left-hand end of the beam at which the maximum bending moment occurs.
9. (continued)

The speed of the turbine is measured using a magnetic sensor and a slotted wheel, as shown below.


As the turbine rotates the following signal is produced.


A Schmitt trigger circuit is used to remove the electrical noise from the signal.


Minimum $\mathrm{V}_{\text {out }}=0.00 \mathrm{~V}$
Maximum $\mathrm{V}_{\text {out }}=5.20 \mathrm{~V}$
(c) Calculate, using nodal analysis, suitable values for $R_{1}$ and $R_{2}$.
9. (continued)

A program is written to further process the signal from the slotted wheel and to display the speed of the turbine in revs $\mathrm{min}^{-1}$. The slotted wheel has 20 slots.
Section $A$ of the program counts the number of pulses on pin 0 which go from a low level to a high level over a period of 3 seconds. The value of 3000 in section A produces this delay.

| Arduino | PBasic |
| :---: | :---: |
| volatile int counter $=0$; <br> int displayrpm $=0$; <br> int $\mathrm{rpm}=0$; | symbol displayrpm $=\mathrm{w} 1$ <br> symbol rpm = w2 <br> symbol counter $=$ w0 |
| Section A | Section A |
| ```void setup() { Serial.begin(9600); attachInterrupt(0, count, RISING);} void count() {counter = counter + 1;} void loop() { delay(3000);``` | main: count b.0, 3000, counter |
| Section B | Section B |
| ```rpm = counter; Serial.print(rpm); counter = 0; }``` | ```rpm = counter serout 7, N2400, (rpm) counter = 0 goto main``` |

Note: There is no synchronisation between the input signal and the beginning of the timing cycle.
Ideal scenario - timing begins just as a rising edge occurs.


This would produce a reading of 3 revs $\mathrm{min}^{-1}$.
9. (continued)

Non-ideal scenario — timing begins just after a rising edge.


This would produce an output speed reading of 2 revs $\mathrm{min}^{-1}$.
(d) Calculate the maximum percentage error the program could produce for a turbine speed of 10 revs $\mathrm{min}^{-1}$.

In operation the turbine runs at rotational speeds in the range 5 to 9 revs $\mathrm{min}^{-1}$. The manufacturer wants to guarantee a speed reading with a $5 \%$ tolerance for a minimum speed of 5 revs $\mathrm{min}^{-1}$. They also want to display the speed with three digits as shown below. The decimal point is printed on the screen.

(e) Describe how the program, shown opposite, could be adapted to ensure a speed reading is displayed as described above, and within tolerance.

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