H

National Qualifications

2022

Mathematics

Paper 2

Friday, 6 May

Instructions to Candidates

Candidates should enter their surname, forename(s), date of birth, Scottish candidate number and the name and level of the subject at the top of their first answer sheet.
Total marks – 65
Attempt ALL Questions
You may use a calculator.
To earn full marks you must show your working in your answers.
State the units for your answer where appropriate.
Questions marked with an asterisk differ in some respects from those in the printed paper.
Tactile diagrams are produced in a separately bound booklet.
You must clearly identify the question number you are attempting on your answer sheet.
Marks are shown in square brackets at the end of each question or part question.
An ow in the margin indicates a new question.
A separate formula sheet is provided.

[Braille page 2] Total marks — 65

Attempt ALL questions

ow * 1. Refer to the diagram for Question 1.

Triangle ABC has vertices A(−1, −1), B(2, −4) and C(7, 3).

ow 1. (a) Find the equation of the altitude through C. [3 marks]

ow 1. (b) Find the equation of the median through B. [3 marks]

ow 1. (c) Determine the coordinates of the point of intersection of the altitude through C and the median through B. [2 marks]

ow 2. The equation $2x^2 - 8x + (4-p) = 0$ has two real and distinct roots.

Determine the range of values for p. [3 marks]

ow 3. (a) Express $4\sin x + 5\cos x$ in the form $k\sin(x+a)$ where $k > 0$ and $0 < a < 2\pi$. [4 marks]

ow 3. (b) Hence solve $4\sin x + 5\cos x = 5.5$ for $0 \leq x < 2\pi$. [3 marks]

ow * 4. Refer to the diagram for Question 4.

[Braille page 3] The graph shown has equation $y = x^3 - 5x^2 + 2x + 8$.

The total shaded area is bounded by the curve and the x-axis.

ow 4. (a) Calculate the shaded area above the x-axis. [4 marks]

ow 4. (b) Hence calculate the total shaded area. [3 marks]

ow 5. Functions f and g are given by $f(x) = x^2 - 2$ and $g(x) = 3x + 5$, $x \in \mathbb{R}$.

ow 5. (a) Find expressions for:

(i) $f(g(x))$ and [2 marks]

(ii) $g(f(x))$. [1 mark]

ow 5. (b) Determine the range of values of x for which $f(g(x)) < g(f(x))$. [4 marks]

ow 6. A curve with equation $y = f(x)$ is such that $dy/dx = 1 - (3/x^2)$, where $x > 0$.

The curve passes through the point (3, 6).

Express y in terms of x. [5 marks]

ow * 7. Refer to the diagram Question 7.

Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown.

Find the values of k and n. [5 marks]

ow *8. Refer to the diagram for Question 8.

A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.

The total area of the pond and path together is 150 square metres.

ow 8. (a) Show that the area of the pond, A square metres, is given by

$A(x) = 156 - 2x - 450x$. [3 marks]

ow 8. (b) Determine the maximum area of the pond. [6 marks]

ow *9. Refer to the diagrams for Questions 9 (a) and 9 (b)

The line $y = 3x + 7$ intersects the circle $x^2 + y^2 - 4x - 6y - 7 = 0$ at the points P and Q.

ow 9. (a) Find the coordinates of P and Q. [5 marks]

PQ is a tangent to a second, smaller circle.

This circle is concentric with the first.

ow 9. (b) Determine the equation of the smaller circle. [4 marks]

ow 10. The heptathlon is an athletics contest made up of seven events. Athletes score points for each event.

In the 200 metres event, the points are calculated using the formula

$P = 4.99087(42.5 - T)^{1.81}$

where P is the number of points awarded, and T is the athlete's time, in seconds.

ow 10. (a) Calculate how many points would be awarded for a time of 24.55 seconds in the 200 metres event. [1 mark]

In the long jump event, the points are calculated using the formula

$P = 0.188807(D - 210)^{k}$

where P is the number of points awarded, D is the distance

[Braille page 4] jumped, in centimetres, and k is a constant.

ow 10. (b) Given that 850 points are awarded for a jump of 600 cm, calculate the value of k. [4 marks]

[END OF QUESTION PAPER]