TUESDAY, 16 MAY
1:00 PM - 3:30 PM

## Total marks - 75

You may refer to the Advanced Higher Engineering Science Data Booklet.

SECTION 1 - 35 marks
Attempt ALL questions.

## SECTION 2 - 40 marks

Attempt ALL questions.

Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.
For questions 1 (a), 8 (a) and 8 (e), write your answers clearly in the worksheets provided in the answer booklet.

Show all working and units where appropriate.
The number of significant figures expressed in a final answer should be equivalent to the least significant data value given in the question. Answers that have two more figures or one less figure than this will be accepted.

Use blue or black ink. Sketches, diagrams and graphs may be drawn in pencil.
Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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## SECTION 1 - 35 marks

Attempt ALL questions

1. A precedence table for the stages in part of a civil engineering construction project is given below. The duration of each stage is measured in weeks.

| Stage | Precedence | Duration <br> (weeks) |
| :---: | :---: | :---: |
| A | -- | 2 |
| B | -- | 4 |
| C | -- | 6 |
| D | A | 8 |
| E | B, C | 3 |
| F | D | 1 |
| G | E | 6 |
| H | E | 5 |

(a) Complete an activity network diagram for the project using the worksheet for question 1 (a).
(b) Identify the critical path for the project.

The project life cycle is defined in four phases: initiation, planning, implementation and closing.
(c) Identify which of these four phases would include the production of a precedence table and an activity network diagram.
(d) Explain the need for one activity that a project manager would complete during the closing phase of a project.
2. The diagram below represents the structure of the UK National Grid.

The UK national grid

(a) Explain the need for step-up transformers at power stations.
(b) Give an example of a form of power generation that would be suited to supplying base load but not responding to peak load and explain why this would be the case.
2. (continued)

Wind turbines contribute significantly to meeting the UK's requirement for electricity. A block diagram for the electricity generation system in a wind turbine is given below.

(c) Name the sub-systems $A$ and $B$ and, for each, describe its function within the system.
3. An electronic amplifier stage is given in the circuit below. For component calculations, ignore the greyed-out components.


Circuit specifications:

- $\mathrm{V}_{\mathrm{CC}}=12 \mathrm{~V}$
- $I_{C}=2.4 \mathrm{~mA}$
- $\mathrm{V}_{\text {out }}=7.2 \mathrm{~V}$
- $\mathrm{V}_{\mathrm{E}}=1.25 \mathrm{~V}$
- $\mathrm{h}_{\mathrm{FE}(\min )}=210$
- $\mathrm{V}_{\mathrm{BE}}=0.62 \mathrm{~V}$

Standard values of 5\% tolerance resistors are given in the table opposite.
(a) Determine the required 5\% resistor values for $R_{c}$ and $R_{e}$. The values selected must be supported by calculations and the table on the opposite page.

The circuit design uses an $82 \mathrm{k} \Omega$ resistor for $R_{1}$ and a $16 \mathrm{k} \Omega$ resistor for $R_{2}$. The measured base current is $11.1 \mu \mathrm{~A}$.

Using nodal analysis at the node between $\mathrm{R}_{1}, \mathrm{R}_{2}$ and the base of the transistor:
(b) calculate the base voltage.

For the values given in the circuit specification shown above:
(c) determine the intended power dissipation in the transistor.
3. (continued)

| Standard resistor values ( $\pm 5 \%$ ) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 10 | 100 | 1.0 k | 10 k | 100 k | 1.0 M |
| 1.1 | 11 | 110 | 1.1 k | 11 k | 110 k | 1.1 M |
| 1.2 | 12 | 120 | 1.2 k | 12 k | 120 k | 1.2 M |
| 1.3 | 13 | 130 | 1.3 k | 13 k | 130 k | 1.3 M |
| 1.5 | 15 | 150 | 1.5 k | 15 k | 150 k | 1.5 M |
| 1.6 | 16 | 160 | 1.6 k | 16 k | 160 k | 1.6 M |
| 1.8 | 18 | 180 | 1.8 k | 18 k | 180 k | 1.8 M |
| 2.0 | 20 | 200 | 2.0 k | 20 k | 200 k | 2.0 M |
| 2.2 | 22 | 220 | 2.2 k | 22 k | 220 k | 2.2 M |
| 2.4 | 24 | 240 | 2.4 k | 24 k | 240 k | 2.4 M |
| 2.7 | 27 | 270 | 2.7 k | 27 k | 270 k | 2.7 M |
| 3.0 | 30 | 300 | 3.0 k | 30 k | 300 k | 3.0 M |
| 3.3 | 33 | 330 | 3.3 k | 33 k | 330 k | 3.3 M |
| 3.6 | 36 | 360 | 3.6 k | 36 k | 360 k | 3.6 M |
| 3.9 | 39 | 390 | 3.9 k | 39 k | 390 k | 3.9 M |
| 4.3 | 43 | 430 | 4.3 k | 43 k | 430 k | 4.3 M |
| 4.7 | 47 | 470 | 4.7 k | 47 k | 470 k | 4.7 M |
| 5.1 | 51 | 510 | 5.1 k | 51 k | 510 k | 5.1 M |
| 5.6 | 56 | 560 | 5.6 k | 56 k | 560 k | 5.6 M |
| 6.2 | 62 | 620 | 6.2 k | 62 k | 620 k | 6.2 M |
| 6.8 | 68 | 680 | 6.8 k | 68 k | 680 k | 6.8 M |
| 7.5 | 75 | 750 | 7.5 k | 75 k | 750 k | 7.5 M |
| 8.2 | 82 | 820 | 8.2 k | 82 k | 820 k | 8.2 M |
| 9.1 | 91 | 910 | 9.1 k | 91 k | 910 k | 9.1 M |
|  |  |  |  |  |  |  |

4. A student uses an integrator to produce a triangular wave of regular amplitude and frequency.


The student uses $a \pm 3.3 \mathrm{~V}$ square wave with a mark-space ratio of 1:1.
(a) Write an expression for the change in the output voltage, $\mathrm{V}_{\text {out }}$, in its simplest form while the input is +3.3 V . Assume that the output reaches +9.0 V as the square wave switches from -3.3 V to +3.3 V at $\mathrm{t}=0 \mathrm{~s}$.
(b) Calculate the time that it would take for the output to change by 18 V .
(c) Describe the effect on the output voltage, $\mathrm{V}_{\text {out }}$, of using a $22 \mu \mathrm{~F}$ capacitor instead of the $2.2 \mu \mathrm{~F}$ capacitor.
5. A homeowner is considering which of two systems to install to heat the household water: either photovoltaic panels to generate electricity to power a heating element in the water tank, or flat-plate collectors through which the water runs to absorb solar radiation.


The photovoltaic panels are $22 \%$ efficient and the heating element is $98 \%$ efficient.
A flat-plate collector has a surface area of $0.505 \mathrm{~m}^{2}$. A pump circulates a water and antifreeze mixture having a specific heat capacity of $3730 \mathrm{Jkg}^{-1} \mathrm{~K}^{-1}$ at a mass flowrate of $0.025 \mathrm{~kg} \mathrm{~s}^{-1}$. The difference between inlet and outlet temperature of the flow in the collector is $4.0^{\circ} \mathrm{C}$ on a day when the solar radiation is $1100 \mathrm{~W} \mathrm{~m}^{-2}$. Power losses amount to 25 W in this system.

$$
\eta_{\text {flat plate collector }}=\frac{\text { rate of heat transfer to water }}{\text { total power supplied by solar radiation }+ \text { system power losses }}
$$

Determine which system is more efficient when converting the sun's energy to heat water in the household tank under these conditions. Show all working.
6. A castellated beam is manufactured from a standard I-beam by the process shown below.

matched halves welded at all points of contact

final castellated beam trimmed to length

An I-beam of depth 160 mm is cut with an offset of 20 mm either side of the centre line and then welded to produce a castellated beam of depth 200 mm , as shown.

section X-X
all dimensions measured to the nearest mm
6. (continued)

Using the information in the diagram on the opposite page:
(a) determine the depth of the hexagonal hole, $d$

| B | $\mathbf{D}$ | $\mathbf{T}$ | t | $\mathbf{I}_{\mathrm{x}-\mathrm{x}}$ |
| :---: | :---: | :---: | :---: | :---: |
| mm | mm | mm | mm | $\mathrm{mm}^{4}$ |
|  |  |  |  |  |
| 75 | 150 | 10 | 5 | 8277917 |
| 75 | 160 | 10 | 5 | 9593333 |
| 75 | 170 | 10 | 5 | 11018750 |
| 75 | 180 | 10 | 5 | 12556667 |
| 75 | 190 | 10 | 5 | 14209583 |
| 75 | 200 | 10 | 5 | 15980000 |
| 75 | 210 | 10 | 5 | 17870417 |
| 75 | 220 | 10 | 5 | 19883333 |
| 75 | 230 | 10 | 5 | 22021250 |
| 75 | 240 | 10 | 5 | 24286667 |


not to scale
[Turn over
7. An industrial photobioreactor uses vertical arrays of identical glass tubes filled with water to grow algae. The glass tubes are supported at points along their lengths by a frame.


The outer diameter of the tube is 54 mm and the second moment of area of the tube's cross-section is $101 \times 10^{3} \mathrm{~mm}^{4}$.
The ultimate tensile stress of the glass is $7.2 \mathrm{Nmm}^{-2}$ and the ultimate compressive stress is $995 \mathrm{~N} \mathrm{~mm}^{-2}$.
The design requires a factor of safety of 3 .
(a) Determine the maximum permissible bending moment in the tube.

Each glass tube has a self-weight of $6.5 \mathrm{Nm}^{-1}$ and the water in a filled tube has a weight of $19.6 \mathrm{~N} \mathrm{~m}^{-1}$. A revised estimate of the maximum permissible bending moment set a value of $8.5 \times 10^{3} \mathrm{Nmm}$.

(b) Determine the maximum span, L, permitted between simple supports for a tube.

## SECTION 2 - 40 marks

## Attempt ALL questions

8. A simply supported beam within a building is loaded as shown.

(a) On the worksheet for question 8 (a) draw the shear force diagram for this beam. Note that the horizontal components of the inclined forces oppose each other and have no effect.
(b) Write the equation for the bending moment as a function of distance, x , measured in metres from the left-hand end of the beam for the range $9<x \leq 24$.
(c) Calculate the magnitude of the maximum bending moment within this region, and its position from the left-hand end of the beam.

An engineer calculated the bending moments at support points $A$ and $B$ as being -100 kN m and -60 kN m respectively.
(d) Explain, with reference to the shear force diagram, why the engineer would have decided to calculate the bending moment at these points, as well as for the point found in part (c).
8. (continued)

Four strain gauges A-D are added to the beam in the positions shown below on a section of the beam where the part of the beam above its neutral axis is in compression and the part below the neutral axis is in tension.


Gauges $A$ and $B$ are equidistant from the neutral axis of the beam.
Gauge $C$ is aligned with gauge $A$ and gauge $D$ is aligned with gauge $B$.
The resistance of a strain gauge changes in proportion to the strain it is subject to. A tensile strain produces an increase in resistance and a compressive strain produces a decrease in resistance.

The circuit below is used to produce a signal from the strain gauges.


The output voltage is given by the formula:

$$
V_{\text {out }}=\left(1+\frac{2 R_{1}}{R_{2}}\right) \frac{R_{4}}{R_{3}}\left(V_{2}-V_{1}\right)
$$

The strain gauge system is designed to produce a positive value of $\mathrm{V}_{\text {out }}$ when the beam is in tension in the region below the neutral axis on the diagram.
8. (continued)
(e) On the worksheet for question 8 (e), complete the table to show which of the four strain gauges $A, B, C$ and $D$ indicated on the diagram of the beam would be
connected in each of the four positions $R_{G 1}, R_{G 2}, R_{G 3}$ and $R_{G 4}$ in the circuit.

The strain gauges have a resistance of $120 \Omega$ when not subject to strain. The maximum change in their resistance is $24 \mathrm{~m} \Omega$.
(f) Determine the maximum value of input voltage $\left(V_{2}-V_{1}\right)$.

The internal resistances of the integrated circuit have a $0.1 \%$ tolerance and the value of the $R_{1}$ resistance is $25.2 \mathrm{k} \Omega . \mathrm{R}_{2}$ is also a $0.1 \%$ resistor that is connected to the integrated circuit to set the voltage gain of the amplifier.

For this system the voltage gain, $\mathrm{A}_{\mathrm{V}}$, is given by:

$$
A_{V}=\frac{V_{\text {out }}}{V_{2}-V_{1}}
$$

When $R_{2}$ is not connected $\left(1+\frac{2 R_{1}}{R_{2}}\right)$ becomes 1 and the voltage gain is exactly 1.
(g) Determine the value of $\mathrm{R}_{2}$ required to produce a voltage gain of 999 .

An analogue-to-digital converter (ADC) is used to convert the voltage from the integrated circuit, $\mathrm{V}_{\text {out }}$, to an 8-bit digital value. The peak input voltage for the ADC is specified as 2.5 V .
(h) Determine:
(i) the resolution of the ADC, giving your answer in millivolts ( mV )
(ii) the digital equivalent of a 2.4 V input signal to the ADC .
9. The electric motor on an e-bike is used to supplement the power the rider supplies when pedalling. The force applied to the pedals rotates the crankshaft which drives the main sprocket for the chain drive.


At the instant shown in the diagram above, the crankshaft is turning at a constant rotational speed of 42 revs $\mathrm{min}^{-1}$. The motor is connected to the crankshaft via meshed gears. The driving gear on the motor has 18 teeth and a pitch circle diameter of 45 mm , while the driven gear has 54 teeth and a pitch circle diameter of 135 mm . The gears have a pressure angle of $20.0^{\circ}$. The gears transmit 235 W of power from the motor to the crankshaft.
(a) Calculate the tangential and radial components of force on the 54-tooth gear at the mesh.

At the same instant, the rider applies a downwards force on the pedal, $\mathrm{F}_{\mathrm{p}}$, of 225 N . The effective radii of the pedal crank, $r_{p}$, and the chain sprocket, $r_{c}$, are 180 mm and 78 mm respectively.
(b) Calculate the horizontal force, $\mathrm{F}_{\text {chain }}$, being applied to the chain drive.

At another constant speed condition, the forces on the pedal, in the chain drive and at the gear mesh are shown in the diagram below, along with their relative lines of action on the crankshaft.

(c) Determine the magnitude of the reaction force at bearing $A$.

The bike frame support for the back axle has a shock absorber that includes a coil spring. The deflection, $\delta$, of a coil spring having cross-sectional diameter, d , is approximated by the following relationship:

$$
\delta=\frac{8 F D^{3} N}{d^{4} G}
$$

The value G is a property of the coil material called the 'modulus of rigidity', N is the number of 'active coils', $D$ is the mean diameter of the coil when $D_{o}$ and $D_{i}$ are the outer and inner coil diameters respectively. The dimensions are shown in the diagrams below.


A coil spring made from spring steel with a modulus of rigidity of $77.2 \mathrm{GN} \mathrm{m}^{-2}$ is specified as having an outer coil diameter of 63.5 mm , an inner diameter of 47.5 mm and 4 active coils.
(d) Calculate the force, F, required to produce a deflection of 18 mm in this spring.
9. (continued)

A student experiments with a microcontroller-based control system for a model e-bike motor.

Driven by the battery, the brushless DC motor is geared to the crankshaft so that it runs six times faster than the crankshaft. The motor assists the cyclist when pedalling at a rate between 10 revs $\mathrm{min}^{-1}$ and 90 revs $\mathrm{min}^{-1}$.
Three Hall sensors spaced at $120^{\circ}$ detect the magnetic field in the motor and the controller uses the signals to control the rotation of the motor. They produce a 5 V signal or a 0 V signal.


The command given below can be used to time the length of a pulse.

| Arduino | PBasic |
| :--- | :--- |
| pulseln (pin, value) | PULSIN pin, state, wordvariable |
| Reads a pulse (either HIGH or LOW) on a |  |
| pin. Returns the length of the pulse in |  |
| microseconds ( $\mu \mathrm{s}$ ). | Reads a pulse (either HIGH or LOW) on a <br> pin. Returns the length of the pulse in <br> multiples of 10 microseconds ( $\mu \mathrm{s}$ ). |
| When value is HIGH, measures the time |  |
| interval between the positive edge and |  |
| the negative edge of a pulse. | When state = 1, measures the time <br> interval between the positive edge and <br> the negative edge of a pulse. |
| pin: the number of the pin on which you | pin: the number of the pin on which you <br> want to read the pulse. |
| walt read the pulse. |  |
| value: type of pulse to read: | stape type of pulse to read: <br> either HIGH or LOW |
| The result can be stored in an 'unsigned |  |
| long' variable (0-4294967295). The |  |
| number represents multiples of 1 $\mu \mathrm{s}$. | wordvariable: the 16-bit register in <br> which to store the result (0-65535). The <br> number represents multiples of 10 $\mu \mathrm{s}$. <br> unsigned long length; |
| length = pulseln (3, HIGH); | symbol length = w0 |
| pulsin B.3, 1, length |  |

9. (continued)

The test program below stores the speed of the motor, measured in revs s ${ }^{-1}$, in the variable 'speed’.

| Arduino | PBasic |
| :--- | :--- |
| unsigned long length = 0; <br> unsigned long interval = 500000; <br> void setup()\{ <br> pinMode(4, INPUT); <br> $\}$ | symbol length = w0 <br> symbol speed = b4 <br> symbol interval = w1 |
| void loop()\{ |  |
| length = pulseln (4, HIGH); <br> int speed = interval/length; <br> let interval = 50000 |  |
| delay(1000); | main: pulsin B.4, 1, length <br> speed = interval/length |
| pause(1000) |  |

(e) (i) Explain the value assigned to the variable 'interval', and the values that the variables 'length' and 'speed' would be for the maximum and minimum specified pedalling speeds.
(ii) Explain the value that the variable 'speed' would be if the pedalling speed fell below the minimum value of 10 revs $\mathrm{min}^{-1}$.

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