National Qualifications 2026

X823/77/11 Engineering Science

FRIDAY, 24 APRIL
1:00 PM – 3:30 PM

Total marks — 75

You may refer to the Advanced Higher Engineering Science Data Booklet.

SECTION 1 — 35 marks

Attempt ALL questions.

SECTION 2 — 40 marks

Attempt ALL questions.

Write your answers clearly in the answer booklet provided. In the answer booklet, you must clearly identify the question number you are attempting.

For questions 2 (a) and 6 (a), write your answers clearly in the worksheets provided in the answer booklet.

Show all working and units where appropriate.

The number of significant figures expressed in a final answer should be equivalent to the least significant data value given in the question. Answers that have two more figures or one less figure than this will be accepted.

Use **blue** or **black** ink. Sketches, diagrams and graphs may be drawn in pencil.

You must leave your answer booklet on your desk; if you do not, you could lose all the marks for this paper.

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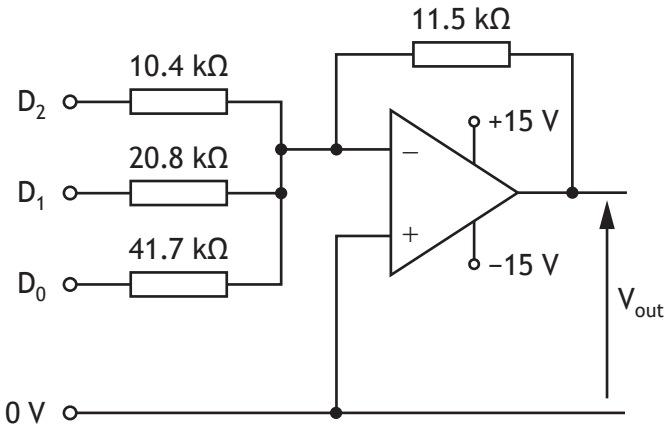
MARKS

SECTION 1 — 35 marks

Attempt ALL questions

1. A student designs and constructs a 3-bit digital to analogue converter (DAC) using 0.1% tolerance resistors, as shown below.



All three digital inputs, D_0, D_1 and D_2, are 5.00 V when turned on and 0.00 V when turned off.

The operational amplifier (op-amp) output saturates at ±14.4 V.

(a) Calculate the output voltage, V_{out}, when all three inputs are turned on. **2**

In error, the student uses a 115 kΩ resistor instead of the 11.5 kΩ resistor when constructing a test circuit but selects the other components correctly.

(b) Determine the measured output voltage when the binary input to the DAC is:

(i) 001 **1**

(ii) 010 **2**

MARKS

2. A project manager uses critical path analysis to help manage a 10-stage short project.

Stage	Precedent	Duration (weeks)
A	—	1
B	—	2
C	A	1
D	A	4
E	B	4
F	C, D	2
G	E	1
H	F	4
I	G	1
J	H, I	3

(a) Using this precedence table and the **worksheet for question 2 (a)**:

(i) complete the activity network diagram by adding precedence arrows to stages A and C to I **1**

(ii) add the earliest and latest finish times, the earliest and latest start times, and the float for each of the stages A and C to I **3**

(iii) identify the critical path. **1**

Project Management includes several processes and phases, one of which is the **Closing Phase**.

This may involve several steps, some of which are listed here:

- analyse project performance
- complete paperwork
- reallocate/release resources
- sign off project.

For any two of the steps above:

(b) Describe one activity a Project Manager would complete for each chosen step. **2**

[Turn over

MARKS

3. A heat exchanger is a device that transfers heat from one fluid to another. A hot fluid cools down and a separate cold fluid heats up.

Heat balance:

$$(\dot{m}c\Delta T)_{hot} = (\dot{m}c\Delta T)_{cold}$$

where $\dot{m}$ is the mass flow rate in $kg\,s^{-1}$.

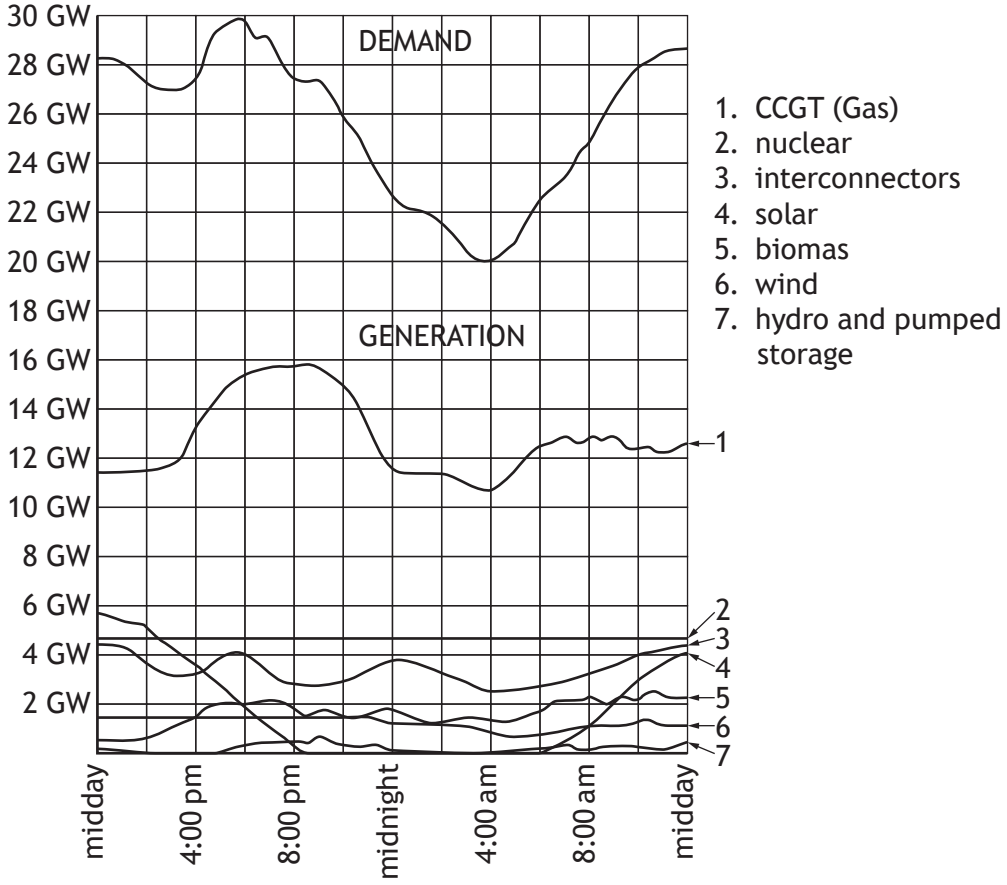
Hot oil flowing at $1.20\ kg\,min^{-1}$ cools from 150.0 °C to 100.0 °C and heats a flow of water from 20.0 °C to 78.3 °C.

The oil has a specific heat capacity that is half that of water.

Calculate the flowrate of the water in $kg\,s^{-1}$. **3**

MARKS

4. The electricity demand in the UK over the course of a typical day is met from a variety of sources. The graphs below show demand and production during a day in April.



(a) Discuss, with reference to the graphs above:

(i) what the data tells us about demand **1**

(ii) how the unpredictable capacity from certain generation methods is compensated for. **2**

A transmission cable runs between a 33 kV–11 kV transformer and a 11 kV–415 V transformer. The power transmitted from the first transformer via the cable is 110 kVA when its secondary winding is at 11 kV. The transmission cable has a resistance of 494 $\mu\Omega m^{-1}$.

The two transformers are separated by 12 km.

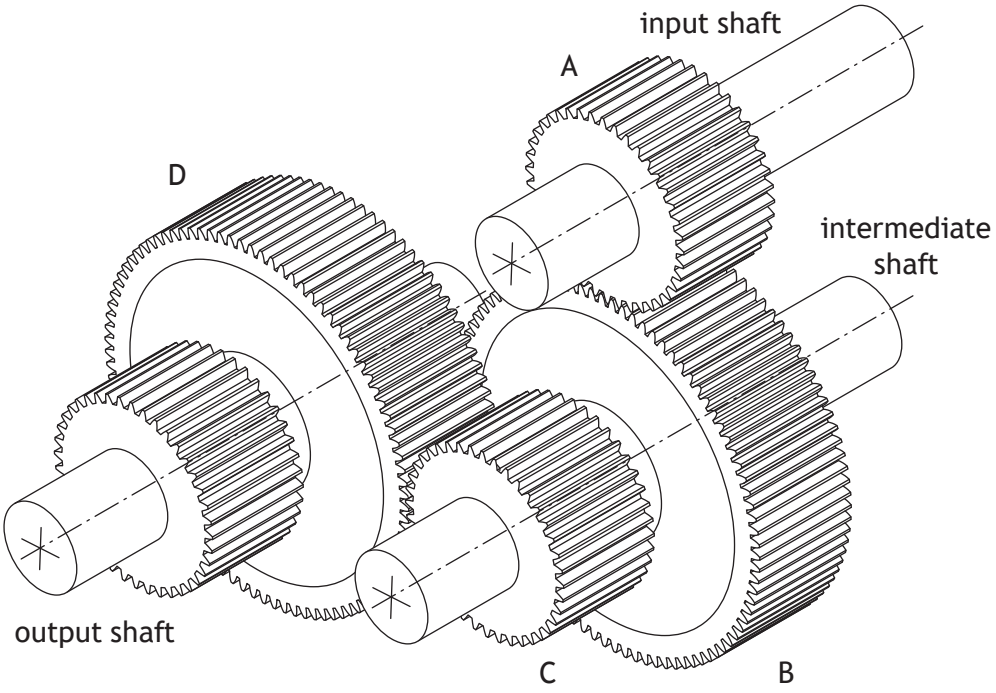
Note that 1 kVA is equivalent to 1 kW.

(b) Calculate the efficiency of the power transmission between the two transformers. **3**

[Turn over

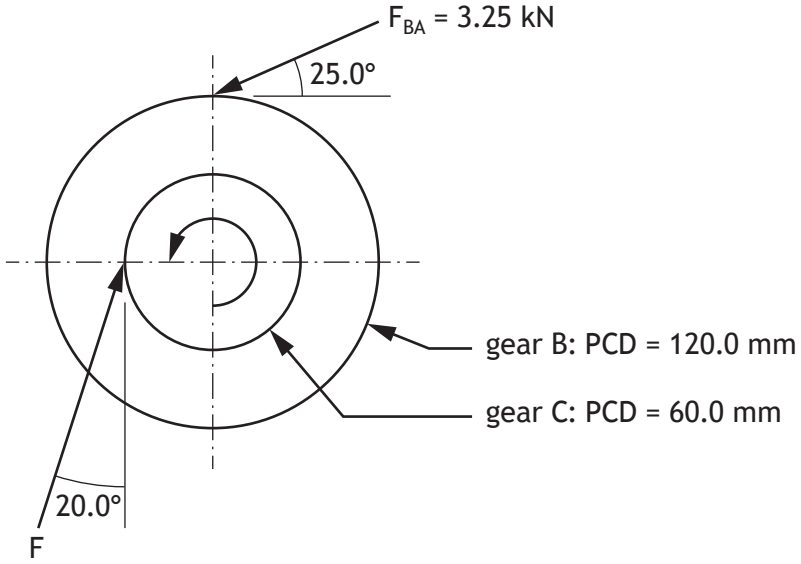
MARKS

5. Gears on an intermediate shaft connect the input shaft to the output shaft of a gear box.



The intermediate shaft rotates at a constant speed of 450 revs min^{-1}.

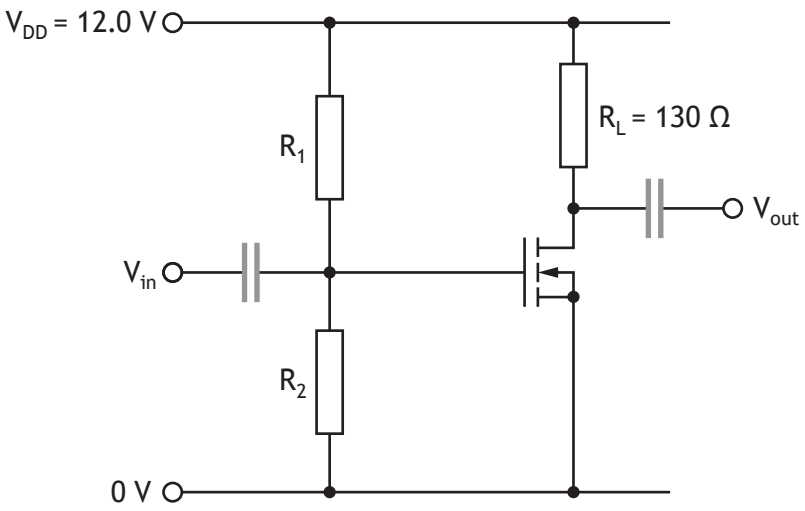
The two contact forces at the gear meshes that act on the intermediate shaft, and the pitch circle diameters (PCD) of gears B and C are shown below.



Calculate the magnitude of the contact force, F, acting on gear C. **3**

MARKS

6. A student designs a MOSFET circuit to act as an amplifier. The circuit diagram is shown below.



The graph on **worksheet for question 6 (a)** shows the operational characteristics of the MOSFET circuit. The MOSFET is biased so that V_{GS} = 4.75 V at the operating point, which must lie in the saturation region.

(a) Draw the load line for the MOSFET on **worksheet for question 6 (a)** and hence, or otherwise, identify the value of the drain-source voltage, V_{DS}, and the drain current, I_D, at the quiescent point (Q-point). **2**

A design rule is also used: when R_1 and R_2 are connected in parallel, their effective resistance is 650 kΩ.

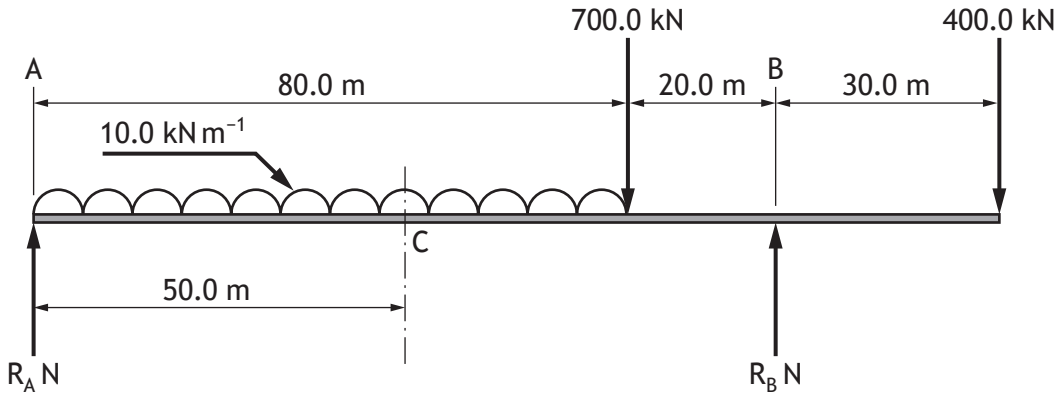
All resistors in the circuit have a tolerance of 1%, so their values have three significant figures.

(b) Calculate suitable values for R_1 and R_2. **3**

[Turn over

MARKS

7. A large civil engineering structure is simplified for analysis. The beam is simply supported at A and B, as shown in the free-body diagram below.



The designer wishes to investigate the stress on the structure at the section shown at point C, 50 m from the left-hand end, A.

(a) Calculate the shear force at point C. Show all working. **3**

(b) Calculate the bending moment at point C. Show all working. **3**

[Turn over for next question

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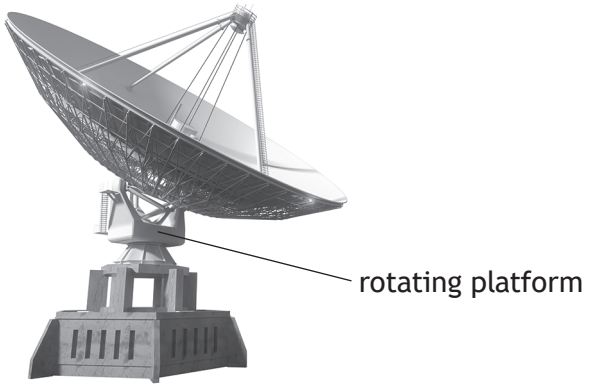
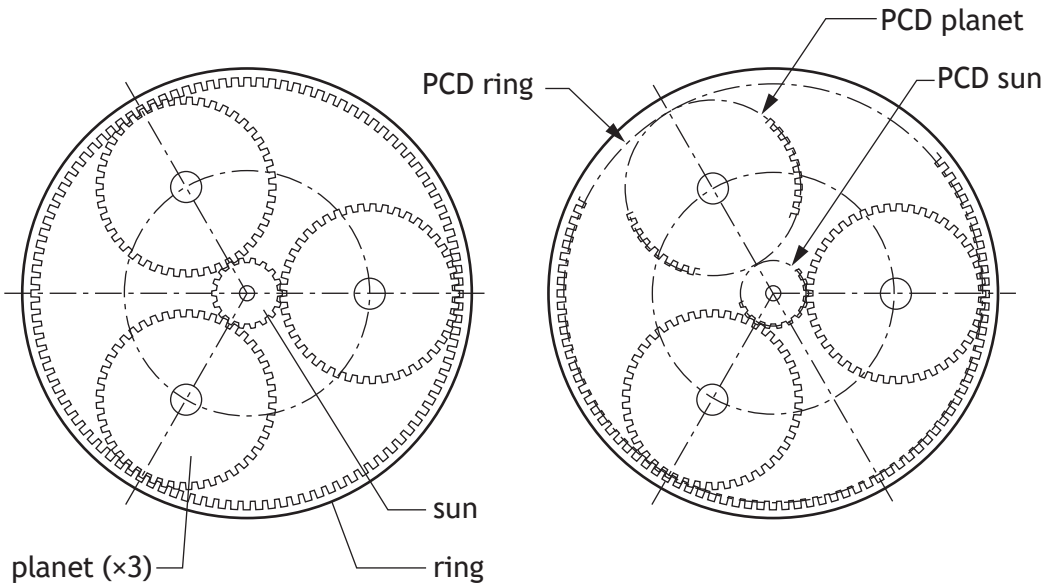
SECTION 2 — 40 marks

Attempt ALL questions

8. Radio telescope dishes must rotate to pinpoint sources of radio frequency signals in space.



The drive for the rotating platform includes an epicyclic gear system; three planet gears each mesh with both a sun gear and a ring gear, as shown below.



The gear module, m, is defined as the ratio between the pitch circle diameter (PCD) of a gear and the number of its teeth, N.

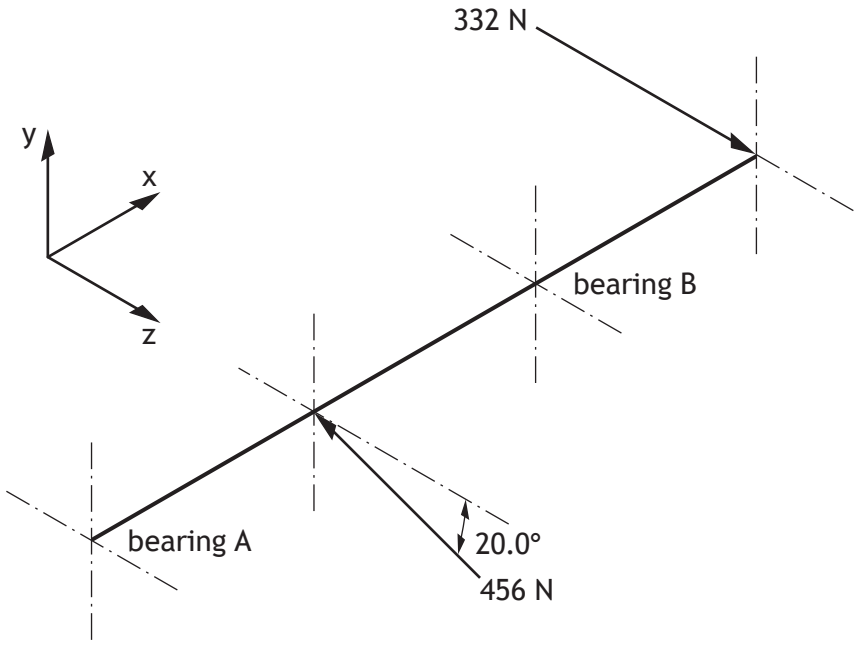
$$PCD = m \times N$$

In this system, the module is three for all gears. The sun gear has 18 teeth, and each planet gear has 45 teeth.

(a) Determine the number of teeth required on the ring gear. **3**

8. (continued)

The carrier shaft for the three planet gears is driven from an electric motor and a spur gear mesh. The arrangement of the loads on the simply supported shaft is given in the diagram below.



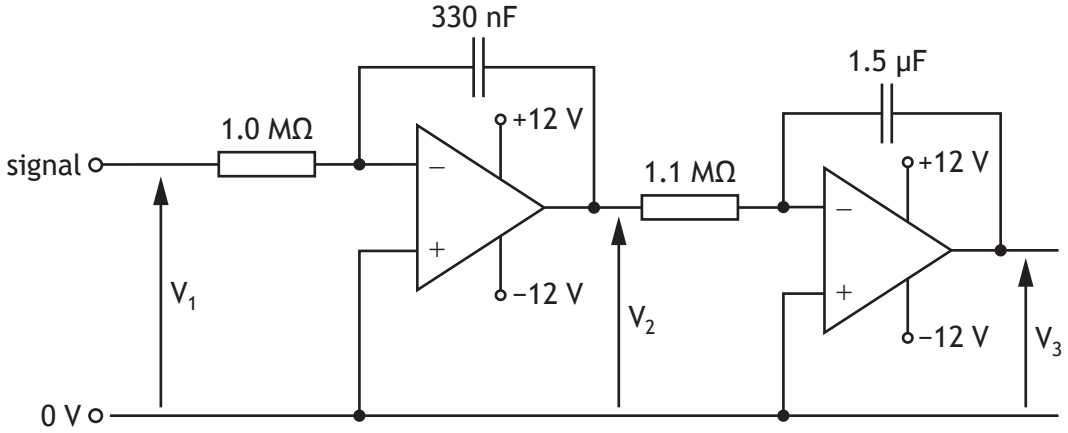
The four points along the shaft at which forces act are all 0.225 m apart.

(b) Calculate the magnitude and direction of the reaction at bearing B. **6**

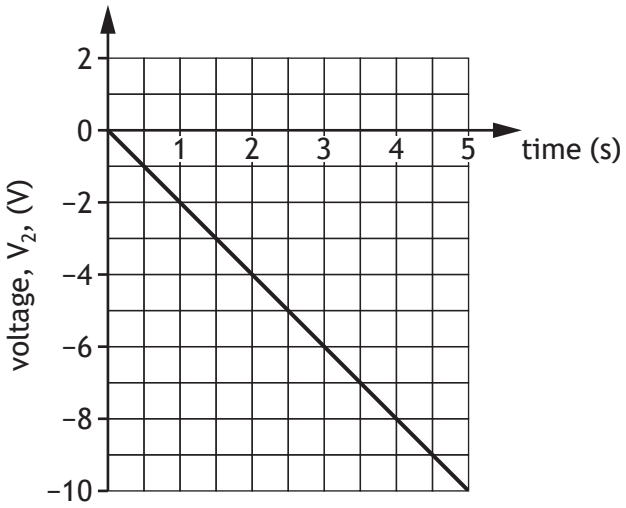
[Turn over

8. (continued)

To monitor the vibration of the housing of bearing B, an accelerometer produces an analogue voltage, V_1, proportional to the housing's acceleration. This voltage is processed by the two-stage integrator circuit shown below.



V_1 is integrated with respect to time to determine V_2. A second integration determines V_3, representing the displacement of the housing as it vibrates. The response of the first integrator to a constant voltage, V_1, is shown below.



(c) (i) Write, referring to the graph above, the equation for V_2 in terms of time, t. **1**

(ii) Write, referring to the circuit diagram above, the equation for V_3 in its simplest form, when its initial value is 0 V. **2**

(iii) Sketch the graph of V_3 with respect to time for $0 < t < 5$ s.

Assume that $V_3 = 0$ V when $t = 0$ s and that the operational amplifiers saturate at ±10.6 V. **3**

(iv) Calculate the value of V_1 for $0 < t < 5$ s. **2**

8. (continued)

A student develops a microcontroller-based program to produce an analogue output signal on pin 6 to soft-start the electric motor. A section of the program is shown below.

ARDUINO	PBASIC

void setup(){
 	
int x=5;
}
 	
void loop() {
 	
analogWrite(6,x);
delay (100);
 	
if (x<50)
{
x=12*x/10;
}
 	
else if (x<246)
{
x=x+10;
}
 	
else
{
x=255;
}
}
	
	x=5
	
	main:
	
	pwmduty B.6, x
	pause 100
	
	if x<50 then
	x=12*x/10
	
	else if x<246 then
	x=x+10
	
	else
	x=255
	
	end if
	
	goto main

The program section produces the following successive values of the variable x:

5, 6, 7, 8, 9, 10, 12, 14, 16, 19, 22, 26, 31, 37, . . .

(d) Determine the next **four** values of the variable x produced by the program section. **2**

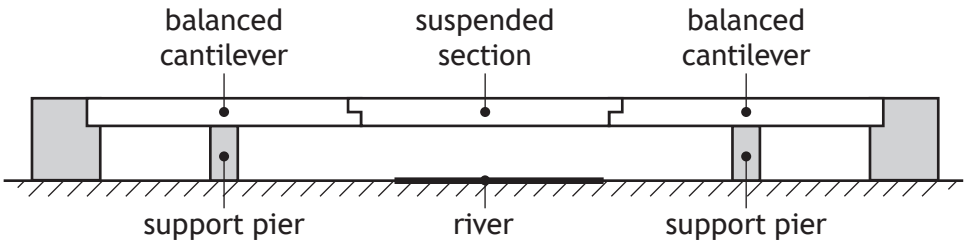
Within the program section, the line x=12*x/10 is replaced with x=11*x/10.

(e) Describe the effect that this has on the sequence of values of the variable x. **1**

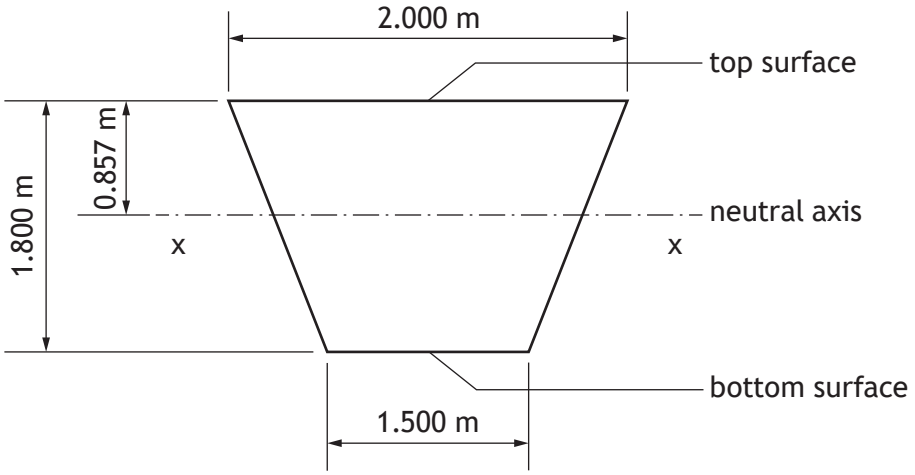
[Turn over

MARKS

9. A balanced cantilever bridge design is used to span a river for a high-speed rail link.



A concrete component within the bridge's structure has a uniform trapezoidal cross-section and experiences a maximum bending moment of 415 kNm.



The second moment of area of a trapezoidal cross-section about the neutral axis (I_{xx}) is given by the formula

$$I_{xx} = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)}$$

where: h is the height

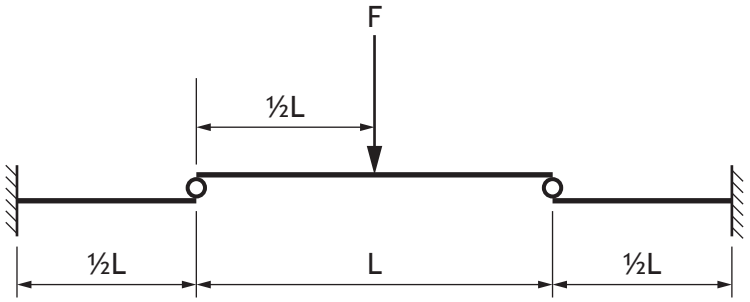
a is the width of the bottom surface

b is the width of the top surface.

(a) Calculate the maximum stress in the cross-section and state where it occurs. **5**

9. (continued)

A student analyses the deflection of the bridge. The suspended section, modelled as a simply supported beam, sits on top of two cantilever supports, built-in at the support piers. A point load, F, is applied at the mid-span, as shown below.



All three components are made from the same material and have the same uniform cross-sectional properties.

(b) Show that the deflection of the point where the point load acts is given by

$$\delta = \frac{FL^3}{24EI}$$ **4**

The span is supporting a load equivalent to a point load of 5.15 MN, when the sections are made from mild steel, having a second moment of area, I = 23.5 m^4, and length, L = 60.0 m.

(c) Calculate the deflection where the point load acts, using the formula above. **2**

[Turn over

9. (continued)

An analogue to digital converter (ADC) that operates on the same principle as the ADC given in the data booklet is used to convert the amplified output voltage from a strain gauge bridge to a 10-bit digital value. The peak input value for the ADC is specified as 2.55 V.

(d) Determine:

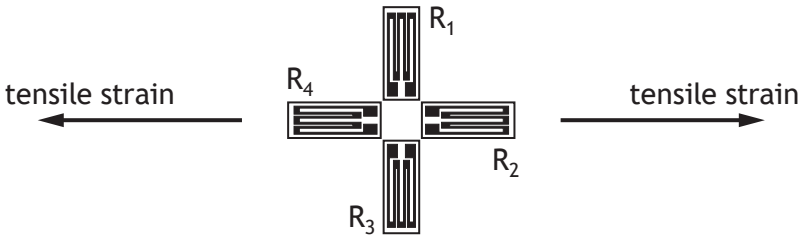
(i) the resolution of the ADC, giving your answer in millivolts (mV) **1**

(ii) the digital equivalent of a 2.15 V input. **3**

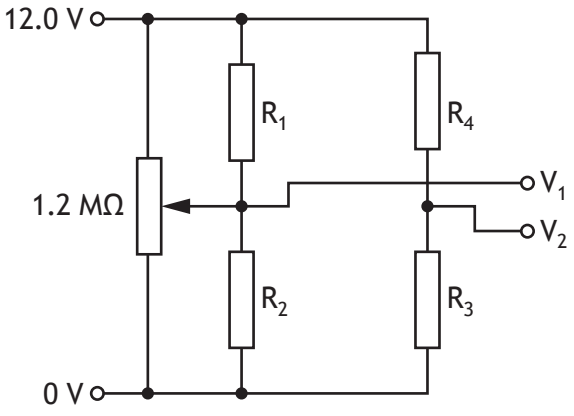
Four strain gauges, R_1–R_4 are bonded to the bottom surface of the suspended section of the bridge in the pattern shown below to monitor the tensile strain.

The resistance of R_2 and R_4 increase in proportion to the tensile strain; they are active.

The resistance of R_1 and R_3 does not change significantly because of the tensile strain; they are passive.



The strain gauges each have a manufactured resistance of 120 Ω ±0.5%. They are connected to the bridge circuit shown below.

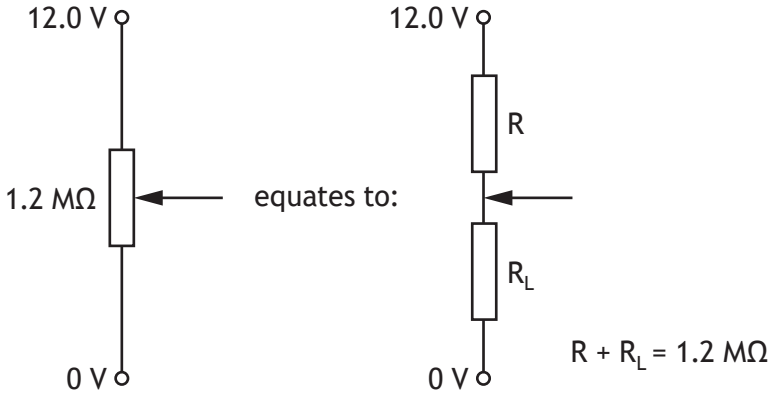


(e) Explain why two passive strain gauges are included in the positions shown in this circuit. **2**

9. (continued)

The potentiometer is used to adjust the bridge circuit so that the output voltages have the same value when there is no strain on the active gauges.

R is the resistance of the rail of the 1.2 MΩ potentiometer between its connection to the 12.0 V supply rail and the potentiometer wiper.



(f) Use nodal analysis to write an expression for R in the form

$$\frac{1}{R} - \frac{1}{a - R} - b = 0$$

where a and b are constant values, and when:

$R_1 = 120.6\ \Omega$

$R_2 = 119.4\ \Omega$

$R_3 = R_4 = 120.0\ \Omega$

$V_1 = V_2$ **3**

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