National
Qualifications
RESOURCE

X803/77/12

Statistics
Paper 2

Marking Instructions

Please note that these marking instructions have not been standardised based on candidate responses. You may therefore need to agree within your centre how to consistently mark an item if a candidate response is not covered by the marking instructions.

SQA

General marking principles for Advanced Higher Statistics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$ Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $y = 5$ and $y = -7$ $\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4\frac{}{}5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3 + 2x^2 + 3x + 2\right)\left(2x + 1\right)$ *written as*

  $\left(x^3 + 2x^2 + 3x + 2\right) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Section 1

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>1.</td><td>(a)</td><td></td><td>•[1] calculate k

•[2] tabulate probability distribution</td><td>•[1] 30

•[2]
<table><tr><td>s</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>P($S = s$)</td><td>$\frac{1}{15}$</td><td>$\frac{2}{15}$</td><td>$\frac{1}{5}$</td><td>$\frac{4}{15}$</td><td>$\frac{1}{3}$</td></tr></table></td><td>2</td></tr>
<tr><td></td><td>(b)</td><td></td><td>•[3] calculate E(S)

•[4] calculate V(S)</td><td>•[3] $\frac{22}{3}$

•[4] $\frac{56}{9}$</td><td>2</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		$\bullet^1$ correct probability $\bullet^2$ correct probability	$\bullet^1$ 0·36 $\bullet^2$ 0·94	**2**
	(b)	(i)	$\bullet^3$ appropriate strategy $\bullet^4$ calculate probability	$\bullet^3$ $\frac{30}{50}\times\ldots+\frac{10}{50}\times\ldots+\frac{10}{50}\times\ldots$ $\bullet^4\left(\frac{30}{50}\times 0\cdot 8+\frac{10}{50}\times 0\cdot 5+\frac{10}{50}\times 0\cdot 3\right)$ $=0\cdot 64$	**2**
		(ii)	$\bullet^5$ appropriate denominator $\bullet^6$ appropriate strategy $\bullet^7$ calculate probability	$\bullet^5$ $\frac{\ldots}{1-0\cdot 64}$ $\bullet^6$ $\frac{\frac{10}{50}\times 0\cdot 5}{1-0\cdot 64}$ $\bullet^7$ $\frac{5}{18}$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
3.			$\bullet^1$ appropriate assumption	$\bullet^1$ the distribution of the mite count is symmetrical	**8**
			$\bullet^2$ appropriate hypothesis	$\bullet^2$ H_0: The median mite count is 7 H_1: The median mite count is greater than 7	
			$\bullet^3$ deal with zero difference	$\bullet^3$ discard the zero difference	
			$\bullet^4$ process ranking	$\bullet^4$ ranks processed (see below)	
			$\bullet^5$ calculate statistics	$\bullet^5$ n = 13, W_- = 14, W_+ = 77	
			$\bullet^6$ correct critical value	$\bullet^6$ 5% cv is 21	
			$\bullet^7$ deal with H_0	$\bullet^7$ $14 < 21$ so we reject H_0 at the 5% level of significance	
			$\bullet^8$ appropriate conclusion	$\bullet^8$ and conclude there is evidence of a risk to colony health	

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)	(i)	•[1] correct strategy •[2] calculate probability	•[1] 1−P($W \leq 10$) •[2] 0·0137	**2**
		(ii)	•[3] correct distribution •[4] calculate probability	•[3] $F + M \sim$ Po(3·5) •[4] P($F + M$ = 2) = 0·1849	**2**
	(b)		•[5] correct strategy •[6] correct approximation •[7] correct strategy •[8] continuity correction •[9] calculate probability	•[5] $T \sim$ Po(357) •[6] $T \approx$ N(357,357) •[7&8] $\mathrm{P}(T < 340) = \mathrm{P}\left(Z < \frac{339 \cdot 5 - 357}{\sqrt{357}}\right)$ •[9] 0·1762	**5**

Question			Generic scheme	Illustrative scheme	Max mark
5.			•[1] calculate s^2 •[2] calculate t •[3] correct cv •[4] deal with H_0 •[5] appropriate conclusion	•[1] $s^2 = 81$ •[2] $t = 1{\cdot}959$ •[3] $t_{24,0{\cdot}975} = 2{\cdot}064$ •[4] $1{\cdot}959 < 2{\cdot}064$ so we do not reject H_0 at the 5% level of significance •[5] and conclude there is no difference in warm up time between brands	**5**

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] state sampling method	•[1] stratified random sampling	**1**
	(b)		•[2] state sampling method •[3] appropriate reason	•[2] convenience sampling •[3] it is possible that the parents who subscribe to the newsletter are particularly involved with their child's education and the selected students are therefore not representative of the population	**2**
	(c)		•[4] correct strategy •[5] calculate interval	•[4] $409 \pm 1{\cdot}96\frac{130}{\sqrt{25}}$ •[5] (358·0, 460·0)	**2**
	(d)		•[6] start explanation •[7] continue explanation	•[6] 458 minutes is not captured by the 90% interval but it is by the 95% interval •[7] they have presented the evidence that would lead to a reward being given	**2**

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•[1] correct probability	•[1] $\frac{0\cdot 38}{5} = 0\cdot 0760$	**1**
	(b)		•[2] correct mean and variance	•[2] $E(X) = 80\cdot 5$ $V(X) = \frac{25}{12} = 2\cdot 0833$	**3**
			•[3] approximate distribution of $\bar{X}$	•[3] $\overline{X} \approx N\left(80\cdot 5, \frac{1}{36}\right)$	
			•[4] appropriate justification	•[4] since $n \geq 20$ the CLT can be used	
	(c)		•[5] appropriate strategy	•[5] $P(Z \leq 1\cdot 98) - P(Z \leq -0\cdot 30)$	**2**
			•[6] calculate probability	•[6] $0\cdot 5940$	

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] appropriate hypotheses	•[1] $H_0 : \mu = 15 \quad H_1 : \mu > 15$	**8**
			•[2] correct distribution	•[2] $\bar{X} \sim N\left(15, \frac{4}{50}\right)$	
			•[3] calculate z	•[3] $z = \frac{16 \cdot 1 - 15}{2/\sqrt{50}} = 3 \cdot 89$	
			•[4] correct critical value	•[4] 5% cv is 1·64	
			•[5] deal with H_0	•[5] 3·89>1·64 so we reject H_0 at the 5% level of significance	
			•[6] appropriate conclusion	•[6] conclude that there is cause for concern	
			•[7] appropriate explanation	•[7] the task may be too complex for the 15-minute time limit	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] correct expectation •[2] correct variance	•[1] $E(C) = 2{\cdot}50 - 1{\cdot}00 = 1{\cdot}50$ •[2] $V(C) = 4^2 + 5^2 = 41$	**2**
	(b)		•[3] correct description	•[3] how much more profit policy A generates than policy B, for each £10 premium	**1**
	(c)		•[4] appropriate strategy •[5] appropriate application of variance law •[6] calculate standard deviation	•[4] total, $T = A_1 + \ldots + A_{33} + B_1 + \ldots B_{26}$ •[5] $V(T) = V(A_1) + \ldots + V(A_{33}) + V(B_1) + \ldots + V(B_{26})$ $= 33V(A) + 26V(B)$ •[6] $= 33 \times 4^2 + 26 \times 5^2$ $= 1178$ $\Rightarrow SD(T) = (£)34{\cdot}32$	**3**

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		$\bullet^1$ correct strategy	$\bullet^1$ $\hat{p} \pm z\sqrt{\frac{\hat{p}\hat{q}}{n}}$	**3**
			$\bullet^2$ appropriate substitution	$\bullet^2$ $0 \cdot 26 \pm 1 \cdot 96\sqrt{\frac{0 \cdot 26 \times 0 \cdot 74}{50}}$	
			$\bullet^3$ calculate interval	$\bullet^3$ (0·1384,0·3816)	
	(b)		$\bullet^4$ correct strategy	$\bullet^4$ $\hat{p} \pm 0 \cdot 02$	**3**
			$\bullet^5$ appropriate substitution	$\bullet^5$ $0 \cdot 02 = 1 \cdot 96\sqrt{\frac{0 \cdot 26 \times 0 \cdot 74}{n}}$	
			$\bullet^6$ calculate sample size	$\bullet^6$ 1848 shops	

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		$\bullet^1$ appropriate comment	$\bullet^1$ distribution appears to be normally distributed	**1**
	(b)		$\bullet^2$ appropriate assumption	$\bullet^2$ scores for workers in group B are independent of those in group C	**7**
			$\bullet^3$ correct hypotheses	$\bullet^3$ $H_0 : \mu_B = \mu_C$ $H_1 : \mu_B \neq \mu_C$	
			$\bullet^4$ correct test statistic	$\bullet^4$ $Z = \dfrac{\overline{X}_B - \overline{X}_C - (\mu_B - \mu_C)}{\sqrt{\dfrac{\sigma_B^2}{n_B} + \dfrac{\sigma_C^2}{n_C}}}$	
			$\bullet^5$ correct z-value	$\bullet^5$ $z = \dfrac{55 \cdot 4 - 51 \cdot 8}{\sqrt{\dfrac{10 \cdot 08^2}{70} + \dfrac{10 \cdot 49^2}{60}}} = 1 \cdot 99$	
			$\bullet^6$ calculate p-value	$\bullet^6$ $\begin{cases} p - \text{value} = 2\text{P}(Z > 1 \cdot 99) \\ = 2(1 - 0 \cdot 9767) \\ = 0 \cdot 0466 \end{cases}$	
			$\bullet^7$ deal with H_0	$\bullet^7$ $0 \cdot 0466 < 0 \cdot 05$, so we can reject H_0 at the 5% level of significance	
			$\bullet^8$ appropriate conclusion	$\bullet^8$ and conclude that there is evidence that the mean score for group B is different to that for group C	
	(c)		$\bullet^9$ correct strategy	$\bullet^9$ $z = \dfrac{56 \cdot 0 - \overline{x}_B}{\sqrt{\dfrac{10 \cdot 71^2}{120} + \dfrac{10 \cdot 08^2}{70}}}$	**3**
			$\bullet^{10}$ correct critical value	$\bullet^{10}$ $z = 1 \cdot 28$	
			$\bullet^{11}$ calculate maximum score	$\bullet^{11}$ $\overline{x}_B = 54 \cdot 01$	

Section 2 - Part A

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] correct strategy •[2] calculate interval	•[1] $\overline{x} \pm 1\frac{\sigma}{\sqrt{n}}$ •[2] $500 \pm 1\frac{5 \cdot 73}{\sqrt{5}} = (497 \cdot 4,\ 502 \cdot 6)$	**2**
	(b)	(i)	•[3] calculate probability •[4] correct strategy •[5] know to double •[6] calculate probability	•[3] P($Z > 1$)=0·1587 •[4] $X \sim \text{B}(3,\ 0 \cdot 1587)$ •[5] 2P($X \geq 2$) •[6] 0·1351	**4**
		(ii)	•[7] appropriate explanation	•[7] The probability is considerably larger than normal WECO thresholds so the machine will regularly meet this criteria.	**1**

Section 2 - Part B

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] appropriate explanation	•[1] the lecturers were perhaps not equally good	**1**
	(b)		•[2] deal with small expected frequencies •[3] calculate test statistic •[4] correct critical value •[5] deal with H_0 •[6] appropriate conclusion	•[2] combine the last two columns so the E_i are now: 10·4 | 9·2 | 7·3 | 9·2 6·6 | 5·8 | 4·7 | 5·8 •[3] $X^2 = 6 \cdot 055$ •[4] $\chi^2_{3,0\cdot90} = 6 \cdot 251$ •[5] as $6 \cdot 05 < 6 \cdot 251$ we do not have evidence to reject H_0 •[6] conclude (differently) that Statistics grades awarded are independent of the course subject studied.	**5**
	(c)		•[7] identification consistent with previous working	•[7] Biology grade D+E combined	**1**

[END OF MARKING INSTRUCTIONS]