Next Generation Higher National Unit Specification

Engineering Mathematics 5 (SCQF level 8)

Unit code: J7LB 48

SCQF level: 8 (8 SCQF credit points)

Valid from: session 2024 to 25

Prototype unit specification for use in pilot delivery only (version 1.0) February 2024

This unit specification provides detailed information about the unit to ensure consistent and transparent assessment year on year.

This unit specification is for teachers and lecturers and contains all the mandatory information required to deliver and assess the unit.

This edition: February 2024 (version 1.0)

Unit purpose

This unit develops the mathematical skills learners need to progress from the Higher National Diploma (HND) in Engineering to degree study. It provides opportunities to develop the knowledge, understanding and skills to:

- solve second-order, constant coefficient differential equations
- use partial differentiation and double integration techniques to solve a range of mathematical problems
- solve first- and second-order differential equations using Laplace transforms
- use eigenvalues and eigenvectors to solve linear system equations

The target learner group for the unit is learners who want to further develop their knowledge of mathematics to support a career in engineering fields such as:

- electrical engineering
- mechanical engineering
- systems engineering
- manufacturing engineering
- measurement and control engineering

It is also aimed at learners who want to develop the practical, personal and professional skills required for a successful career as an engineering technician.

Entry to the unit is at your centre's discretion. However, we recommend that learners have:

- a good knowledge and understanding of differential and integral calculus, including techniques for solving first-order differential equations
- a sound knowledge and understanding of basic matrix techniques
- strong numerical and algebraic skills

This may be evidenced by having completed the SCQF level 7 unit Engineering Mathematics 3 and the SCQF level 8 unit Engineering Mathematics 4.

Unit outcomes

Learners who complete this unit can:

1 solve second-order, constant coefficient differential equations
2 solve mathematical problems using partial differentiation
3 solve mathematical problems using double integration techniques
4 solve differential equations using Laplace transforms
5 use eigenvalues and eigenvectors to solve linear system equations

Evidence requirements

You should use a sampling approach to assess knowledge and skills. Learners must provide written or recorded oral evidence to demonstrate their knowledge and skills across all outcomes. Where evidence for outcomes is assessed on a sample basis, you must teach the whole of the content listed in the 'Knowledge and skills' section and make it available for assessment.

We recommend you assess all five outcomes using a single end-of-unit assessment. You must use different assessment instruments, and a different sample of knowledge and skills, for re-assessments.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions. Learners must not use computer algebra in assessments for the unit.

To successfully achieve the unit, learners must provide written and/or oral recorded evidence that they have completed the following outcomes.

Outcome 1

Sample any three of the four required items:

- Determine the complementary function of one second-order differential equation.
- Determine the particular integral of one second-order differential equation.
- Determine the particular solution of one second-order, constant coefficient differential equation given initial conditions.
- Use a Taylor linear approximation and solve the resulting second-order non-linear differential equation.

Outcome 2

Sample any three of the four required items:

- Solve one mathematical problem involving first-order partial derivatives.
- Solve one partial differentiation problem involving the use of the chain, product or quotient rules.
- Solve one problem involving higher-order partial differentiation.
- Solve one problem that involves finding the location and nature of a stationary point (s) for a function of the form $f(x, y)$

Outcome 3

Sample any three of the five required items:

- Define the domain and limits of integration.
- Solve a double integral in the rectangular domain.
- Solve a double integral after transforming the integrand to the polar domain.
- Solve a double integral by changing the order in the double integration.
- Use double integration to determine the volume or surface area of a shape or length of a curve.

Outcome 4

Sample any three of the five required items:

- Determine the Laplace transform of a function $f(t)$ from a table of Laplace transforms.
- Determine the inverse Laplace transform of a function $f(s)$ using the completing the square or partial fraction methods.
- Solve one problem that involves the use of the first or second shift theorems.
- Solve one problem that involves the use of the Dirac delta function.
- Solve a first- or second-order differential equation with initial conditions using Laplace transforms.

Outcome 5

Learners must provide evidence of the following knowledge and skills each time this outcome is assessed:

- Determine eigenvalue and eigenvector for one 3×3 matrix.
- Determine a diagonalisation transform to solve a problem of A^n.

Knowledge and skills

The following table shows the knowledge and skills covered by the unit outcomes:

Knowledge	Skills
Outcome 1 Learners should understand: ♦ in relation to second-order, constant coefficient differential equations: — complementary functions integration — constant coefficient differential equation — Taylor linear approximation	**Outcome 1** Learners can: ♦ determine the complementary function of one second-order differential equation ♦ determine the particular integral of one second-order differential equation ♦ determine the particular solution of one second-order, constant coefficient differential equation given initial conditions ♦ use a Taylor linear approximation and solve the resulting second-order non-linear differential equation
Outcome 2 Learners should understand: ♦ partial differentiation ♦ chain, product and quotient rules ♦ higher-order partial derivatives ♦ stationary points	**Outcome 2** Learners can: ♦ solve one mathematical problem involving first-order partial derivatives ♦ solve one partial differentiation problem involving the use of the chain, product or quotient rules ♦ solve one problem involving higher-order partial differentiation ♦ solve one problem that involves finding the location and nature of a stationary point (s) for a function of the form $f(x,y)$

Knowledge	Skills
Outcome 3 Learners should understand: ♦ for double integration techniques: — rectangular domain — polar domain — change order of integration in double integrals — volumes or surface areas	**Outcome 3** Learners can: ♦ define the domain and limits of integration ♦ solve a double integral in the rectangular domain ♦ solve a double integral after transforming the integrand to the polar domain ♦ solve a double integral by changing the order in the double integration ♦ use double integration to determine the volume or surface area of a shape or length of a curve
Outcome 4 Learners should understand: ♦ Laplace transforms ♦ inverse Laplace transforms ♦ shift theorems ♦ Dirac delta function ♦ first- and second-order differential equations	**Outcome 4** Learners can: ♦ determine the Laplace transform of a function $f(t)$ from a table of Laplace transforms ♦ determine the inverse Laplace transform of a function $f(s)$ using the completing the square or partial fraction methods ♦ solve one problem that involves using the first or second shift theorems ♦ solve one problem that involves using the Dirac delta function ♦ solve a first- or second-order differential equation with initial conditions using Laplace transforms
Outcome 5 Learners should understand: ♦ for linear system equations: — eigenvalues and eigenvectors — eigenvalue-related problems	**Outcome 5** Learners can: ♦ determine eigenvalue and eigenvector for one 3×3 matrix ♦ determine a diagonalisation transform to solve a problem of A^n

Meta-skills

Throughout the unit, learners develop meta-skills to enhance their employability in the engineering sector.

Self-management

Learners develop the meta-skills of focusing, adapting and initiative as they solve engineering problems.

Social intelligence

Learners develop the meta-skill of communicating as they ask questions and receive information from lecturers.

Innovation

Learners develop the meta-skills of curiosity, sense-making and critical thinking as they apply mathematical techniques to problem solving.

Literacies

Numeracy

Learners develop their numeracy skills by solving engineering problems using mathematics.

Communication

Learners develop their communication skills by studying the course material, and engaging with other learners, and their teachers or lecturers.

Digital

Learners develop digital literacy by accessing the course material through a virtual learning environment (VLE) and using software tools to solve engineering problems.

Delivery of unit

This unit provides core mathematical principles and processes that underpin Higher National Certificate (HNC) and HND Engineering units across a range of disciplines. We recommend you deliver the unit towards the beginning of these awards.

This unit is one of a suite of five units in mathematics developed for Higher National Qualifications across a range of engineering disciplines. The five units are:

- Engineering Mathematics 1 at SCQF level 6
- Engineering Mathematics 2 at SCQF level 7
- Engineering Mathematics 3 at SCQF level 7
- Engineering Mathematics 4 at SCQF level 8
- Engineering Mathematics 5 at SCQF level 8

The amount of time you allocate to this unit is at your centre's discretion. However, the notional design length is 40 hours.

The amount of time you allocate to each outcome is also at your discretion. We suggest the following distribution of time, including assessment:

Outcome 1 — Solve second-order, constant coefficient differential equations (6 hours)
Outcome 2 — Solve mathematical problems using partial differentiation (5 hours)
Outcome 3 — Solve mathematical problems using double integration techniques (5 hours)
Outcome 4 — Solve differential equations using Laplace transforms (6 hours)
Outcome 5 — Use eigenvalues and eigenvectors to solve linear system equations (6 hours)

Additional guidance

The guidance in this section is not mandatory.

Content and context for this unit

We strongly recommend you use the following list of topics to ensure continuity of learning and teaching, and that learners are prepared for assessment.

Solve second-order, constant coefficient differential equations (outcome 1)

- Introduce the general form of a constant coefficient, second-order linear differential equation as follows:

  $$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f\left(x\right)$$

  where a, b, c are constants.

- Provide examples of second-order differential equations from engineering.
- Explain that solving a second-order linear differential equation involves the following three stages:
  - finding the complementary function, y_{cf}
  - finding a particular integral, y_{pi}
  - determining the general solution by adding together the complementary function and the particular integral $\left(y=y_{\text{cf}}+y_{\text{pi}}\right)$ and applying initial conditions where known
- Explain the difference between homogeneous and non-homogeneous in the context of second-order differential equations.
- Introduce the auxiliary equation (characteristic equation) and explain that when finding roots of the auxiliary equation there are three cases to consider.
- Explain that to find the complementary function one is finding the solution to the equation:

  $$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$$

- Find the complementary functions of second-order differential equations using solutions of the form $\mathrm{y}=\mathrm{e}^{\mathrm{kx}}$
- Explore the various forms a complementary function can take.
- Explain that the particular integral is any function that satisfies the following equation:

  $$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f\left(x\right)$$

- Explain that for special classes of $f\left(x\right)$ we can use the method of undetermined coefficients to find the particular integral.
- Find the general solution of second-order differential equations including applying initial conditions.

Solve mathematical problems using partial differentiation (outcome 2)

- Introduce partial differentiation as the process of differentiating functions of two or more variables.
- Identify the notation used in partial differentiation.
- For any function $f\left(x,y\right)$ differentiate with respect to treating all terms in y as constant.
- For any function $f\left(x,y\right)$ differentiate with respect to treating all terms in x as constant.
- Undertake partial differentiation involving the use of the chain, product or quotient rules.
- Introduce higher-order partial derivatives.
- Demonstrate how to determine higher-order derivatives.
- Introduce examples of partial differential equations (solutions not required) that occur in engineering.
- Explain and demonstrate how to find the location and nature of a stationary point for a function of the form $z=f\left(x,y\right)$

Solve mathematical problems using double integration techniques (outcome 3)

- Explain that double integration involves integrating a function $f\left(x,y\right)$ as follows:

  $$\iint_R f\left(x,y\right)\,dy\,dx$$

  where R is the region of integration in the $x-y$ plane.
- Explain that the process of double integration normally comprises the following three stages:
  - work out the limits of integration if these are not known
  - determine the inner integral assuming terms in x are constant
  - determine the outer integral
- Perform double integration where the limits are known and where they have to be determined.
- Demonstrate that changing the order of integration can sometimes make double integration easier or possible to perform.
- Demonstrate the way in which double integration may be performed by transforming variables from the rectangular domain to the polar domain.
- Use double integration to determine the volume or surface areas of objects or the length of a curve.

Solve differential equations using Laplace transforms (outcome 4)

- Explain that the Laplace transform method is an integral transform method in which a linear constant coefficient differential equation is transformed into an algebraic equation. The corresponding algebraic equation is then solved and the transform reversed to find the solution of the differential equation.
- Introduce the Laplace transform of $f(t)$ as:

  $F(s) = \int_0^\infty e^{-st} f(t)\,dt$ (non-assessable).
- Determine one or two simple Laplace transforms using the above equation.
- Direct learners to tables of Laplace transforms and inverse Laplace transforms.
- Demonstrate techniques for finding the inverse of functions of $F(s)$ using completing the square and partial fractions.
- Introduce the first and second shift theorems.
- Demonstrate the application of the two theorems.
- Introduce the Dirac delta function (simple treatment only) — provide engineering analogies (for example, a power supply spike or a hammer striking an object).
- State the Laplace transform of the Dirac delta function as $L(\delta(t-a)) = \mathrm{e}^{-as}$
- Solve differential equations involving the use of the Dirac delta function.
- Solve first- and second-order differential equations with initial conditions using Laplace transforms.
- Solve systems of linear differential equations using Laplace transforms if time permits.

Use eigenvalues and eigenvectors to solve linear system equations (outcome 5)

- Introduce the idea of a trivial and non-trivial solution in the context of matrix theory.
- Define the concept of an eigenvalue in terms of, for example, the characteristic equation $|A - \lambda| = 0$
- Determine eigenvalues for 2 × 2 and 3 × 3 matrices.
- Introduce the concept of eigenvectors as the non-trivial solutions X of the equation $AX = \lambda X$
- Find the eigenvectors for 2 × 2 and 3 × 3 matrices.
- Solve eigenvalue and eigenvector problems (for example, diagonalisation matrices and transformation matrices).

Approaches to delivery

The unit provides many of the core mathematical principles and processes learners need to study engineering at a more advanced level. Given the subject matter, we recommend you deliver this unit after learners have studied Engineering Mathematics 4.
You can deliver the outcomes in any order.

We recommend that you deliver the unit using a mainly didactic approach. You should supplement teaching with formative assessment where learners have opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the unit. If appropriate, you could deliver the unit as a series of lectures supported by tutorial sessions to help learners prepare for degree-level study.

You can use computer software and computer algebra to support learning (for example, to confirm the solutions of mathematical problems), but we strongly recommend that these learning resources are only used in a supportive capacity and not as the principal means of delivering unit content.

Approaches to assessment

We recommend using an examination question paper. This should comprise an appropriate balance of short-answer, restricted-response and structured questions. You should not group or label questions by outcome.

Learners should not see the assessment paper before the assessment takes place. You must ensure the security, integrity and confidentiality of assessment papers at all times. You should conduct assessments under closed-book, controlled and invigilated conditions.

The total assessment time for the five outcomes should not exceed 2 hours and 30 minutes. When assessing learners' responses in a summative assessment, you should concentrate principally on their ability to apply the correct mathematical technique and processes. You should not penalise learners for making simple numerical errors. You can set an appropriate threshold score for assessing the unit.

You should provide learners with a formulae sheet appropriate to the content of the unit for them to use during the assessment. Learners must not use computer algebra in the assessment.

It is the learners' responsibility to ensure that any calculators they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- symbolic differentiation or integration
- communication with other machines or the internet

In addition, calculators must not have retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematic formulae

Using a range of assessment methods helps learners to develop different skills that should be transferable to work or further and higher education.

Opportunities for e-assessment

Assessment that is supported by information and communication technology (ICT), such as e-testing or the use of e-portfolios or social software, may be appropriate for some assessments in this unit.

If you want to use e-assessment, you must ensure that you apply the national standard to all evidence and that conditions of assessment (as specified in the evidence requirements) are met, regardless of the mode of gathering evidence.

Equality and inclusion

This unit is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

You should take into account the needs of individual learners when planning learning experiences, selecting assessment methods or considering alternative evidence.

Guidance on assessment arrangements for disabled learners and/or those with additional support needs is available on the assessment arrangements web page.

Information for learners

Engineering Mathematics 5 (SCQF level 8)

This information explains:

- what the unit is about
- what you should know or be able to do before you start
- what you need to do during the unit
- opportunities for further learning and employment

Unit information

This unit is one of five mathematics units developed for Higher National Certificate (HNC) and Higher National Diploma (HND) in Engineering. These units help you develop the mathematical skills you need for workplace roles and for more advanced studies in engineering, for example to progress to degree study.

You learn a range of techniques for solving second-order differential equations. These equations appear frequently in many areas of engineering. You develop the knowledge, understanding and skills to perform partial differentiation and double integration. You learn about the very powerful Laplace transforms method for solving a wide range of differential equations. You are also introduced to eigenvalues and eigenvectors, which are used in the solution of linear system equations.

The unit is likely to involve significant teaching input from your lecturer. This is supplemented by tutorial exercises that allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes you cover to a range of engineering problems. If appropriate, they could choose to teach the unit as a series of lectures supported by tutorial sessions to help you prepare for degree-level study.

You are assessed by an examination under closed-book, controlled and invigilated conditions.

To take the unit, you are expected to have passed the SCQF level 7 unit Engineering Mathematics 3 or an equivalent qualification. We also recommend that you have passed the SCQF level 8 unit Engineering Mathematics 4.

Meta-skills

Throughout the unit, you can develop meta-skills to enhance your employability in the engineering sector.

These skills include self-management, social intelligence and innovation.

Self-management

You develop the meta-skills of focusing, adapting and initiative as you solve engineering problems.

Social intelligence

You develop the meta-skill of communicating as you ask questions and receive information from lecturers.

Innovation

You develop the meta-skills of curiosity, sense-making and critical thinking as you apply mathematical techniques to problem solving.

Administrative information

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Superclass: RB

History of changes

Version	Description of change	Date

Note: please check SQA's website to ensure you are using the most up-to-date version of this document.