

X847/77/11

Mathematics Paper 1 (Non-calculator)

MONDAY, 12 MAY 9:00 AM – 10:00 AM



Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use blue or black ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.





FORMULAE LIST

Standard derivatives	
f(x)	f'(x)
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
tan ⁻¹ x	$\frac{1}{1+x^2}$
tan x	sec ² x
$\cot x$	$-\csc^2 x$
sec x	sec x tan x
cosec x	$-\csc x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
f(x)	$\int f(x)dx$
$sec^2(ax)$	$\frac{1}{a}\tan(ax) + c$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x}$	$\ln x + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$

Summations

$$S_n = \frac{1}{2}n \Big[2a + (n-1)d \Big]$$

$$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$
 where $\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \dots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Vector product

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ , about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

Total marks — 35 Attempt ALL questions

1. Use the binomial theorem to expand $\left(\frac{1}{x} - 3x\right)^4$. Simplify your answer.

4

2. Given $f(x) = \frac{2x^3 + x}{3 + 2x}$, find f'(x). Simplify your answer.

3

3. Two complex numbers are defined as z = 11 + 10i and w = 3 - 2i.

Find
$$\frac{z}{w}$$
 in the form $a+bi$, where $a, b \in \mathbb{R}$.

2

4. Matrices A and B are defined by $A = \begin{pmatrix} -3 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 \\ 5 & \lambda \end{pmatrix}$ where $\lambda \in \mathbb{R}$.

(a) Find 3A + 2B.

1

(b) (i) Find A'B, where A' is the transpose of A.

2

(ii) Find an expression for the determinant of A'B.

1

(iii) Determine the value of λ such that A'B is singular.

2

 $5. \quad \text{Find } \int \frac{1}{1+4x^2} \, dx.$

2

6. On a suitable domain a curve is given by the equation y = f(x), where

$$f(x) = \frac{x^2 + x + 5}{x - 2}.$$

- (a) Express f(x) in the form $Ax + B + \frac{C}{x-2}$, where A, B and C are constants.
- (b) State the equations of the vertical and non-vertical asymptotes of the curve. 2
- 7. Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{2x-1}, x, y > 1,$$

given that y = 12 when x = 5. Express y in terms of x.

8. Three planes are defined by

$$\pi_1$$
: $x - y + 4z = -6$

$$\pi_2$$
: $2x + 3y + z = 15$

$$\pi_3$$
: $3x + 2y - 2z = 16$

(a) Use Gaussian elimination to find T, the point of intersection of the three planes. 4

The line L_1 is defined by $\frac{x+4}{3} = \frac{y-7}{2} = \frac{z-4}{1}$.

(b) Find P, the point of intersection of the line L_1 and the plane π_3 .

The line L_2 passes through points T and P.

(c) Find, in parametric form, the equations of the line L_2 .

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